

UNIVERSITY OF SWAZILAND

SUPPLEMENTARY EXAMINATION PAPER 2010

TITLE OF PAPER : DESIGN AND ANALYSIS OF EXPERIMENTS

COURSE CODE : ST404

TIME ALLOWED : TWO (2) HOURS

REQUIREMENTS : CALCULATOR AND STATISTICAL TABLES

INSTRUCTIONS : ATTEMPT ALL QUESTIONS

Question 1

[20 marks, 3+4+3+3+3+4]

- (a) Write down a 4×4 Graeco-Latin square.
- (b) Suppose that you wish to investigate 3 fertilizer types, by applying them to crops in 3 regions of land. Each region is only large enough for two crops, so that a Balanced Incomplete Block design is called for. Write down an appropriate design. What is λ for your design? Give both the meaning of λ , and its numerical value.
- (c) Consider a 2^2 factorial design with factors **A** and **B**, for which the effects model is that the expected value of the k^{th} observation with **A** at level i ($i = 1$ for low, $i = 2$ for high) and **B** at level j ($j = 1$ for low, $j = 2$ for high) is

$$E[y_{ijk}] = \mu + A_i + B_j + (AB)_{ij}.$$

In terms of y_{ijk} and appropriate averages, give expressions for

- (i) the estimate of the main effect of the high level **A**,
 - (ii) the estimate of $(AB)_{ij}$.
- (d) An agricultural experiment, to test the effectiveness of certain crop stimulants on certain crops, is carried out as follows. Three 100 hectare plots of land are selected and randomly labelled 1,2,3. In plot 1, wheat is grown. In plot 2, rye is grown and in plot 3, canola is grown. Then each plot is divided into four 25 hectare fields and the four possible stimulants are applied, one to a field, in random order. This entire experiment is then replicated 4 times.
- (i) What is the name of the design being used here?
 - (ii) Viewing replicates as a random factor, **clearly** write down the effects model which you would use to analyze these data. Be sure to describe any constraints on, or distributions of, the terms in your model.

Question 2

[20 marks, 8+8+4]

In an experimental study of the efficacy of treatments for depression, patients attending a clinic were allocated at random to one of a number different drug treatments and other therapies. The response measurement was the decrease in Hamilton Rating Depression Score (HRDS) over a three month course of treatment. The data recorded are tabulated below; entries in the rows of the table are the decrease in HRDS for each treatment. The final group was treated using an inactive treatment (or placebo).

Group					
Therapy 1	5.20	3.65	7.03	5.63	4.57
Therapy 2	7.05	7.91	1.46	4.11	4.18
Drug A	4.71	3.75	2.85	4.89	6.09
Drug B	6.82	6.77	8.31	6.57	6.55
Placebo	5.13	6.07	3.55	5.69	3.97

- (a) What kind of design is being used in this study? Identify the *factor* being investigated, and for each factor state the number of levels that that factor has. Is the study *balanced*? Is it *complete*? Justify your answers.

(b) Using the data, an ANOVA analysis is to be carried out. The ANOVA table below contains some missing entries marked by the notation?. Write out the ANOVA table in full, filling in the missing values **using the information already given in the table.**

Source	<i>SS</i>	<i>DF</i>	<i>MS</i>	<i>F</i> ₀
Group	19.613	?	?	?
Error	?	?	2.295	
Total	?	?		

(c) What is the conclusion of the ANOVA analysis? State clearly the null and alternative hypothesis, the test statistic, the null distribution, and the conclusion.

Question 3

[20 marks, 8+8+4]

The article “Responsiveness of Food Sales to Shelf Space Changes in Supermarkets” (*J. Mktg. Res.* 1969: 63-67) described an experiment to assess the effect of allotted shelf space sales of “Tang”. Six stores were used in the experiment, and six different shelf-space allotments of 6, 9, 12, 15, 18 and 21 feet were tried for 1 week. The author speculated that changes in shelf space would affect sales. Data on the number of containers of Tang sold is given in the following table.

Store	Shelf space (feet)					
	6	9	12	15	18	21
1	30	35	25	27	38	31
2	47	59	43	62	65	48
3	47	55	48	54	36	54
4	29	19	41	27	33	39
5	17	11	25	23	24	26
6	22	9	19	18	25	22

- (a) Is there any effect of shelf-space (as a factor) on sales, allowing for different sales in different stores? State clearly your null and alternative hypothesis, decision rule and present you conclusions.

(b) Test for equality of variance by store. State clearly your null and alternative hypothesis, decision rule and present you conclusions.

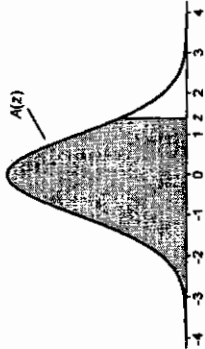
(c) Compute 95% family-wise confidence intervals for the mean difference in sales between Store 1 and 6. Use the most efficient approach.

TABLE A.1

Cumulative Standardized Normal Distribution

$A(z)$ is the integral of the standardized normal distribution from $-\infty$ to z (in other words, the area under the curve to the left of z). It gives the probability of a normal random variable not being more than z standard deviations above its mean. Values of z of particular importance:

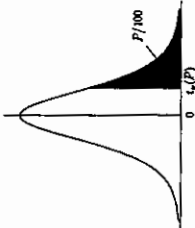
z	$A(z)$	Lower limit of right 5% tail
1.645	0.9500	
1.960	0.9750	Lower limit of right 2.5% tail
2.326	0.9900	Lower limit of right 1% tail
2.576	0.9950	Lower limit of right 0.5% tail
3.090	0.9990	Lower limit of right 0.1% tail
3.291	0.9995	Lower limit of right 0.05% tail



z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9978	0.9979	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998
3.5	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998
3.6	0.9998	0.9998	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999

Percentage Points of the t-Distribution

This table gives the percentage points $t_{\nu}(P)$ for various values of P and degrees of freedom ν , as indicated by the figure to the right.



The lower percentage points are given by symmetry as $-t_{\nu}(P)$, and the probability that $|t| \geq t_{\nu}(P)$ is $2P/100$. The limiting distribution of t as $\nu \rightarrow \infty$ is the normal distribution with zero mean and unit variance.

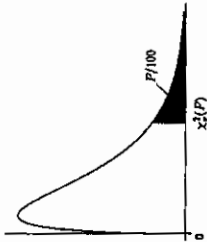
ν	1	5	10	20	50	100	200	500	1000	∞
1	3.078	6.314	12.706	31.821	63.657	128.309	318.309	636.619	1273.757	12.706
2	1.886	2.920	4.303	6.965	9.925	22.327	31.599	42.004	51.802	6.965
3	1.638	2.353	3.182	4.541	5.841	10.215	12.924	15.990	18.476	4.541
4	1.533	2.132	2.776	3.747	4.604	7.173	8.610	10.215	11.716	3.747
5	1.476	2.015	2.571	3.365	4.032	5.893	6.860	8.144	9.277	3.365
6	1.440	1.943	2.447	3.143	3.707	5.208	5.959	6.933	7.709	3.143
7	1.415	1.895	2.365	2.998	3.499	4.785	5.408	6.215	6.851	2.998
8	1.397	1.860	2.306	2.896	3.355	4.501	5.041	5.793	6.358	2.896
9	1.383	1.833	2.262	2.821	3.250	4.297	4.781	5.487	6.066	2.821
10	1.372	1.812	2.228	2.764	3.169	4.144	4.587	5.250	5.835	2.764
11	1.363	1.796	2.201	2.718	3.106	4.025	4.437	5.066	5.616	2.718
12	1.356	1.782	2.179	2.681	3.055	3.930	4.318	4.916	5.438	2.681
13	1.350	1.771	2.160	2.650	3.012	3.852	4.221	4.787	5.277	2.650
14	1.345	1.761	2.145	2.624	2.977	3.787	4.140	4.693	5.156	2.624
15	1.341	1.753	2.131	2.602	2.947	3.733	4.073	4.625	5.087	2.602
16	1.337	1.746	2.120	2.583	2.921	3.686	4.015	4.566	5.027	2.583
18	1.330	1.734	2.101	2.552	2.878	3.610	3.922	4.481	4.952	2.552
21	1.323	1.721	2.080	2.518	2.831	3.527	3.819	4.373	4.844	2.518
25	1.316	1.708	2.060	2.485	2.787	3.450	3.725	4.297	4.766	2.485
30	1.310	1.697	2.042	2.457	2.750	3.385	3.646	4.230	4.690	2.457
40	1.303	1.684	2.021	2.423	2.704	3.307	3.551	4.154	4.614	2.423
50	1.299	1.676	2.009	2.403	2.678	3.261	3.486	4.118	4.578	2.403
70	1.294	1.667	1.994	2.381	2.648	3.211	3.435	4.073	4.533	2.381
100	1.290	1.660	1.984	2.364	2.626	3.174	3.390	4.028	4.488	2.364
∞	1.282	1.645	1.960	2.326	2.576	3.090	3.291	3.959	4.437	2.326

Percentage Points of the χ^2 -Distribution

This table gives the percentage points $\chi^2_\nu(P)$ for various values of P and degrees of freedom ν , as indicated by the figure to the right.

If X is a variable distributed as χ^2 with ν degrees of freedom, $P/100$ is the probability that $X \geq \chi^2_\nu(P)$.

For $\nu > 100$, $\sqrt{2X}$ is approximately normally distributed with mean $\sqrt{2\nu - 1}$ and unit variance.



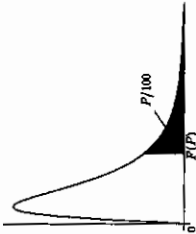
ν	Percentage points P									
	10	5	2.5	1	0.5	0.1	0.05			
1	2.706	3.841	5.024	6.635	7.879	10.828	12.116			
2	4.605	5.991	7.378	9.210	10.597	13.816	15.202			
3	6.251	7.815	9.348	11.345	12.838	16.266	17.730			
4	7.779	9.488	11.143	13.277	14.860	18.467	19.997			
5	9.236	11.070	12.833	15.086	16.750	20.515	22.105			
6	10.645	12.592	14.449	16.812	18.548	22.458	24.153			
7	12.017	14.067	16.013	18.475	20.278	24.322	26.018			
8	13.362	15.507	17.535	20.090	21.955	26.124	27.868			
9	14.684	16.919	19.023	21.666	23.589	27.877	29.666			
10	15.987	18.307	20.483	23.209	25.188	29.588	31.420			
11	17.275	19.675	21.920	24.725	26.757	31.264	33.137			
12	18.549	21.026	23.337	26.217	28.300	32.909	34.821			
13	19.812	22.362	24.736	27.688	29.819	34.528	36.478			
14	21.064	23.685	26.119	29.141	31.319	36.123	38.109			
15	22.307	24.996	27.488	30.578	32.801	37.697	39.719			
16	23.542	26.296	28.845	32.000	34.267	39.252	41.308			
17	24.769	27.587	30.191	33.409	35.718	40.790	42.879			
18	25.989	28.869	31.526	34.805	37.156	42.312	44.434			
19	27.204	30.144	32.852	36.191	38.582	43.820	45.973			
20	28.412	31.410	34.170	37.566	39.997	45.315	47.498			
25	34.382	37.652	40.646	44.314	46.928	52.620	54.947			
30	40.286	43.773	46.979	50.892	53.672	59.703	62.162			
40	51.805	55.758	59.342	63.691	66.766	73.402	76.095			
50	63.167	67.505	71.420	76.154	79.490	86.661	89.561			
80	96.578	101.879	106.629	112.329	116.321	124.839	128.261			

5 Percent Points of the F -Distribution

This table gives the percentage points $F_{\nu_1, \nu_2}(P)$ for $P = 0.05$ and degrees of freedom ν_1, ν_2 , as indicated by the figure to the right.

The lower percentage points, that is the values $F'_{\nu_1, \nu_2}(P)$ such that the probability that $F \leq F'_{\nu_1, \nu_2}(P)$ is equal to $P/100$, may be found using the formula

$$F'_{\nu_1, \nu_2}(P) = 1/F_{\nu_2, \nu_1}(P)$$



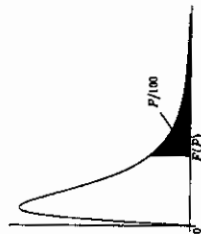
ν_2	ν_1									
	1	2	3	4	5	6	12	24	∞	
2	18.513	19.000	19.164	19.247	19.296	19.330	19.413	19.454	19.496	
3	10.128	9.552	9.277	9.117	9.013	8.941	8.745	8.639	8.526	
4	7.709	6.944	6.591	6.388	6.256	6.163	5.912	5.774	5.628	
5	6.608	5.786	5.409	5.192	5.050	4.950	4.678	4.527	4.365	
6	5.987	5.143	4.757	4.534	4.387	4.284	4.000	3.841	3.669	
7	5.591	4.737	4.347	4.120	3.972	3.866	3.575	3.410	3.230	
8	5.318	4.459	4.066	3.838	3.687	3.581	3.284	3.115	2.928	
9	5.117	4.256	3.863	3.633	3.482	3.374	3.073	2.900	2.707	
10	4.965	4.103	3.708	3.478	3.326	3.217	2.913	2.737	2.538	
11	4.844	3.982	3.587	3.357	3.204	3.095	2.788	2.609	2.404	
12	4.747	3.885	3.490	3.259	3.106	2.996	2.687	2.505	2.296	
13	4.667	3.806	3.411	3.179	3.026	2.915	2.604	2.420	2.206	
14	4.600	3.739	3.344	3.112	2.958	2.848	2.534	2.349	2.131	
15	4.543	3.682	3.287	3.056	2.901	2.790	2.475	2.288	2.066	
16	4.494	3.634	3.239	3.007	2.852	2.741	2.425	2.235	2.010	
17	4.451	3.592	3.197	2.965	2.810	2.699	2.381	2.190	1.960	
18	4.414	3.555	3.160	2.928	2.773	2.661	2.342	2.150	1.917	
19	4.381	3.522	3.127	2.895	2.740	2.628	2.308	2.114	1.878	
20	4.351	3.493	3.098	2.866	2.711	2.599	2.278	2.082	1.843	
25	4.242	3.385	2.991	2.759	2.603	2.490	2.165	1.964	1.711	
30	4.171	3.316	2.922	2.690	2.534	2.421	2.092	1.887	1.622	
40	4.085	3.232	2.839	2.606	2.449	2.336	2.003	1.793	1.509	
50	4.034	3.183	2.790	2.557	2.400	2.286	1.952	1.737	1.438	
100	3.936	3.087	2.696	2.463	2.305	2.191	1.850	1.627	1.283	
∞	3.841	2.996	2.605	2.372	2.214	2.099	1.752	1.517	1.002	

10 Percent Points of the F-Distribution

This table gives the percentage points $F_{\nu_1, \nu_2}(P)$ for $P = 0.10$ and degrees of freedom ν_1, ν_2 , as indicated by the figure to the right.

The lower percentage points, that is the values $F'_{\nu_1, \nu_2}(P)$ such that the probability that $F \leq F'_{\nu_1, \nu_2}(P)$ is equal to $P/100$, may be found using the formula

$$F'_{\nu_1, \nu_2}(P) = 1/F_{\nu_2, \nu_1}(P)$$



ν_2	1	2	3	4	5	6	12	24	∞
2	8.526	9.000	9.162	9.243	9.293	9.326	9.408	9.450	9.491
3	5.538	5.462	5.391	5.343	5.309	5.285	5.216	5.176	5.134
4	4.545	4.325	4.191	4.107	4.051	4.010	3.896	3.831	3.761
5	4.060	3.780	3.619	3.520	3.453	3.405	3.268	3.191	3.105
6	3.776	3.463	3.289	3.181	3.108	3.055	2.905	2.818	2.722
7	3.589	3.257	3.074	2.961	2.883	2.827	2.668	2.575	2.471
8	3.458	3.113	2.924	2.806	2.726	2.668	2.502	2.404	2.293
9	3.360	3.006	2.813	2.693	2.611	2.551	2.379	2.277	2.159
10	3.285	2.924	2.728	2.605	2.522	2.461	2.284	2.178	2.055
11	3.225	2.860	2.660	2.536	2.451	2.389	2.209	2.100	1.972
12	3.177	2.807	2.606	2.480	2.394	2.331	2.147	2.036	1.904
13	3.136	2.763	2.560	2.434	2.347	2.283	2.097	1.983	1.846
14	3.102	2.726	2.522	2.395	2.307	2.243	2.054	1.938	1.797
15	3.073	2.695	2.490	2.361	2.273	2.208	2.017	1.899	1.755
16	3.048	2.668	2.462	2.333	2.244	2.178	1.985	1.866	1.718
17	3.026	2.645	2.437	2.308	2.218	2.152	1.958	1.836	1.686
18	3.007	2.624	2.416	2.286	2.196	2.130	1.933	1.810	1.657
19	2.990	2.606	2.397	2.266	2.176	2.109	1.912	1.787	1.631
20	2.975	2.589	2.380	2.249	2.158	2.091	1.892	1.767	1.607
25	2.918	2.528	2.317	2.184	2.092	2.024	1.820	1.699	1.518
30	2.881	2.489	2.276	2.142	2.049	1.980	1.773	1.658	1.456
40	2.835	2.440	2.226	2.091	1.997	1.927	1.715	1.574	1.377
50	2.809	2.412	2.197	2.061	1.966	1.895	1.680	1.536	1.327
100	2.756	2.356	2.139	2.002	1.906	1.834	1.612	1.460	1.214
∞	2.706	2.303	2.084	1.945	1.847	1.774	1.546	1.383	1.002

TABLE A.4 Studentized Range Statistic

Error df		K = Number of Means or Number of Steps Between Ordered Means										
		α	2	3	4	5	6	7	8	9	10	11
5	(df within)	.05	3.64	4.60	5.22	5.67	6.03	6.33	6.58	6.80	6.99	7.17
		.01	5.70	6.98	7.80	8.42	8.91	9.32	9.67	9.97	10.24	10.48
6		.05	3.46	4.34	4.90	5.30	5.63	5.90	6.12	6.32	6.49	6.65
		.01	5.24	6.33	7.03	7.56	7.97	8.32	8.61	8.87	9.10	9.30
7		.05	3.34	4.16	4.68	5.06	5.36	5.61	5.82	6.00	6.16	6.30
		.01	4.95	5.92	6.54	7.01	7.37	7.68	7.94	8.17	8.37	8.55
8		.05	3.26	4.04	4.53	4.89	5.17	5.40	5.60	5.77	5.92	6.05
		.01	4.75	5.64	6.20	6.62	6.96	7.24	7.47	7.68	7.86	8.03
9		.05	3.20	3.95	4.41	4.76	5.02	5.24	5.43	5.59	5.74	5.87
		.01	4.60	5.43	5.96	6.35	6.66	6.91	7.13	7.33	7.49	7.65
10		.05	3.15	3.88	4.33	4.65	4.91	5.12	5.30	5.46	5.60	5.72
		.01	4.48	5.27	5.77	6.14	6.43	6.67	6.87	7.05	7.21	7.36
11		.05	3.11	3.82	4.26	4.57	4.82	5.03	5.20	5.35	5.49	5.61
		.01	4.39	5.15	5.62	5.97	6.25	6.48	6.67	6.84	6.99	7.13
12		.05	3.08	3.77	4.20	4.51	4.75	4.95	5.12	5.27	5.39	5.51
		.01	4.32	5.05	5.50	5.84	6.10	6.32	6.51	6.67	6.81	6.94
13		.05	3.06	3.73	4.15	4.45	4.69	4.88	5.05	5.19	5.32	5.43
		.01	4.26	4.96	5.40	5.73	5.98	6.19	6.37	6.53	6.67	6.79
14		.05	3.03	3.70	4.11	4.41	4.64	4.83	4.99	5.13	5.25	5.36
		.01	4.21	4.89	5.32	5.63	5.88	6.08	6.26	6.41	6.54	6.66
15		.05	3.01	3.67	4.08	4.37	4.59	4.78	4.94	5.08	5.20	5.31
		.01	4.17	4.84	5.25	5.56	5.80	5.99	6.16	6.31	6.44	6.55
16		.05	3.00	3.65	4.05	4.33	4.56	4.74	4.90	5.03	5.15	5.26
		.01	4.13	4.79	5.19	5.49	5.72	5.92	6.08	6.22	6.35	6.46
17		.05	2.98	3.63	4.02	4.30	4.52	4.70	4.86	4.99	5.11	5.21
		.01	4.10	4.74	5.14	5.43	5.66	5.85	6.01	6.15	6.27	6.38
18		.05	2.97	3.61	4.00	4.28	4.49	4.67	4.82	4.96	5.07	5.17
		.01	4.07	4.70	5.09	5.38	5.60	5.79	5.94	6.08	6.20	6.31
19		.05	2.96	3.59	3.98	4.25	4.47	4.65	4.79	4.92	5.04	5.14
		.01	4.05	4.67	5.05	5.33	5.55	5.73	5.89	6.02	6.14	6.25
20		.05	2.95	3.58	3.96	4.23	4.45	4.62	4.77	4.90	5.01	5.11
		.01	4.02	4.64	5.02	5.29	5.51	5.69	5.84	5.97	6.09	6.19
24		.05	2.92	3.53	3.90	4.17	4.37	4.54	4.68	4.81	4.92	5.01
		.01	3.96	4.55	4.91	5.17	5.37	5.54	5.69	5.81	5.92	6.02
30		.05	2.89	3.49	3.85	4.10	4.30	4.46	4.60	4.72	4.82	4.92
		.01	3.89	4.45	4.80	5.05	5.24	5.40	5.54	5.65	5.76	5.85
40		.05	2.86	3.44	3.79	4.04	4.23	4.39	4.52	4.63	4.73	4.82
		.01	3.82	4.37	4.70	4.93	5.11	5.26	5.39	5.50	5.60	5.69
60		.05	2.83	3.40	3.74	3.98	4.16	4.31	4.44	4.55	4.65	4.73
		.01	3.76	4.28	4.59	4.82	4.99	5.13	5.25	5.36	5.45	5.53
120		.05	2.80	3.36	3.68	3.92	4.10	4.24	4.35	4.47	4.56	4.64
		.01	3.70	4.20	4.50	4.71	4.87	5.01	5.12	5.21	5.30	5.37
∞		.05	2.77	3.31	3.63	3.86	4.03	4.17	4.29	4.39	4.47	4.55
		.01	3.64	4.12	4.40	4.60	4.76	4.88	4.99	5.08	5.16	5.23