# UNIVERSITY OF SWAZILAND

## SUPPLEMENTARY EXAMINATION PAPER 2010

| | | |
|---|---|---|
| TITLE OF PAPER | : | NONPARAMETRIC ANALYSIS |
| COURSE CODE | : | ST409 |
| TIME ALLOWED | : | TWO (2) HOURS |
| REQUIREMENTS | : | CALCULATOR AND STATISTICAL TABLES |
| INSTRUCTIONS | : | ATTEMPT ALL QUESTIONS |

## Question 1 [20 marks, 7+7+6]

(a) At a particular school it is suspected that grade 6 mathematics marks in 2006 have a different distribution from the 2008 marks, in particular that the 2008 marks are poorer. Since the pass mark of 33% historically occurred at the first quartile, test the appropriate hypothesis at the 5% level of significance. Marks obtained from equivalent assessments in 2006 and 2008 are shown next.

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| **2006** | 84 | 51 | 37 | 75 | 61 | 39 | 38 |
| **2008** | 41 | 43 | 39 | 36 | 35 | 48 | 41 |

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| **2006** | 56 | 53 | 53 | 58 | 58 | 52 | 56 |
| **2008** | 35 | 48 | 50 | 35 | 44 | | |

(b) The following data, arising from independent runs of a particular experiment, are a sample of $n = 10$ proportions:

Use an appropriate test to assess at the 10% significance level whether these data can be regarded as being sampled from a distribution having *cdf*

$$F_X(x) = 1 - (1 - x)^3, \qquad 0 < x < 1.$$

(c) In a first phase of a health study in a city, a random sample of size 2000 is to be obtained. The city is comprised (broadly) of five different ethnic subpopulations that make up 40%, 30%, 10%, 10% and 10% of the city population respectively.

A commercial company is employed to obtain the random sample, with the instruction that the sample should reflect the ethnic composition of the city. The sample they return is summarized in the following table.

| | **Ethnic Subpopulation** | | | | |
|---|---|---|---|---|---|
| | **1** | **2** | **3** | **4** | **5** |
| **Number in Sample** | 822 | 638 | 210 | 157 | 173 |

Using a Chi-squared test for this one-way layout, comment on whether the company have fulfilled their remit to produce a sample that reflects the ethnic composition of the city.

## Question 2 [20 marks, 8+8+4]

(a) An experimentor was involved in a study to determine if there is an association between stress and mental health. The Hopkins Symptom Checklist was used to determine the level of symptoms experiences and a scale was developed to measure stress levels. For data obtained, refer to the following table. Use **Kendall's tau** to perform the appropriate test at the 1% level of significance.

| Participant | Stress ($X$) | Symptoms ($Y$) |
|---|---|---|
| 1 | 33 | 100 |
| 2 | 30 | 92 |
| 3 | 11 | 83 |
| 4 | 18 | 69 |
| 5 | 5 | 98 |
| 6 | 16 | 105 |
| 7 | 3 | 65 |
| 8 | 21 | 84 |
| 9 | 21 | 70 |
| 10 | 35 | 120 |

(b) The following data are the annual incomes (in thousands of emalangeni) for a random sample of $n = 10$ adults from a certain area

12.8, 13.6, 14.7, 16.2, 17.9, 18.6, 19.2, 19.8, 20.3, 24.4

Use the *sign* test to test that the median income doesn't exceed SZL20000 in this particular area. Use the exact null distribution of your test statistic to determine the critical value. Report your conclusions.

(c) Where would you use each of there following statistical tests

(i) McNemar's test

(ii) Jonckhere test

(iii) Friedman's test

(iv) Runs test

## Question 3 [20 marks, 10+10]

(a) To determine if a new treatment is effective in treating sleep apnea, individual suffering from sleep apnea were enrolled in a study. The number of hours of uninterrupted sleep prior to and after treatment was measured. The data is provided in the following table. Test the appropriate hypothesis at the 5% level of significance. Use a large sample approximation to the null distribution of your test statistic to determine an approximate critical value. Report your conclusions.

| Individual | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|---|
| **Before** | 6.2 | 4.4 | 5.9 | 6.4 | 5.8 | 6.1 | 6.7 | 7.0 | 5.6 | 4.9 |
| **After** | 7.1 | 8.0 | 4.9 | 7.8 | 6.2 | 8.2 | 5.6 | 7.2 | 6.5 | 5.2 |

| Individual | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |
|---|---|---|---|---|---|---|---|---|---|---|
| **Before** | 8.2 | 7.4 | 4.7 | 7.3 | 4.5 | 5.5 | 5.7 | 3.6 | 3.7 | 5.6 |
| **After** | 7.6 | 8.8 | 5.6 | 6.3 | 6.1 | 6.1 | 7.0 | 4.1 | 3.6 | 4.9 |

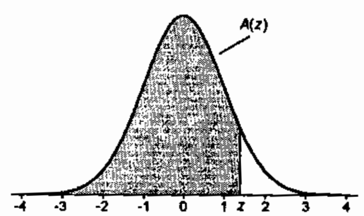
(b) In a comparative study of the lifetimes of three different brands of light bulbs, three independent random samples from each brand were tested to see how long the lasted (in hours) with the following results:

| **Brand** | **Lifetime (hours)** | | | | | |
|---|---|---|---|---|---|---|
| 1 | 80.5 | 82.4 | 88.9 | 95.6 | 102.8 | 111.7 |
| 2 | 81.7 | 84.3 | 86.8 | 92.3 | 98.5 | 101.9 |
| 3 | 98.2 | 108.7 | 118.1 | 124.5 | 130.9 | 138.4 |

Use an appropriate test to show that, at the 10% significance level, there are differences between the brands in terms of median lifetime.

**Table A.1**

**Cumulative Standardized Normal Distribution**




$A(z)$ is the integral of the standardized normal distribution from $-\infty$ to $z$ (in other words, the area under the curve to the left of $z$). It gives the probability of a normal random variable not being more than $z$ standard deviations above its mean. Values of $z$ of particular importance:

| $z$ | $A(z)$ | |
|---|---|---|
| 1.645 | 0.9500 | Lower limit of right 5% tail |
| 1.960 | 0.9750 | Lower limit of right 2.5% tail |
| 2.326 | 0.9900 | Lower limit of right 1% tail |
| 2.576 | 0.9950 | Lower limit of right 0.5% tail |
| 3.090 | 0.9990 | Lower limit of right 0.1% tail |
| 3.291 | 0.9995 | Lower limit of right 0.05% tail |

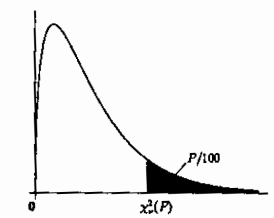
| $z$ | 0.00 | 0.01 | 0.02 | 0.03 | 0.04 | 0.05 | 0.06 | 0.07 | 0.08 | 0.09 |
|---|---|---|---|---|---|---|---|---|---|---|
| 0.0 | 0.5000 | 0.5040 | 0.5080 | 0.5120 | 0.5160 | 0.5199 | 0.5239 | 0.5279 | 0.5319 | 0.5359 |
| 0.1 | 0.5398 | 0.5438 | 0.5478 | 0.5517 | 0.5557 | 0.5596 | 0.5636 | 0.5675 | 0.5714 | 0.5753 |
| 0.2 | 0.5793 | 0.5832 | 0.5871 | 0.5910 | 0.5948 | 0.5987 | 0.6026 | 0.6064 | 0.6103 | 0.6141 |
| 0.3 | 0.6179 | 0.6217 | 0.6255 | 0.6293 | 0.6331 | 0.6368 | 0.6406 | 0.6443 | 0.6480 | 0.6517 |
| 0.4 | 0.6554 | 0.6591 | 0.6628 | 0.6664 | 0.6700 | 0.6736 | 0.6772 | 0.6808 | 0.6844 | 0.6879 |
| 0.5 | 0.6915 | 0.6950 | 0.6985 | 0.7019 | 0.7054 | 0.7088 | 0.7123 | 0.7157 | 0.7190 | 0.7224 |
| 0.6 | 0.7257 | 0.7291 | 0.7324 | 0.7357 | 0.7389 | 0.7422 | 0.7454 | 0.7486 | 0.7517 | 0.7549 |
| 0.7 | 0.7580 | 0.7611 | 0.7642 | 0.7673 | 0.7704 | 0.7734 | 0.7764 | 0.7794 | 0.7823 | 0.7852 |
| 0.8 | 0.7881 | 0.7910 | 0.7939 | 0.7967 | 0.7995 | 0.8023 | 0.8051 | 0.8078 | 0.8106 | 0.8133 |
| 0.9 | 0.8159 | 0.8186 | 0.8212 | 0.8238 | 0.8264 | 0.8289 | 0.8315 | 0.8340 | 0.8365 | 0.8389 |
| 1.0 | 0.8413 | 0.8438 | 0.8461 | 0.8485 | 0.8508 | 0.8531 | 0.8554 | 0.8577 | 0.8599 | 0.8621 |
| 1.1 | 0.8643 | 0.8665 | 0.8686 | 0.8708 | 0.8729 | 0.8749 | 0.8770 | 0.8790 | 0.8810 | 0.8830 |
| 1.2 | 0.8849 | 0.8869 | 0.8888 | 0.8907 | 0.8925 | 0.8944 | 0.8962 | 0.8980 | 0.8997 | 0.9015 |
| 1.3 | 0.9032 | 0.9049 | 0.9066 | 0.9082 | 0.9099 | 0.9115 | 0.9131 | 0.9147 | 0.9162 | 0.9177 |
| 1.4 | 0.9192 | 0.9207 | 0.9222 | 0.9236 | 0.9251 | 0.9265 | 0.9279 | 0.9292 | 0.9306 | 0.9319 |
| 1.5 | 0.9332 | 0.9345 | 0.9357 | 0.9370 | 0.9382 | 0.9394 | 0.9406 | 0.9418 | 0.9429 | 0.9441 |
| 1.6 | 0.9452 | 0.9463 | 0.9474 | 0.9484 | 0.9495 | 0.9505 | 0.9515 | 0.9525 | 0.9535 | 0.9545 |
| 1.7 | 0.9554 | 0.9564 | 0.9573 | 0.9582 | 0.9591 | 0.9599 | 0.9608 | 0.9616 | 0.9625 | 0.9633 |
| 1.8 | 0.9641 | 0.9649 | 0.9656 | 0.9664 | 0.9671 | 0.9678 | 0.9686 | 0.9693 | 0.9699 | 0.9706 |
| 1.9 | 0.9713 | 0.9719 | 0.9726 | 0.9732 | 0.9738 | 0.9744 | 0.9750 | 0.9756 | 0.9761 | 0.9767 |
| 2.0 | 0.9772 | 0.9778 | 0.9783 | 0.9788 | 0.9793 | 0.9798 | 0.9803 | 0.9808 | 0.9812 | 0.9817 |
| 2.1 | 0.9821 | 0.9826 | 0.9830 | 0.9834 | 0.9838 | 0.9842 | 0.9846 | 0.9850 | 0.9854 | 0.9857 |
| 2.2 | 0.9861 | 0.9864 | 0.9868 | 0.9871 | 0.9875 | 0.9878 | 0.9881 | 0.9884 | 0.9887 | 0.9890 |
| 2.3 | 0.9893 | 0.9896 | 0.9898 | 0.9901 | 0.9904 | 0.9906 | 0.9909 | 0.9911 | 0.9913 | 0.9916 |
| 2.4 | 0.9918 | 0.9920 | 0.9922 | 0.9925 | 0.9927 | 0.9929 | 0.9931 | 0.9932 | 0.9934 | 0.9936 |
| 2.5 | 0.9938 | 0.9940 | 0.9941 | 0.9943 | 0.9945 | 0.9946 | 0.9948 | 0.9949 | 0.9951 | 0.9952 |
| 2.6 | 0.9953 | 0.9955 | 0.9956 | 0.9957 | 0.9959 | 0.9960 | 0.9961 | 0.9962 | 0.9963 | 0.9964 |
| 2.7 | 0.9965 | 0.9966 | 0.9967 | 0.9968 | 0.9969 | 0.9970 | 0.9971 | 0.9972 | 0.9973 | 0.9974 |
| 2.8 | 0.9974 | 0.9975 | 0.9976 | 0.9977 | 0.9977 | 0.9978 | 0.9979 | 0.9979 | 0.9980 | 0.9981 |
| 2.9 | 0.9981 | 0.9982 | 0.9982 | 0.9983 | 0.9984 | 0.9984 | 0.9985 | 0.9985 | 0.9986 | 0.9986 |
| 3.0 | 0.9987 | 0.9987 | 0.9987 | 0.9988 | 0.9988 | 0.9989 | 0.9989 | 0.9989 | 0.9990 | 0.9990 |
| 3.1 | 0.9990 | 0.9991 | 0.9991 | 0.9991 | 0.9992 | 0.9992 | 0.9992 | 0.9992 | 0.9993 | 0.9993 |
| 3.2 | 0.9993 | 0.9993 | 0.9994 | 0.9994 | 0.9994 | 0.9994 | 0.9994 | 0.9995 | 0.9995 | 0.9995 |
| 3.3 | 0.9995 | 0.9995 | 0.9995 | 0.9996 | 0.9996 | 0.9996 | 0.9996 | 0.9996 | 0.9996 | 0.9997 |
| 3.4 | 0.9997 | 0.9997 | 0.9997 | 0.9997 | 0.9997 | 0.9997 | 0.9997 | 0.9997 | 0.9997 | 0.9998 |
| 3.5 | 0.9998 | 0.9998 | 0.9998 | 0.9998 | 0.9998 | 0.9998 | 0.9998 | 0.9998 | 0.9998 | 0.9998 |
| 3.6 | 0.9998 | 0.9998 | 0.9999 | | | | | | | |

# Percentage Points of the $\chi^2$-Distribution

This table gives the percentage points $\chi^2_\nu(P)$ for various values of $P$ and degrees of freedom $\nu$, as indicated by the figure to the right.

If $X$ is a variable distributed as $\chi^2$ with $\nu$ degrees of freedom, $P/100$ is the probability that $X \geq \chi^2_\nu(P)$.

For $\nu > 100$, $\sqrt{2X}$ is approximately normally distributed with mean $\sqrt{2\nu - 1}$ and unit variance.




| $\nu$ | Percentage points $P$ | | | | | | |
|---|---|---|---|---|---|---|---|
| | 10 | 5 | 2.5 | 1 | 0.5 | 0.1 | 0.05 |
| 1 | 2.706 | 3.841 | 5.024 | 6.635 | 7.879 | 10.828 | 12.116 |
| 2 | 4.605 | 5.991 | 7.378 | 9.210 | 10.597 | 13.816 | 15.202 |
| 3 | 6.251 | 7.815 | 9.348 | 11.345 | 12.838 | 16.266 | 17.730 |
| 4 | 7.779 | 9.488 | 11.143 | 13.277 | 14.860 | 18.467 | 19.997 |
| 5 | 9.236 | 11.070 | 12.833 | 15.086 | 16.750 | 20.515 | 22.105 |
| 6 | 10.645 | 12.592 | 14.449 | 16.812 | 18.548 | 22.458 | 24.103 |
| 7 | 12.017 | 14.067 | 16.013 | 18.475 | 20.278 | 24.322 | 26.018 |
| 8 | 13.362 | 15.507 | 17.535 | 20.090 | 21.955 | 26.124 | 27.868 |
| 9 | 14.684 | 16.919 | 19.023 | 21.666 | 23.589 | 27.877 | 29.666 |
| 10 | 15.987 | 18.307 | 20.483 | 23.209 | 25.188 | 29.588 | 31.420 |
| 11 | 17.275 | 19.675 | 21.920 | 24.725 | 26.757 | 31.264 | 33.137 |
| 12 | 18.549 | 21.026 | 23.337 | 26.217 | 28.300 | 32.909 | 34.821 |
| 13 | 19.812 | 22.362 | 24.736 | 27.688 | 29.819 | 34.528 | 36.478 |
| 14 | 21.064 | 23.685 | 26.119 | 29.141 | 31.319 | 36.123 | 38.109 |
| 15 | 22.307 | 24.996 | 27.488 | 30.578 | 32.801 | 37.697 | 39.719 |
| 16 | 23.542 | 26.296 | 28.845 | 32.000 | 34.267 | 39.252 | 41.308 |
| 17 | 24.769 | 27.587 | 30.191 | 33.409 | 35.718 | 40.790 | 42.879 |
| 18 | 25.989 | 28.869 | 31.526 | 34.805 | 37.156 | 42.312 | 44.434 |
| 19 | 27.204 | 30.144 | 32.852 | 36.191 | 38.582 | 43.820 | 45.973 |
| 20 | 28.412 | 31.410 | 34.170 | 37.566 | 39.997 | 45.315 | 47.498 |
| 25 | 34.382 | 37.652 | 40.646 | 44.314 | 46.928 | 52.620 | 54.947 |
| 30 | 40.256 | 43.773 | 46.979 | 50.892 | 53.672 | 59.703 | 62.162 |
| 40 | 51.805 | 55.758 | 59.342 | 63.691 | 66.766 | 73.402 | 76.095 |
| 50 | 63.167 | 67.505 | 71.420 | 76.154 | 79.490 | 86.661 | 89.561 |
| 80 | 96.578 | 101.879 | 106.629 | 112.329 | 116.321 | 124.839 | 128.261 |

# Percentage Points of the Wilcoxon Signed Rank Distribution

This table gives the lower percentage points of $W^+$, the sum of the ranks of the positive observations in a ranking in order of increasing absolute magnitude of a random sample of size $n$ from a continuous distribution which is symmetric about zero. The function tabulated $x(P)$ is the largest $x$ such that $P(W^+ < x) \leq P/100$.

| | P | | | | | | P | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| $n$ | 5 | 2.5 | 1 | 0.5 | 0.1 | $n$ | 5 | 2.5 | 1 | 0.5 | 0.1 |
| 8 | 6 | 4 | 2 | 1 | 0 | 43 | 337 | 311 | 282 | 262 | 223 |
| 9 | 9 | 6 | 4 | 2 | 0 | 44 | 354 | 328 | 297 | 277 | 236 |
| 10 | 11 | 9 | 6 | 4 | 1 | 45 | 372 | 344 | 313 | 292 | 250 |
| 11 | 14 | 11 | 8 | 6 | 2 | 46 | 390 | 362 | 329 | 308 | 264 |
| 12 | 18 | 14 | 10 | 8 | 3 | 47 | 408 | 379 | 346 | 323 | 278 |
| 13 | 22 | 18 | 13 | 10 | 5 | 48 | 427 | 397 | 363 | 340 | 293 |
| 14 | 26 | 22 | 16 | 13 | 7 | 49 | 447 | 416 | 380 | 356 | 308 |
| 15 | 31 | 26 | 20 | 16 | 9 | 50 | 467 | 435 | 398 | 374 | 324 |
| 16 | 36 | 30 | 24 | 20 | 12 | 51 | 487 | 454 | 417 | 391 | 340 |
| 17 | 42 | 35 | 28 | 24 | 15 | 52 | 508 | 474 | 435 | 409 | 356 |
| 18 | 48 | 41 | 33 | 28 | 19 | 53 | 530 | 495 | 455 | 428 | 373 |
| 19 | 54 | 47 | 38 | 33 | 22 | 54 | 551 | 515 | 474 | 446 | 390 |
| 20 | 61 | 53 | 44 | 38 | 27 | 55 | 574 | 537 | 494 | 466 | 408 |
| 21 | 68 | 59 | 50 | 43 | 31 | 56 | 596 | 558 | 515 | 485 | 426 |
| 22 | 76 | 66 | 56 | 49 | 36 | 57 | 619 | 580 | 536 | 505 | 444 |
| 23 | 84 | 74 | 63 | 55 | 41 | 58 | 643 | 603 | 557 | 526 | 463 |
| 24 | 92 | 82 | 70 | 62 | 46 | 59 | 667 | 626 | 579 | 547 | 483 |
| 25 | 101 | 90 | 77 | 69 | 52 | 60 | 691 | 649 | 601 | 568 | 502 |
| 26 | 111 | 99 | 85 | 76 | 59 | 61 | 716 | 673 | 624 | 590 | 522 |
| 27 | 120 | 108 | 93 | 84 | 65 | 62 | 742 | 698 | 647 | 612 | 543 |
| 28 | 131 | 117 | 102 | 92 | 72 | 63 | 768 | 722 | 670 | 635 | 564 |
| 29 | 141 | 127 | 111 | 101 | 80 | 64 | 794 | 748 | 694 | 658 | 585 |
| 30 | 152 | 138 | 121 | 110 | 87 | 65 | 821 | 773 | 719 | 682 | 607 |
| 31 | 164 | 148 | 131 | 119 | 95 | 66 | 848 | 799 | 743 | 706 | 629 |
| 32 | 176 | 160 | 141 | 129 | 104 | 67 | 876 | 826 | 769 | 730 | 652 |
| 33 | 188 | 171 | 152 | 139 | 113 | 68 | 904 | 853 | 794 | 755 | 675 |
| 34 | 201 | 183 | 163 | 149 | 122 | 69 | 932 | 880 | 820 | 780 | 698 |
| 35 | 214 | 196 | 174 | 160 | 132 | 70 | 961 | 908 | 847 | 806 | 722 |
| 36 | 228 | 209 | 186 | 172 | 142 | 71 | 991 | 937 | 874 | 832 | 746 |
| 37 | 242 | 222 | 199 | 183 | 152 | 72 | 1021 | 965 | 902 | 859 | 771 |
| 38 | 257 | 236 | 212 | 195 | 163 | 73 | 1051 | 995 | 929 | 885 | 796 |
| 39 | 272 | 250 | 225 | 208 | 174 | 74 | 1082 | 1024 | 958 | 913 | 822 |
| 40 | 287 | 265 | 239 | 221 | 186 | 75 | 1113 | 1054 | 987 | 941 | 848 |
| 41 | 303 | 280 | 253 | 234 | 198 | 76 | 1145 | 1085 | 1016 | 969 | 874 |
| 42 | 320 | 295 | 267 | 248 | 210 | 77 | 1177 | 1116 | 1045 | 998 | 901 |
| 43 | 337 | 311 | 282 | 262 | 223 | 78 | 1210 | 1148 | 1076 | 1027 | 928 |



# Upper Critical Values for the Kruskal-Wallis Test

| Group Sizes | | | Nominal size $\alpha$ | | | |
|---|---|---|---|---|---|---|
| | | | 0.10 | 0.05 | 0.025 | 0.01 |
| 2 | 2 | 2 | 4.571 (.06667) | - - | - - | - - |
| 3 | 2 | 1 | 4.286 (.10000) | - - | - - | - - |
| 3 | 2 | 2 | 4.500 (.06667) | 4.714 (.04762) | - - | - - |
| 3 | 3 | 1 | 4.571 (.10000) | 5.143 (.04286) | - - | - - |
| 3 | 3 | 2 | 4.556 (.10000) | 5.361 (.03214) | 5.556 (.02500) | - - |
| 3 | 3 | 3 | 4.622 (.10000) | 5.600 (.05000) | 5.956 (.02500) | 7.200 (.00357) |
| 4 | 2 | 1 | 4.500 (.07619) | - - | - - | - - |
| 4 | 2 | 2 | 4.458 (.10000) | 5.333 (.03333) | 5.500 (.02381) | - - |
| 4 | 3 | 1 | 4.056 (.09286) | 5.208 (.05000) | 5.833 (.02143) | - - |
| 4 | 3 | 2 | 4.511 (.09841) | 5.444 (.04603) | 6.000 (.02381) | 6.444 (.00794) |
| 4 | 3 | 3 | 4.709 (.09238) | 5.791 (.04571) | 6.155 (.02476) | 6.745 (.01000) |
| 4 | 4 | 1 | 4.167 (.08254) | 4.967 (.04762) | 6.167 (.02222) | 6.667 (.00952) |
| 4 | 4 | 2 | 4.555 (.09778) | 5.455 (.04571) | 6.327 (.02413) | 7.036 (.00571) |
| 4 | 4 | 3 | 4.545 (.09905) | 5.598 (.04866) | 6.394 (.02476) | 7.144 (.00970) |
| 4 | 4 | 4 | 4.654 (.09662) | 5.692 (.04866) | 6.615 (.02424) | 7.654 (.00762) |
| 5 | 2 | 1 | 4.200 (.09524) | 5.000 (.04762) | - - | - - |
| 5 | 2 | 2 | 4.373 (.08995) | 5.160 (.03439) | 6.000 (.01852) | 6.533 (.00794) |
| 5 | 3 | 1 | 4.018 (.09524) | 4.960 (.04762) | 6.044 (.01984) | - - |
| 5 | 3 | 2 | 4.651 (.09127) | 5.251 (.04921) | 6.004 (.02460) | 6.909 (.00873) |
| 5 | 3 | 3 | 4.533 (.09697) | 5.648 (.04892) | 6.315 (.02121) | 7.079 (.00866) |
| 5 | 4 | 1 | 3.987 (.09841) | 4.985 (.04444) | 5.858 (.02381) | 6.955 (.00794) |
| 5 | 4 | 2 | 4.541 (.09841) | 5.273 (.04877) | 6.068 (.02482) | 7.205 (.00895) |
| 5 | 4 | 3 | 4.549 (.09892) | 5.656 (.04863) | 6.410 (.02496) | 7.445 (.00974) |
| 5 | 4 | 4 | 4.668 (.09817) | 5.657 (.04906) | 6.673 (.02429) | 7.760 (.00946) |
| 5 | 5 | 1 | 4.109 (.08586) | 5.127 (.04618) | 6.000 (.02165) | 7.309 (.00938) |
| 5 | 5 | 2 | 4.623 (.09704) | 5.338 (.04726) | 6.346 (.02489) | 7.338 (.00962) |
| 5 | 5 | 3 | 4.545 (.09965) | 5.705 (.04612) | 6.549 (.02436) | 7.578 (.00968) |
| 5 | 5 | 4 | 4.523 (.09935) | 5.666 (.04931) | 6.760 (.02490) | 7.823 (.00978) |
| 5 | 5 | 5 | 4.560 (.09952) | 5.780 (.04878) | 6.740 (.02475) | 8.000 (.00946) |

## Kolmogorov-Smirnov One-Sided Test

| $n$ | 0.1 | 0.05 | 0.025 | 0.01 | 0.005 |
|---|---|---|---|---|---|
| 1 | 0.9000 | 0.9500 | 0.9750 | 0.9900 | 0.9950 |
| 2 | 0.6838 | 0.7764 | 0.8419 | 0.9000 | 0.9293 |
| 3 | 0.5648 | 0.6360 | 0.7076 | 0.7846 | 0.8290 |
| 4 | 0.4927 | 0.5652 | 0.6239 | 0.6889 | 0.7342 |
| 5 | 0.4470 | 0.5094 | 0.5633 | 0.6272 | 0.6685 |
| 6 | 0.4104 | 0.4680 | 0.5193 | 0.5774 | 0.6166 |
| 7 | 0.3815 | 0.4361 | 0.4834 | 0.5384 | 0.5758 |
| 8 | 0.3583 | 0.4096 | 0.4543 | 0.5065 | 0.5418 |
| 9 | 0.3391 | 0.3875 | 0.4300 | 0.4796 | 0.5133 |
| 10 | 0.3226 | 0.3687 | 0.4092 | 0.4566 | 0.4889 |
| 11 | 0.3083 | 0.3524 | 0.3912 | 0.4367 | 0.4677 |
| 12 | 0.2958 | 0.3382 | 0.3754 | 0.4192 | 0.4490 |
| 13 | 0.2847 | 0.3255 | 0.3614 | 0.4036 | 0.4325 |
| 14 | 0.2748 | 0.3142 | 0.3489 | 0.3897 | 0.4176 |
| 15 | 0.2659 | 0.3040 | 0.3376 | 0.3771 | 0.4042 |
| 16 | 0.2578 | 0.2947 | 0.3273 | 0.3657 | 0.3920 |
| 17 | 0.2504 | 0.2863 | 0.3180 | 0.3553 | 0.3809 |
| 18 | 0.2436 | 0.2785 | 0.3094 | 0.3457 | 0.3706 |
| 19 | 0.2373 | 0.2714 | 0.3014 | 0.3369 | 0.3612 |
| 20 | 0.2316 | 0.2647 | 0.2941 | 0.3287 | 0.3524 |
| 21 | 0.2262 | 0.2586 | 0.2872 | 0.3210 | 0.3443 |
| 22 | 0.2212 | 0.2528 | 0.2809 | 0.3139 | 0.3367 |
| 23 | 0.2165 | 0.2475 | 0.2749 | 0.3073 | 0.3295 |
| 24 | 0.2120 | 0.2424 | 0.2693 | 0.3010 | 0.3229 |
| 25 | 0.2079 | 0.2377 | 0.2640 | 0.2952 | 0.3166 |
| 26 | 0.2040 | 0.2332 | 0.2591 | 0.2896 | 0.3106 |
| 27 | 0.2003 | 0.2290 | 0.2544 | 0.2844 | 0.3050 |
| 28 | 0.1968 | 0.2250 | 0.2499 | 0.2794 | 0.2997 |
| 29 | 0.1935 | 0.2212 | 0.2457 | 0.2747 | 0.2947 |
| 30 | 0.1903 | 0.2176 | 0.2417 | 0.2702 | 0.2899 |
| 31 | 0.1873 | 0.2141 | 0.2379 | 0.2660 | 0.2853 |
| 32 | 0.1844 | 0.2108 | 0.2342 | 0.2619 | 0.2809 |
| 33 | 0.1817 | 0.2077 | 0.2308 | 0.2580 | 0.2768 |
| 34 | 0.1791 | 0.2047 | 0.2274 | 0.2543 | 0.2728 |
| 35 | 0.1766 | 0.2018 | 0.2242 | 0.2507 | 0.2690 |
| 36 | 0.1742 | 0.1991 | 0.2212 | 0.2473 | 0.2653 |
| 37 | 0.1719 | 0.1965 | 0.2183 | 0.2440 | 0.2618 |
| 38 | 0.1697 | 0.1939 | 0.2154 | 0.2409 | 0.2584 |
| 39 | 0.1675 | 0.1915 | 0.2127 | 0.2379 | 0.2552 |
| 40 | 0.1655 | 0.1891 | 0.2101 | 0.2349 | 0.2521 |
| > 40 | $1.07/\sqrt{n}$ | $1.22/\sqrt{n}$ | $1.36/\sqrt{n}$ | $1.52/\sqrt{n}$ | $1.63/\sqrt{n}$ |

**Quantiles of the Kendall test statistic $T = N_c - N_d$. Quantiles of Kendall's**
**in parentheses. Lower quantiles are the negative of the upper quantiles,**

| $n$ | $p$ = 0.900 | 0.950 | 0.975 | 0.990 | 0.99 |
|---|---|---|---|---|---|
| 4 | 4 (0.6667) | 4 (0.6667) | 6 (1.0000) | 6 (1.0000) | 6 (1. |
| 5 | 6 (0.6000) | 6 (0.6000) | 8 (0.8000) | 8 (0.8000) | 10 (1.0 |
| 6 | 7 (0.4667) | 9 (0.6000) | 11 (0.7333) | 11 (0.7333) | 13 (0.8 |
| 7 | 9 (0.4286) | 11 (0.5238) | 13 (0.6190) | 15 (0.7143) | 17 (0.8 |
| 8 | 10 (0.3571) | 14 (0.5000) | 16 (0.5714) | 18 (0.6429) | 20 (0.7 |
| 9 | 12 (0.3333) | 16 (0.4444) | 18 (0.5000) | 22 (0.6111) | 24 (0.6 |
| 10 | 15 (0.3333) | 19 (0.4222) | 21 (0.4667) | 25 (0.5556) | 27 (0.6 |
| 11 | 17 (0.3091) | 21 (0.3818) | 25 (0.4545) | 29 (0.5273) | 31 (0.5 |
| 12 | 18 (0.2727) | 24 (0.3636) | 28 (0.4242) | 34 (0.5152) | 36 (0.5 |
| 13 | 22 (0.2821) | 26 (0.3333) | 32 (0.4103) | 38 (0.4872) | 42 (0.5 |
| 14 | 23 (0.2527) | 31 (0.3407) | 35 (0.3846) | 41 (0.4505) | 45 (0.4 |
| 15 | 27 (0.2571) | 33 (0.3143) | 39 (0.3714) | 47 (0.4476) | 51 (0.4 |
| 16 | 28 (0.2333) | 36 (0.3000) | 44 (0.3667) | 50 (0.4167) | 56 (0.4 |
| 17 | 32 (0.2353) | 40 (0.2941) | 48 (0.3529) | 56 (0.4118) | 62 (0.4 |
| 18 | 35 (0.2288) | 43 (0.2810) | 51 (0.3333) | 61 (0.3987) | 67 (0.4 |
| 19 | 37 (0.2164) | 47 (0.2749) | 55 (0.3216) | 65 (0.3801) | 73 (0.4 |
| 20 | 40 (0.2105) | 50 (0.2632) | 60 (0.3158) | 70 (0.3684) | 78 (0.4 |
| 21 | 42 (0.2000) | 54 (0.2571) | 64 (0.3048) | 76 (0.3619) | 84 (0.4 |
| 22 | 45 (0.1948) | 59 (0.2554) | 69 (0.2987) | 81 (0.3506) | 89 (0.3 |
| 23 | 49 (0.1937) | 63 (0.2490) | 73 (0.2885) | 87 (0.3439) | 97 (0.3 |
| 24 | 52 (0.1884) | 66 (0.2391) | 78 (0.2826) | 92 (0.3333) | 102 (0.3 |
| 25 | 56 (0.1867) | 70 (0.2333) | 84 (0.2800) | 98 (0.3267) | 108 (0.3 |
| 26 | 59 (0.1815) | 75 (0.2308) | 89 (0.2738) | 105 (0.3231) | 115 (0.3 |
| 27 | 61 (0.1738) | 79 (0.2251) | 93 (0.2650) | 111 (0.3162) | 123 (0.3 |
| 28 | 66 (0.1746) | 84 (0.2222) | 98 (0.2593) | 116 (0.3069) | 128 (0.3 |
| 29 | 68 (0.1675) | 88 (0.2167) | 104 (0.2562) | 124 (0.3054) | 136 (0.3 |
| 30 | 73 (0.1678) | 93 (0.2138) | 109 (0.2506) | 129 (0.2966) | 143 (0.3 |
| 31 | 75 (0.1613) | 97 (0.2086) | 115 (0.2473) | 135 (0.2903) | 149 (0.3 |
| 32 | 80 (0.1613) | 102 (0.2056) | 120 (0.2419) | 142 (0.2863) | 158 (0.3 |
| 33 | 84 (0.1591) | 106 (0.2008) | 126 (0.2386) | 150 (0.2841) | 164 (0.3 |
| 34 | 87 (0.1551) | 111 (0.1979) | 131 (0.2335) | 155 (0.2763) | 173 (0.3 |
| 35 | 91 (0.1529) | 115 (0.1933) | 137 (0.2303) | 163 (0.2739) | 179 (0.3 |
| 36 | 94 (0.1492) | 120 (0.1905) | 144 (0.2286) | 170 (0.2698) | 188 (0.2 |
| 37 | 98 (0.1471) | 126 (0.1892) | 150 (0.2252) | 176 (0.2643) | 198 (0.2 |

Source: Conover, W.J., *Practical Nonparametric Statistics*, Third edition, Joh
Sons, Inc., 1999.