

UNIVERSITY OF SWAZILAND
2010/2011 SECOND SEMESTER EXAMINATIONS

TITLE OF PAPER: QUANTITATIVE METHODS IN DEMOGRAPHY

COURSE CODE: DEM 206

TIME ALLOWED: TWO (2) HOURS

INSTRUCTIONS: ANSWER ANY THREE (3) QUESTIONS

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THE INVIGILATOR**

Question 1

- a)
- i. What is meant by statistical inference? In your answer include its function and importance? (2)
 - ii. Define the terms parameter and statistic. State the relationship between the two. (2)
 - iii. What is meant by estimation? (1)
 - iv. What is meant by hypothesis testing, and what is its function? (2)
 - v. What is meant by the 95% confidence interval? (1)

b)

You have been offered an internship by the Swaziland Central Statistical Office (CSO). You are told that the office has been tasked with the responsibility of conducting a survey that will investigate sexual behavior amongst school going pupils in Swaziland. Part of your job is to help the CSO sample at least 10000 pupils to be part of the study.

List and describe the probability based sampling methods you would use. You should also include what the sampling units for this survey would be and how you would choose these sampling units. (12)

Question 2

A company manager is interested in knowing whether an employee's educational level has an effect on the knowledge of his or her job. An exam was given to a sample of 120 employees and the manager would like to know if there is a difference in exam performance among the following groups:

- (1) Those with a high school certificate only (2) those with a bachelor's degree only (3) those with a master's degree.

The performance in the exam is classified as being;

- (1) Low (2) Medium (3) High

The observed frequencies are presented in the table below:

	High	Average	Low
Master's	4	20	11
Bachelor's	12	18	15
O'Level	9	22	9

- A. As an analyst you are to answer the question: does the knowledge, as measured by the exam appear to be related to an employee's education? Use a significance level of 0.05 (8).
- B. For a random sample of 400 households, it was found that the average amount spent on electricity bills per month is E295, with a population standard deviation of E39.50.
- i. Find the 95% confidence interval for the average monthly bill for the sample. (3)
 - ii. For a randomly selected household what is the probability that:
 - A. It spends more than E330 on electricity? (2)
 - B. It spends between E265 and E355?
 - C. If the company providing the electricity wants only 35% of the households to exceed a given amount, what amount will this correspond to? (3)
 - D. What must be the mean amount paid per month to ensure that no more than 15% of the households pay more than E400? (4)

Question 3

A demographer is interested in knowing the average number of birth related deaths per year recorded in the major hospitals in each of four regions over the past couple of years.

Hospital A:	12	11	14	10	12	10
Hospital B:	14	12	16	15		
Hospital C:	17	18	20	22	23	
Hospital D:	10	9	13	12		

- a) Using a 95% confidence level, conduct a test of the null hypothesis that the hospital deaths are equal. (10)
- b) Conduct a hypothesis test to compare the means for Hospital A and Hospital B. Use a sample standard deviation of 2.3 and 3.2 for A and B respectively. Use a 5% significance level. (7)
- c) What assumptions did you take into consideration in conducting the tests above? (3)

Question 4

- a) The country is currently faced with a economic meltdown. It is believed that 10% of the households will have the head of the household lose his or her job. It is known that 90% of the households have the heads working for Government. Experience from other records shows that 3% of household heads working for government will lose their jobs. As a demographer you are tasked with calculating:
- i. The probability that for a randomly selected household, the household head works for government and will lose his/her job.
 - ii. The probability that for a randomly selected household, the household head works for government or will lose his/her job.
 - iii. The percentage of employees that lose their jobs and work for government.
 - iv. Given that a household head loses his/her job, what is the probability that the head works for government? (10)
- b) Car ownership soared to its heighest in Swaziland after the arrival of the “grey imports”. The average percentage of the graduates owning a vehicle is 66.2% in Manzini, 50% in Nhlangano and 70% in Mbabane.
- i. Supposing that 10 graduates are chosen in Nhlangano, what is the probability that at least five will own a car?
 - ii. Supposing that 10 graduates are chosen from the sample of the three towns, what is the probability that seven will own a car?
 - iii. If 300 graduates are taken from each town, what are the expected number of car owners in each town? (10)

Question 5

Use the Regression Analysis output to answer this question:

- a) Explain the difference between correlation and regression. (1)
- b) Comment of the correlation of the Sales variable with each one of the other variables. (2)
- c) Formulate a hypothesis for the correlation (your answer should be both in words and symbol form (1).
- d) Write out an equation for the multiple regression model. (2)
- e) What percentage of total variation in sales is explained by the regression model? Explain which statistic was used to come up with the answer and the reason it was chosen over the other(s) (2)
- f) Interpret the meaning of the regression coefficients. (3)
- g) State whether each of the independent variables is significant in determining income. Explain your answer using a 0.05 significance level. (3)
- h) Choose one of the independent variables and write down the hypotheses used in testing its significance for predicting income. What is your conclusion on this variable? (2)
- i) Use the multiple regression model to estimate the likely average number of padlocks for the following values of the independent variable. AdSpend = 4.6, Reps=66, Brands=11 and CrimeIndex=12. (2)
- j) Define multicollinearity, and comment on it in relation to this regression analysis. (2)

Regression Analysis Output (Question 5)

Descriptive Statistics

	Mean	Std. Deviation	N
Sales	168.7846	85.38775	26
AdSpend	5.4077	1.83192	26
Reps	50.0769	12.93344	26
Brands	8.5385	2.26682	26
CrimeIndex	9.3846	4.39160	26

Correlations

		Sales	AdSpend	Reps	Brands	CrimeIndex
Pearson Correlation	Sales	1.000	.174	.591	-.697	.387
	AdSpend	.174	1.000	.138	-.158	-.129
	Reps	.591	.138	1.000	-.147	.457
	Brands	-.697	-.158	-.147	1.000	-.002
	CrimeIndex	.387	-.129	.457	-.002	1.000
Sig. (1-tailed)	Sales	.	.198	.001	.000	.025
	AdSpend	.198	.	.251	.220	.266
	Reps	.001	.251	.	.236	.010
	Brands	.000	.220	.236	.	.497
	CrimeIndex	.025	.266	.010	.497	.
N	Sales	26	26	26	26	26
	AdSpend	26	26	26	26	26
	Reps	26	26	26	26	26
	Brands	26	26	26	26	26
	CrimeIndex	26	26	26	26	26

Model	R	R Square	Adjusted R Square	Std. Error of the Estimate	Change Statistics				
					R Square Change	F Change	df1	df2	Sig. F Change
1	.874 ^a	.763	.718	45.33211	.763	16.925	4	21	.000

a. Predictors: (Constant), CrimelIndex, Brands, AdSpend, Reps

ANOVA^b

Model		Sum of Squares	df	Mean Square	F	Sig.
1	Regression	139121.704	4	34780.426	16.925	.000 ^a
	Residual	43155.010	21	2055.000		
	Total	182276.714	25			

a. Predictors: (Constant), CrimelIndex, Brands, AdSpend, Reps

b. Dependent Variable: Sales

Coefficients^a

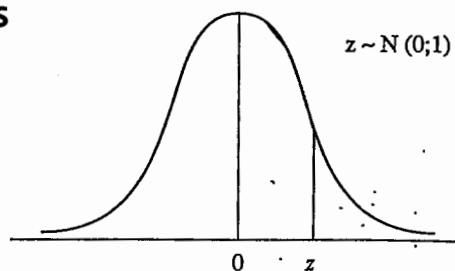
Model	Unstandardized Coefficients	Standardized Coefficients	t	Sig.	95.0% Confidence Interval for B		Correlations			Collinearity Statistics		
					Lower Bound	Upper Bound	Zero-order	Partial	Part	Tolerance	VIF	
1 (Constant)	190.455		3.121	.005	63.561	317.349						
AdSpend	2.180	.047	.422	.677	-8.554	12.913	.174	.092	.045	.919	1.088	
Reps	2.610	.395	3.200	.004	.914	4.306	.591	.573	.340	.739	1.354	
Brands	-23.753	-.631	-	.000	-32.256	-15.250	-.697	-.785	-	.957	1.045	
CrimelIndex	4.119	.212	1.731	.098	-.830	9.068	.387	.353	.184	.753	1.329	

a. Dependent Variable: Sales

STATISTICAL TABLES

TABLE 1
The standard normal distribution (z)

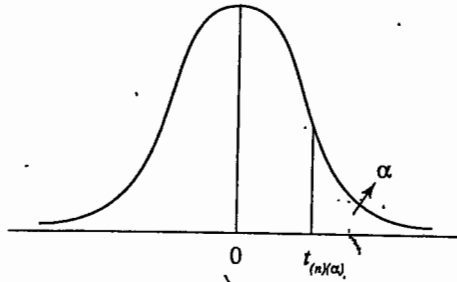
This table gives the area under the standard normal curve between 0 and z , i.e. $P[0 < Z < z]$



Z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.0000	0.0040	0.0080	0.0120	0.0160	0.0199	0.0239	0.0279	0.0319	0.0359
0.1	0.0398	0.0438	0.0478	0.0517	0.0557	0.0596	0.0636	0.0675	0.0714	0.0753
0.2	0.0793	0.0832	0.0871	0.0910	0.0948	0.0987	0.1026	0.1064	0.1103	0.1141
0.3	0.1179	0.1217	0.1255	0.1293	0.1331	0.1368	0.1406	0.1443	0.1480	0.1517
0.4	0.1554	0.1591	0.1628	0.1664	0.1700	0.1736	0.1772	0.1808	0.1844	0.1879
0.5	0.1915	0.1950	0.1985	0.2019	0.2054	0.2088	0.2123	0.2157	0.2190	0.2224
0.6	0.2257	0.2291	0.2324	0.2357	0.2389	0.2422	0.2454	0.2486	0.2517	0.2549
0.7	0.2580	0.2611	0.2642	0.2673	0.2703	0.2734	0.2764	0.2793	0.2823	0.2852
0.8	0.2881	0.2910	0.2939	0.2967	0.2995	0.3023	0.3051	0.3078	0.3106	0.3133
0.9	0.3159	0.3186	0.3212	0.3238	0.3264	0.3289	0.3315	0.3340	0.3365	0.3389
1.0	0.3413	0.3438	0.3461	0.3485	0.3508	0.3531	0.3554	0.3577	0.3599	0.3621
1.1	0.3643	0.3665	0.3686	0.3708	0.3729	0.3749	0.3770	0.3790	0.3810	0.3830
1.2	0.3849	0.3869	0.3888	0.3907	0.3925	0.3944	0.3962	0.3980	0.3997	0.4015
1.3	0.4032	0.4049	0.4066	0.4082	0.4099	0.4115	0.4131	0.4147	0.4162	0.4177
1.4	0.4192	0.4207	0.4222	0.4236	0.4251	0.4265	0.4279	0.4292	0.4306	0.4319
1.5	0.4332	0.4345	0.4357	0.4370	0.4382	0.4394	0.4406	0.4418	0.4429	0.4441
1.6	0.4452	0.4463	0.4474	0.4484	0.4495	0.4505	0.4515	0.4525	0.4535	0.4545
1.7	0.4554	0.4564	0.4573	0.4582	0.4591	0.4599	0.4608	0.4616	0.4625	0.4633
1.8	0.4641	0.4649	0.4656	0.4664	0.4671	0.4678	0.4686	0.4693	0.4699	0.4706
1.9	0.4713	0.4719	0.4726	0.4732	0.4738	0.4744	0.4750	0.4756	0.4761	0.4767
2.0	0.4772	0.4778	0.4783	0.4788	0.4793	0.4798	0.4803	0.4808	0.4812	0.4817
2.1	0.4821	0.4826	0.4830	0.4834	0.4838	0.4842	0.4846	0.4850	0.4854	0.4857
2.2	0.4861	0.4864	0.4868	0.4871	0.4875	0.4878	0.4881	0.4884	0.4887	0.4890
2.3	0.48928	0.48956	0.48983	0.49010	0.49036	0.49061	0.49086	0.49111	0.49134	0.49158
2.4	0.49180	0.49202	0.49224	0.49245	0.49266	0.49286	0.49305	0.49324	0.49343	0.49361
2.5	0.49379	0.49396	0.49413	0.49430	0.49446	0.49461	0.49477	0.49492	0.49506	0.49520
2.6	0.49534	0.49547	0.49560	0.49573	0.49585	0.49598	0.49609	0.49621	0.49632	0.49643
2.7	0.49653	0.49664	0.49674	0.49683	0.49693	0.49702	0.49711	0.49720	0.49728	0.49736
2.8	0.49744	0.49752	0.49760	0.49767	0.49774	0.49781	0.49788	0.49795	0.49801	0.49807
2.9	0.49813	0.49819	0.49825	0.49831	0.49836	0.49841	0.49846	0.49851	0.49856	0.49861
3.0	0.49865	0.49869	0.49874	0.49878	0.49882	0.49886	0.49889	0.49893	0.49897	0.49900
3.1	0.49903	0.49906	0.49910	0.49913	0.49916	0.49918	0.49921	0.49924	0.49926	0.49929
3.2	0.49931	0.49934	0.49936	0.49938	0.49940	0.49942	0.49944	0.49946	0.49948	0.49950
3.3	0.49952	0.49953	0.49955	0.49957	0.49958	0.49960	0.49961	0.49962	0.49964	0.49965
3.4	0.49966	0.49968	0.49969	0.49970	0.49971	0.49972	0.49973	0.49974	0.49975	0.49976
3.5	0.49977	0.49978	0.49978	0.49979	0.49980	0.49981	0.49981	0.49982	0.49983	0.49983
3.6	0.49984	0.49985	0.49985	0.49986	0.49986	0.49987	0.49987	0.49988	0.49988	0.49989
3.7	0.49989	0.49990	0.49990	0.49990	0.49991	0.49991	0.49991	0.49992	0.49992	0.49992
3.8	0.49993	0.49993	0.49993	0.49994	0.49994	0.49994	0.49994	0.49995	0.49995	0.49995
3.9	0.49995	0.49995	0.49996	0.49996	0.49996	0.49996	0.49996	0.49996	0.49997	0.49997
4.0	0.49997	0.49997	0.49997	0.49997	0.49997	0.49997	0.49998	0.49998	0.49998	0.49998

TABLE 2
The t distribution

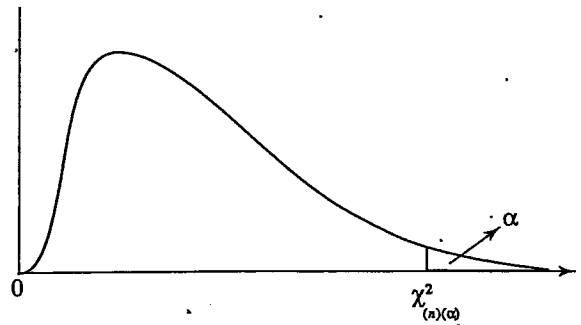
This table gives the value of $t_{(n)(\alpha)}$ where n is the degrees of freedom i.e. $\alpha = P[t \geq t_{(n)(\alpha)}]$



α df	0.100	0.050	0.025	0.010	0.005	0.0025
1	3.078	6.314	12.706	31.821	63.657	127.322
2	1.886	2.920	4.303	6.965	9.925	14.089
3	1.638	2.353	3.182	4.541	5.841	7.453
4	1.533	2.132	2.776	3.747	4.604	5.598
5	1.476	2.015	2.571	3.365	4.032	4.773
6	1.440	1.943	2.447	3.143	3.707	4.317
7	1.415	1.895	2.365	2.998	3.499	4.029
8	1.397	1.860	2.306	2.896	3.355	3.833
9	1.383	1.833	2.262	2.821	3.250	3.690
10	1.372	1.812	2.228	2.764	3.169	3.581
11	1.363	1.796	2.201	2.718	3.106	3.497
12	1.356	1.782	2.179	2.681	3.055	3.428
13	1.350	1.771	2.160	2.650	3.012	3.372
14	1.345	1.761	2.145	2.624	2.977	3.326
15	1.341	1.753	2.131	2.602	2.947	3.286
16	1.337	1.746	2.120	2.583	2.921	3.252
17	1.333	1.740	2.110	2.567	2.898	3.222
18	1.330	1.734	2.101	2.552	2.878	3.197
19	1.328	1.729	2.093	2.539	2.861	3.174
20	1.325	1.725	2.086	2.528	2.845	3.153
21	1.323	1.721	2.080	2.518	2.831	3.135
22	1.321	1.717	2.074	2.508	2.819	3.119
23	1.319	1.714	2.069	2.500	2.807	3.104
24	1.318	1.711	2.064	2.492	2.797	3.091
25	1.316	1.708	2.060	2.485	2.787	3.078
26	1.315	1.706	2.056	2.479	2.779	3.067
27	1.314	1.703	2.052	2.473	2.771	3.057
28	1.313	1.701	2.048	2.467	2.763	3.047
29	1.311	1.699	2.045	2.462	2.756	3.038
30	1.310	1.697	2.042	2.457	2.750	3.030
31	1.309	1.696	2.040	2.453	2.744	3.022
32	1.309	1.694	2.037	2.449	2.738	3.015
33	1.308	1.692	2.035	2.445	2.733	3.008
34	1.307	1.691	2.032	2.441	2.728	3.002
35	1.306	1.690	2.030	2.438	2.724	2.996
36	1.306	1.688	2.028	2.434	2.719	2.990
37	1.305	1.687	2.026	2.431	2.715	2.985
38	1.304	1.686	2.024	2.429	2.712	2.980
39	1.304	1.685	2.023	2.426	2.708	2.976
40	1.303	1.684	2.021	2.423	2.704	2.971
45	1.301	1.679	2.014	2.412	2.690	2.952
50	1.299	1.676	2.009	2.403	2.678	2.937
60	1.296	1.671	2.000	2.390	2.660	2.915
70	1.294	1.667	1.994	2.381	2.648	2.899
80	1.292	1.664	1.990	2.374	2.639	2.887
90	1.291	1.662	1.987	2.369	2.632	2.878
100	1.290	1.660	1.984	2.364	2.626	2.871
110	1.289	1.659	1.982	2.361	2.621	2.865
120	1.289	1.658	1.980	2.358	2.617	2.860
140	1.288	1.656	1.977	2.353	2.611	2.852
160	1.287	1.654	1.975	2.350	2.607	2.847
180	1.286	1.653	1.973	2.347	2.603	2.842
200	1.286	1.653	1.972	2.345	2.601	2.839
∞	1.282	1.645	1.960	2.327	2.576	2.807

TABLE 3
The Chi-square distribution (χ^2)

This table gives the value of $\chi^2_{(n)(\alpha)}$ where n is the degrees of freedom i.e. $\alpha = P[\chi^2 > \chi^2_{(n)(\alpha)}]$

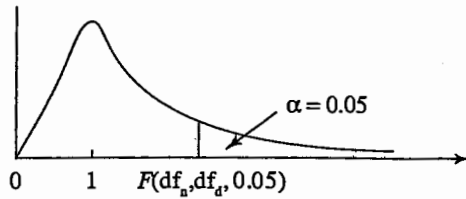


α df	0.100	0.050	0.025	0.01	0.005	0.0025
1	2.707	3.843	5.026	6.637	7.881	9.142
2	4.605	5.991	7.378	9.210	10.597	11.983
3	6.251	7.815	9.348	11.345	12.838	14.321
4	7.779	9.488	11.143	13.277	14.860	16.424
5	9.236	11.071	12.833	15.086	16.750	18.386
6	10.645	12.592	14.449	16.812	18.548	20.249
7	12.017	14.067	16.013	18.475	20.278	22.040
8	13.362	15.507	17.535	20.090	21.955	23.774
9	14.684	16.919	19.023	21.666	23.589	25.462
10	15.987	18.307	20.483	23.209	25.188	27.112
11	17.275	19.675	21.920	24.725	26.757	28.729
12	18.549	21.026	23.337	26.217	28.300	30.318
13	19.812	22.362	24.736	27.688	29.819	31.883
14	21.064	23.685	26.119	29.141	31.319	33.426
15	22.307	24.996	27.488	30.578	32.801	34.950
16	23.542	26.296	28.845	32.000	34.267	36.456
17	24.769	27.587	30.191	33.409	35.718	37.946
18	25.989	28.869	31.526	34.805	37.156	39.422
19	27.204	30.144	32.852	36.191	38.582	40.885
20	28.412	31.410	34.170	37.566	39.997	42.336
21	29.615	32.671	35.479	38.932	41.401	43.775
22	30.813	33.924	36.781	40.289	42.796	45.204
23	32.007	35.172	38.076	41.638	44.181	46.623
24	33.196	36.415	39.364	42.980	45.558	48.034
25	34.382	37.652	40.646	44.314	46.928	49.435
26	35.563	38.885	41.923	45.642	48.290	50.829
27	36.741	40.113	43.195	46.963	49.645	52.215
28	37.916	41.337	44.461	48.278	50.993	53.594
29	39.087	42.557	45.722	49.588	52.336	54.967
30	40.256	43.773	46.979	50.892	53.672	56.332
31	41.422	44.985	48.232	52.191	55.003	57.692
32	42.585	46.194	49.480	53.486	56.328	59.046
33	43.745	47.400	50.725	54.776	57.648	60.395
34	44.903	48.602	51.966	56.061	58.964	61.738
35	46.059	49.802	53.203	57.342	60.275	63.076
36	47.212	50.998	54.437	58.619	61.581	64.410
37	48.363	52.192	55.668	59.892	62.883	65.739
38	49.513	53.384	56.896	61.162	64.181	67.063
39	50.660	54.572	58.120	62.428	65.476	68.383
40	51.805	55.758	59.342	63.691	66.766	69.699
45	57.505	61.656	65.410	69.957	73.166	76.233
50	63.167	67.505	71.420	76.154	79.490	82.664
60	74.399	79.087	83.305	88.386	91.957	95.357
70	85.529	90.537	95.031	100.432	104.222	107.812
80	96.581	101.885	106.636	112.336	116.329	120.107
90	107.568	113.151	118.144	124.125	128.307	132.262
100	118.501	124.348	129.570	135.815	140.178	144.300
110	129.388	135.487	140.925	147.423	151.958	156.238
120	140.571	146.572	152.222	159.202	163.672	168.122
140	168.618	174.659	180.174	186.875	191.604	196.099
160	190.522	196.926	202.766	209.852	214.845	219.588
180	212.310	219.056	225.200	232.647	237.890	242.866

Table A(2)

F distribution ($\alpha = 0.05$)

The entries in this table are critical values of F for which the area under the curve to the right is equal to 0.05.



Degrees of Freedom for Denominator	Degrees of Freedom for Numerator									
	1	2	3	4	5	6	7	8	9	10
1	161.4	199.5	215.7	224.6	230.2	234	236.8	238.9	240.5	241.9
2	18.5	19.0	19.2	19.2	19.3	19.3	19.4	19.4	19.4	19.4
3	10.1	9.55	9.28	9.12	9.01	8.94	8.89	8.85	8.81	8.79
4	7.71	6.94	6.59	6.39	6.26	6.16	6.09	6.04	6.00	5.96
5	6.61	5.79	5.41	5.19	5.05	4.95	4.88	4.82	4.77	4.74
6	5.99	5.14	4.76	4.53	4.39	4.28	4.21	4.15	4.10	4.06
7	5.59	4.74	4.35	4.12	3.97	3.87	3.79	3.73	3.68	3.64
8	5.32	4.46	4.07	3.84	3.69	3.58	3.50	3.44	3.39	3.35
9	5.12	4.26	3.86	3.63	3.48	3.37	3.29	3.23	3.18	3.14
10	4.96	4.10	3.71	3.48	3.33	3.22	3.14	3.07	3.02	2.98
11	4.84	3.98	3.59	3.36	3.20	3.09	3.01	2.95	2.90	2.85
12	4.75	3.89	3.49	3.26	3.11	3.00	2.91	2.85	2.80	2.75
13	4.67	3.81	3.41	3.18	3.03	2.92	2.83	2.77	2.71	2.67
14	4.60	3.74	3.34	3.11	2.96	2.85	2.76	2.70	2.65	2.60
15	4.54	3.68	3.29	3.06	2.90	2.79	2.71	2.64	2.59	2.54
16	4.49	3.63	3.24	3.01	2.85	2.74	2.66	2.59	2.54	2.49
17	4.45	3.59	3.20	2.96	2.81	2.70	2.61	2.55	2.49	2.45
18	4.41	3.55	3.16	2.93	2.77	2.66	2.58	2.51	2.46	2.41
19	4.38	3.52	3.13	2.90	2.74	2.63	2.54	2.48	2.42	2.38
20	4.35	3.49	3.10	2.87	2.71	2.60	2.51	2.45	2.39	2.35
21	4.32	3.47	3.07	2.84	2.68	2.57	2.49	2.42	2.37	2.32
22	4.30	3.44	3.05	2.82	2.66	2.55	2.46	2.40	2.34	2.30
23	4.28	3.42	3.03	2.80	2.64	2.53	2.44	2.37	2.32	2.27
24	4.26	3.40	3.01	2.78	2.62	2.51	2.42	2.36	2.30	2.25
25	4.24	3.39	2.99	2.76	2.60	2.49	2.40	2.34	2.28	2.24
30	4.17	3.32	2.92	2.69	2.53	2.42	2.33	2.27	2.21	2.16
40	4.08	3.23	2.84	2.61	2.45	2.34	2.25	2.18	2.12	2.08
60	4.00	3.15	2.76	2.53	2.37	2.25	2.17	2.10	2.04	1.99
120	3.92	3.07	2.68	2.45	2.29	2.18	2.09	2.02	1.96	1.91
∞	3.84	3.00	2.60	2.37	2.21	2.10	2.01	1.94	1.88	1.83

TABLE 4(a) (continued)
F distribution ($\alpha = 0.05$)

	Degrees of Freedom for Numerator								
	12	15	20	24	30	40	60	120	∞
1	243.9	245.9	248	249.1	250.1	251.1	252.2	253.3	254.3
2	19.4	19.4	19.4	19.5	19.5	19.5	19.5	19.5	19.5
3	8.74	8.70	8.66	8.64	8.62	8.59	8.57	8.55	8.53
4	5.91	5.86	5.80	5.77	5.75	5.72	5.69	5.66	5.63
5	4.68	4.62	4.56	4.53	4.50	4.46	4.43	4.40	4.37
6	4.00	3.94	3.87	3.84	3.81	3.77	3.74	3.70	3.67
7	3.57	3.51	3.44	3.41	3.38	3.34	3.30	3.27	3.23
8	3.28	3.22	3.15	3.12	3.08	3.04	3.01	2.97	2.93
9	3.07	3.01	2.94	2.90	2.86	2.83	2.79	2.75	2.71
10	2.91	2.85	2.77	2.74	2.70	2.66	2.62	2.58	2.54
11	2.79	2.72	2.65	2.61	2.57	2.53	2.49	2.45	2.40
12	2.69	2.62	2.54	2.51	2.47	2.43	2.38	2.34	2.30
13	2.60	2.53	2.46	2.42	2.38	2.34	2.30	2.25	2.21
14	2.53	2.46	2.39	2.35	2.31	2.27	2.22	2.18	2.13
15	2.48	2.40	2.33	2.29	2.25	2.20	2.16	2.11	2.07
16	2.42	2.35	2.28	2.24	2.19	2.15	2.11	2.06	2.01
17	2.38	2.31	2.23	2.19	2.15	2.10	2.06	2.01	1.96
18	2.34	2.27	2.19	2.15	2.11	2.06	2.02	1.97	1.92
19	2.31	2.23	2.16	2.11	2.07	2.03	1.98	1.93	1.88
20	2.28	2.20	2.12	2.08	2.04	1.99	1.95	1.90	1.84
21	2.25	2.18	2.10	2.05	2.01	1.96	1.92	1.87	1.81
22	2.23	2.15	2.07	2.03	1.98	1.94	1.89	1.84	1.78
23	2.20	2.13	2.05	2.01	1.96	1.91	1.86	1.81	1.76
24	2.18	2.11	2.03	1.98	1.94	1.89	1.84	1.79	1.73
25	2.16	2.09	2.01	1.96	1.92	1.87	1.82	1.77	1.71
30	2.09	2.01	1.93	1.89	1.84	1.79	1.74	1.68	1.62
40	2.00	1.92	1.84	1.79	1.74	1.69	1.64	1.58	1.51
60	1.92	1.84	1.75	1.70	1.65	1.59	1.53	1.47	1.39
120	1.83	1.75	1.66	1.61	1.55	1.50	1.43	1.35	1.25
∞	1.75	1.67	1.57	1.52	1.46	1.39	1.32	1.22	1.00