UNIVERSITY OF SWAZILAND

DEPARTMENT OF STATISTICS AND DEMOGRAPHY

SUPPLEMENTARY EXAMINATION, 2010/11

COURSE TITLE:	MATHEMATICS FOR STATISTICS
COURSE CODE:	ST 202
TIME ALLOWED:	TWO (2) HOURS
INSTRUCTION:	ANSWER ANY THREE QUESTIONS ALL QUESTIONS CARRY EQUAL MARKS (20 MARKS)
SPECIAL REQUIREMENTS:	SCIENTIFIC CALCULATORS

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Question 1

If x is a **Gamma-distributed** random variable (with $\alpha = 2$ and $\beta = 1$), its probability density function is given by

$$f(x) = xe^{-x}, \text{ for } x \geq 0$$

(a) Determine all critical values and critical points of $f'(x)$ to find the relative/absolute maxima and minima of this function, and then indicate the direction of the slope around each extremum.

(10 marks)

(b) Determine all critical values and critical points of $f''(x)$ to locate any point(s) of inflection for the function, and then indicate the concavity around the point(s).

(5 marks)

(c) Now, sketch the graph of the density function. [Hint: Note that $x \geq 0$, which means that $f(x) \geq 0$ as well.] (Be sure to mark and label any axis intercepts along with critical points of the function.)

(5 marks)

Question 2

(a) A fast-food restaurant's profit function for hamburgers is given by:

$P = 2.44x - x^2/20,000 - 5,000, \quad 0 \leq x \leq 50,000$

Find the sales level that yields a maximum profit and determine the maximum profit.

(5 marks)

(b) The marketing department of a business has determined that the demand for a product can be modelled by

$p = 50/\sqrt{x}$,

The cost of producing x units is given by C=0.5x + 500. What price will yield a maximum profit?

(7 marks)

(c) Find the second derivative of the function

$f(x) = x \ln\sqrt{x} + 2x$ **(3 marks)**

(d) Find the first partial derivatives of $f(x,y)=xe^{xy}$ and evaluate it at the point (1, ln2)

(5 marks)

Question 3

(a) A baseball fan examined the record of a favourite baseball player's performance during his last 50 games. The numbers of games in which the player obtained zero, one, two, three, and four hits are recorded in the table shown below:

Number of hits (X):	0	1	2	3	4
Frequency (f):	14	26	7	2	1

(i) Tabulate the probability distribution of X; where x is the number of hits.
(ii) Use the table in part (a) to find $P(1 \leq X \leq 3)$
(iii) Determine E(X), V(X), and σ.

(2+3+5 marks)

(b) (i) Determine whether the following function *f* represents a probability density function over the given interval:

$f(x) = 1/3e^{-x/3}$, $[0,\infty]$ **(2 marks)**

(ii) The daily demand 'X' for water (in millions of gallons) in a town is a random variable with the probability density function $f(x) = 1/9xe^{-x/3}$, $[0,\infty]$

Determine the expected value and standard deviation of the demand. Also find the probability that the demand is greater than 4 million gallons on a given day.

(2+ 4+2 marks)

Question 4

(a) Write the following system of equations in matrix form and use determinants to solve for x, y and z.

$$2x + 5y + 4z = 4$$
$$x + 4y + 3z = 1$$
$$x - 3y - 2z = 5$$

(7 marks)

(b) Solve the following system of equations using the Gauss-Jordan elimination method:

$$x + y + 2z = 9$$
$$2x + 4y - 3z = 1$$
$$3x + 6y - 5z = 0$$

(7 marks)

(c) Suppose $x = \begin{bmatrix} 2 \\ 0 \\ -4 \end{bmatrix}$ and $y = \begin{bmatrix} 0 \\ -1 \\ -3 \end{bmatrix}$

Use vector algebra to find the least squares regression line through the set of points determined by vectors x and y.

(6 marks)

Question 5

(a) Find eigenvalues and eigenvectors of matrix $A = \begin{bmatrix} 2 & 1 & 0 \\ 0 & 1 & 0 \\ -1 & 0 & 3 \end{bmatrix}$

(10 marks)

(b) Find the adjoint of the following matrix A and use the adjoint to find the inverse of this matrix.

$$A = \begin{bmatrix} 1 & 3 & 3 \\ 1 & 4 & 3 \\ 1 & 3 & 4 \end{bmatrix}$$

(10 marks)

END OF EXAM!!