UNIVERSITY OF SWAZILAND

SUPPLEMENTARY EXAMINATION PAPER 2011

TITLE OF PAPER	:	INFERENTIAL STATISTICS
COURSE CODE	:	ST 220
TIME ALLOWED	:	TWO (2) HOURS
REQUIREMENTS	:	CALCULATOR AND STATISTICAL TABLES
INSTRUCTIONS	:	THIS PAPER HAS SIX (6) QUESTIONS AND TWO SECTIONS. ANSWER ALL QUESTIONS IN SECTION ONE, ANY THREE (3) QUESTIONS IN SECTION TWO

SECTION ONE

(ANSWER ALL QUESTIONS)

Question 1 [10 marks, 1 mark each]

Choose the correct answer from the alternatives provided.

1. Which of the following is not a property of the normal distribution?
   - (a) It is symmetric about its mean
   - (b) It is bell-shaped
   - (c) It is common
   - (d) It is unimodal

2. The government claims that students earn an average of SZL4500 during their summer break from studies. A random sample of students gave a sample average of SZL3975 and a 95% confidence interval was found to be (SZL3525 $< \mu <$ SZL4425). This interval is interpreted to mean that:
   - (a) if the study were to be repeated many times, there is a 95% probability that the true average summer earnings is not SZL4500 as the government claims.
   - (b) because our specific confidence interval does not contain the value SZL4500 there is a 95% probability that the true average summer earnings is not SZL4500.
   - (c) if we were to repeat our survey many times, then about 95% of all the confidence intervals will contain the value SZL4500.
   - (d) if we repeat our survey many times, then about 95% of our confidence intervals will contain the true value of the average earnings of students.
   - (e) there is a 95% probability that the true average earnings are between SZL3525 and SZL4425 for all students.

3. Which of the following statements about confidence intervals is *incorrect*?
   - (a) If we keep the sample size fixed, the confidence interval gets wider as we increase the confidence coefficient.
   - (b) A confidence interval for a mean always contains the sample mean.
   - (c) If we keep the confidence coefficient fixed, the confidence interval gets narrower as we increase the sample size.
   - (d) If the population standard deviation increases, the confidence interval decreases in width.
   - (e) If the confidence intervals for two means do not overlap very much, there is evidence that the two population means are different.

4. A 95 percent confidence interval for the mean time taken to process new insurance policies is (11, 12) days. This interval can be interpreted to mean that:
   - (a) only 5 percent of all policies take less than 11 or more than 12 days to process
   - (b) only 5 percent of all policies take between 11 and 12 days to process

(c) about 95 out of every 100 such intervals constructed from random samples of the same size will contain the population mean processing time

(d) the probability is .95 that all policies take between 11 and 12 days to process

(e) none of the above

5. A turkey producer knows from previous experience that profits are maximized by selling turkeys when their average weight is 12 kilograms. Before determining whether to put all their full grown turkeys on the market this month, the producer wishes to estimate their mean weight. Prior knowledge indicates that turkey weights have a standard deviation of around 1.5 kilograms. The number of turkeys that must be sampled in order to estimate their true mean weight to within 0.5 kilograms with 95% confidence is:

   (a) 35
   (b) 5
   (c) 65
   (d) 10
   (e) 150

6. A confidence statement includes what two things?

   (a) margin of error and bias
   (b) bias and variability
   (c) bias and confidence level
   (d) confidence level and margin of error

7. I read an advertisement recently in which a credit card company promised that I could reduce my debt by 150 percent. Which of the following statements is (are) true?

   (a) This is possible if my debt is more than 150 dollars.
   (b) This is possible if my debt has recently increased by at least 150 percent.
   (c) The company's claim makes no sense.
   (d) Both (a) and (b).

8. The alternative hypothesis for the Chi-square test of independence is that the variables are

   (a) dependent
   (b) related
   (c) independent
   (d) always zero

9. The diameter of ball bearings are known to be normally distributed with unknown mean and variance. A random sample of size 25 gave a mean 2.5 cm. The 95% confidence interval had length 4 cm. Then

   (a) The sample variance is 4.86.
   (b) The sample variance is 26.03.

(c) The population variance is 4.84.

(d) The population variance is 23.47.

(e) The sample variance is 23.47.

10. The 0.01 level of significance is used in an experiment and a two-tailed hypothesis test applied. Computed z is found to be -2.0. This indicates:

    (a) H_0 should be accepted

    (b) We should reject H_0 and accept H_1

    (c) We should have used the 0.05 level of significance.

    (d) None of these is correct.

Question 2 **[5 marks, 1 mark each]**

State whether each of these statements is true or false, giving brief reasons why this is so (*Note that no marks will be awarded for a simple true/false reply*)

1. When using a large random sample, we cannot assume that its mean forms part of a normal distribution.
2. The least squares regression line minimizes the sum of absolute deviations.
3. The power of a test is the probability of a type 2 error.
4. If two variables are correlated then they must have a linear relationship.
5. The sampling distribution of the mean is distributed the same way as the original observations.

SECTION TWO

(ANSWER ANY THREE QUESTIONS)

Question 3 [20 marks, 6+4+6+4]

(a) Music Technologies, an electronics retail company in Durban has kept records of the number of ipods sold within a week of placing advertisements in the *Mercury*. The following table shows the *number of ipods sold* and the corresponding *number of advertisements placed* in the *Mercury* for 12 randomly selected weeks over the past year.

Ads	4	4	3	2	5	2	4	3	5	5	3	4
Sales	26	28	24	18	35	24	36	25	31	37	30	32

(i) Estimate the linear regression line ($\sum x^2 = 174$, $\sum xy = 1324$ and $\sum y^2 = 10336$).

(ii) Compute and interpret the coefficient of determination.

(iii) Is the *relationship* between the number of *newspaper advertisements* placed and *ipod sales* meaningful (or significant)? Use $\alpha = 0.05$.

(b) Every morning, Duncan bakes 30 scones to sell in his cafe'. If any scones are unsold at the end of the day, Duncan throws them away. The number of scones requested during a day may be modelled by a Poisson distribution with mean 27. Estimate the probability that Duncan does not have enough scones to satisfy all the requests on a particular day (*Use normal approximation to the Poisson distribution*).

Question 4 [20 marks, 8+8+4]

(a) It is suspected that the number of faulty products produced by a manufacturing industry varies depending on which day of the week they were made. In order to test this a random sample of 200 products is taken from the warehouse, and after inspection yields the following results.

	Day of manufacture		
Product	**Monday**	**Tuesday-Thursday**	**Friday**
Perfect	32	94	34
Faulty	3	21	16

Use the χ^2 test to decide if these results indicate that there is a connection between faulty products and when they were made.

(b) The production manager of Raylite batteries, a car battery manufacturer, wants to know whether the three machines used for this process (labelled A, B and C) produce equal amount of rejects. A random sample of shifts for each machine was selected and the number of rejects produced per shift was recorded.

Machine A	Machine B	Machine C
11	7	14
9	10	13
6	8	11
12	13	16
14		16
11		

Can the production manager of Raylite batteries concluded that the three machines used to manufacture car batteries produce rejects at the *same average rate* per shift (NB: $SS_T = 129.6$). Use $\alpha = 0.05$ and show the ANOVA table.

Question 5 **[20 marks, 8+7+5]**

(a) In a study of the effectiveness of physical exercise in weigh reduction, a group of 16 persons engaged in a prescribed program of physical exercise for one month showed the following results:

Subject	Weight before (pounds)	Weight after (pounds)
1	209	196
2	178	171
3	169	170
4	212	207
5	180	177
6	192	190
7	158	159
8	180	180

Use the 0.01 level of significance to test the null hypothesis that the prescribed program of exercise is not effective in reducing weight.

(b) The manager of a power tool rental company wants to know whether the true average number of power tools rented per day is 25.0. If in a random sample of 36 days the average number of power tools rented is 22.8 with a standard deviation of 6.9 rentals, is there, at the 0.05 level of significance, sufficient evidence to reject the null hypothesis $\mu = 25.0$?

(c) A crossword puzzle enthusiast has exactly 30 minutes to complete a puzzle each day during her morning to commute on the train. She knows from experience that the puzzle published in newspaper A takes an average of 25.2 minutes to complete with a standard deviation of 3.9 minutes. The puzzle published in newspaper B also takes an average of 25.2 minutes but it has a standard deviation of 1.9 minutes. State with reason, which of these two newspapers should she buy in order to maximize her chances of completing the crossword puzzle.

Question 6 [20 marks, 10+4+6]

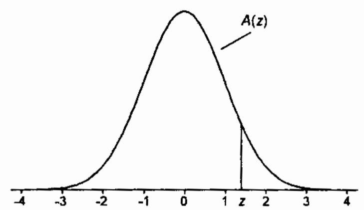
(a) Ten randomly selected oil wells in a large field of oil wells produced 21, 19, 20, 22, 24, 21, 19, 22, 22, and 20 barrels of crude oil per day. Is this evidence at the 0.01 level of significance that the oil wells are not producing an average of 22.5 barrels of crude oil per day?

(b) Suppose the probability is 0.30 that any given student in a large class can provide the answer to an assigned problem. What is the probability that the fourth student randomly selected by the instructor will be the first one who can provide the answer to the problem?

(c) A careful analysis of the causes for absences in a certain factory shows that the probability that an employee will be absent because of substance abuse is 0.03; the probability that the factory manager correctly attributes the absence to substance abuse is 0.80, and the probability that the factor manager incorrectly attributes the absence of substance abuse is 0.05. What is the probability that an absence is attributed to substance abuse by the factory manager is actually due to substance abuse?

TABLE A.1

Cumulative Standardized Normal Distribution



$A(z)$ is the integral of the standardized normal distribution from $-\infty$ to z (in other words, the area under the curve to the left of z). It gives the probability of a normal random variable not being more than z standard deviations above its mean. Values of z of particular importance:

z	$A(z)$	
1.645	0.9500	Lower limit of right 5% tail
1.960	0.9750	Lower limit of right 2.5% tail
2.326	0.9900	Lower limit of right 1% tail
2.576	0.9950	Lower limit of right 0.5% tail
3.090	0.9990	Lower limit of right 0.1% tail
3.291	0.9995	Lower limit of right 0.05% tail

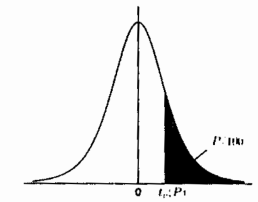
z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998
3.5	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998
3.6	0.9998	0.9998	0.9999							

Percentage Points of the t-Distribution

This table gives the percentage points $t_\nu(P)$ for various values of P and degrees of freedom ν, as indicated by the figure to the right.

The lower percentage points are given by symmetry as $-t_\nu(P)$, and the probability that $|t| \geq t_\nu(P)$ is $2P/100$.

The limiting distribution of t as $\nu \to \infty$ is the normal distribution with zero mean and unit variance.



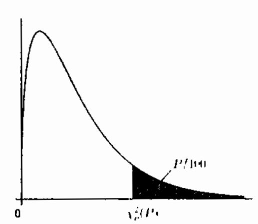
ν	Percentage points P						
	10	5	2.5	1	0.5	0.1	0.05
1	3.078	6.314	12.706	31.821	63.657	318.309	636.619
2	1.886	2.920	4.303	6.965	9.925	22.327	31.599
3	1.638	2.353	3.182	4.541	5.841	10.215	12.924
4	1.533	2.132	2.776	3.747	4.604	7.173	8.610
5	1.476	2.015	2.571	3.365	4.032	5.893	6.869
6	1.440	1.943	2.447	3.143	3.707	5.208	5.959
7	1.415	1.895	2.365	2.998	3.499	4.785	5.408
8	1.397	1.860	2.306	2.896	3.355	4.501	5.041
9	1.383	1.833	2.262	2.821	3.250	4.297	4.781
10	1.372	1.812	2.228	2.764	3.169	4.144	4.587
11	1.363	1.796	2.201	2.718	3.106	4.025	4.437
12	1.356	1.782	2.179	2.681	3.055	3.930	4.318
13	1.350	1.771	2.160	2.650	3.012	3.852	4.221
14	1.345	1.761	2.145	2.624	2.977	3.787	4.140
15	1.341	1.753	2.131	2.602	2.947	3.733	4.073
16	1.337	1.746	2.120	2.583	2.921	3.686	4.015
18	1.330	1.734	2.101	2.552	2.878	3.610	3.922
21	1.323	1.721	2.080	2.518	2.831	3.527	3.819
25	1.316	1.708	2.060	2.485	2.787	3.450	3.725
30	1.310	1.697	2.042	2.457	2.750	3.385	3.646
40	1.303	1.684	2.021	2.423	2.704	3.307	3.551
50	1.299	1.676	2.009	2.403	2.678	3.261	3.496
70	1.294	1.667	1.994	2.381	2.648	3.211	3.435
100	1.290	1.660	1.984	2.364	2.626	3.174	3.390
∞	1.282	1.645	1.960	2.326	2.576	3.090	3.291

Percentage Points of the χ^2-Distribution

This table gives the percentage points $\chi^2_\nu(P)$ for various values of P and degrees of freedom ν, as indicated by the figure to the right.

If X is a variable distributed as χ^2 with ν degrees of freedom, $P/100$ is the probability that $X > \chi^2_\nu(P)$.

For $\nu > 100$, $\sqrt{2X}$ is approximately normally distributed with mean $\sqrt{2\nu - 1}$ and unit variance.



Percentage points P

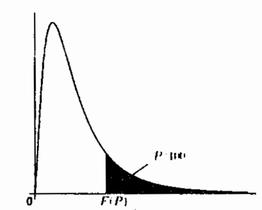
ν	10	5	2.5	1	0.5	0.1	0.05
1	2.706	3.841	5.024	6.635	7.879	10.828	12.116
2	4.605	5.991	7.378	9.210	10.597	13.816	15.202
3	6.251	7.815	9.348	11.345	12.838	16.266	17.730
4	7.779	9.488	11.143	13.277	14.860	18.467	19.997
5	9.236	11.070	12.833	15.086	16.750	20.515	22.105
6	10.645	12.592	14.449	16.812	18.548	22.458	24.103
7	12.017	14.067	16.013	18.475	20.278	24.322	26.018
8	13.362	15.507	17.535	20.090	21.955	26.124	27.868
9	14.684	16.919	19.023	21.666	23.589	27.877	29.666
10	15.987	18.307	20.483	23.209	25.188	29.588	31.420
11	17.275	19.675	21.920	24.725	26.757	31.264	33.137
12	18.549	21.026	23.337	26.217	28.300	32.909	34.821
13	19.812	22.362	24.736	27.688	29.819	34.528	36.478
14	21.064	23.685	26.119	29.141	31.319	36.123	38.109
15	22.307	24.996	27.488	30.578	32.801	37.697	39.719
16	23.542	26.296	28.845	32.000	34.267	39.252	41.308
17	24.769	27.587	30.191	33.409	35.718	40.790	42.879
18	25.989	28.869	31.526	34.805	37.156	42.312	44.434
19	27.204	30.144	32.852	36.191	38.582	43.820	45.973
20	28.412	31.410	34.170	37.566	39.997	45.315	47.498
25	34.382	37.652	40.646	44.314	46.928	52.620	54.947
30	40.256	43.773	46.979	50.892	53.672	59.703	62.162
40	51.805	55.758	59.342	63.691	66.766	73.402	76.095
50	63.167	67.505	71.420	76.154	79.490	86.661	89.561
80	96.578	101.879	106.629	112.329	116.321	124.839	128.261

5 Percent Points of the F-Distribution

This table gives the percentage points $F_{\nu_1,\nu_2}(P)$ for $P = 0.05$ and degrees of freedom ν_1, ν_2, as indicated by the figure to the right.

The lower percentage points, that is the values $F'_{\nu_1,\nu_2}(P)$ such that the probability that $F < F'_{\nu_1,\nu_2}(P)$ is equal to $P/100$, may be found using the formula

$$F'_{\nu_1,\nu_2}(P) = 1/F_{\nu_1,\nu_2}(P)$$



ν_2 \ ν_1	1	2	3	4	5	6	12	24	∞
2	18.513	19.000	19.164	19.247	19.296	19.330	19.413	19.454	19.496
3	10.128	9.552	9.277	9.117	9.013	8.941	8.745	8.639	8.526
4	7.709	6.944	6.591	6.388	6.256	6.163	5.912	5.774	5.628
5	6.608	5.786	5.409	5.192	5.050	4.950	4.678	4.527	4.365
6	5.987	5.143	4.757	4.534	4.387	4.284	4.000	3.841	3.669
7	5.591	4.737	4.347	4.120	3.972	3.866	3.575	3.410	3.230
8	5.318	4.459	4.066	3.838	3.687	3.581	3.284	3.115	2.928
9	5.117	4.256	3.863	3.633	3.482	3.374	3.073	2.900	2.707
10	4.965	4.103	3.708	3.478	3.326	3.217	2.913	2.737	2.538
11	4.844	3.982	3.587	3.357	3.204	3.095	2.788	2.609	2.404
12	4.747	3.885	3.490	3.259	3.106	2.996	2.687	2.505	2.296
13	4.667	3.806	3.411	3.179	3.025	2.915	2.604	2.420	2.206
14	4.600	3.739	3.344	3.112	2.958	2.848	2.534	2.349	2.131
15	4.543	3.682	3.287	3.056	2.901	2.790	2.475	2.288	2.066
16	4.494	3.634	3.239	3.007	2.852	2.741	2.425	2.235	2.010
17	4.451	3.592	3.197	2.965	2.810	2.699	2.381	2.190	1.960
18	4.414	3.555	3.160	2.928	2.773	2.661	2.342	2.150	1.917
19	4.381	3.522	3.127	2.895	2.740	2.628	2.308	2.114	1.878
20	4.351	3.493	3.098	2.866	2.711	2.599	2.278	2.082	1.843
25	4.242	3.385	2.991	2.759	2.603	2.490	2.165	1.964	1.711
30	4.171	3.316	2.922	2.690	2.534	2.421	2.092	1.887	1.622
40	4.085	3.232	2.839	2.606	2.449	2.336	2.003	1.793	1.509
50	4.034	3.183	2.790	2.557	2.400	2.286	1.952	1.737	1.438
100	3.936	3.087	2.696	2.463	2.305	2.191	1.850	1.627	1.283
∞	3.841	2.996	2.605	2.372	2.214	2.099	1.752	1.517	1.002