UNIVERSITY OF SWAZILAND

SUPPLEMENTARY EXAMINATION PAPER 2011

TITLE OF PAPER : TOPICS IN STATISTICS (ADVANCED CATEGORICAL DATA ANALYSIS)

COURSE CODE : ST 405

TIME ALLOWED : THREE (3) HOURS

REQUIREMENTS : CALCULATOR AND STATISTICAL TABLES

INSTRUCTIONS : ANSWER ANY FOUR QUESTIONS

(ALL QUESTION CARRY EQUAL MARKS)

Question 1

A construction company makes concrete beams from cement mixed with gravel. The company wishes to compare the relative strengths of the concrete made from the different types of cement available. There are four different types of cement and three types of gravel. From each of the 12 different combinations of cement and gravel, equal test beams were made and tested for all combinations. The following table gives the number of destroyed beam per combination.

		Cement type			
		A	*B*	*C*	*D*
Gravel type	*1*	10	12	16	8
	2	14	15	18	10
	3	18	22	26	20

Carry out a suitable analysis of these data and write a report for the manager of the construction company who is not trained in statistics.

(20 Marks)

Question 2

In a psychological experiment to investigate the effects of stress on the ability to perform simple tasks, 90 volunteers were asked to perform a simple puzzle assembly task under normal conditions and under conditions of stress. Each subject was given three minutes to complete the task and on each occasion it was recorded whether or not they were successful. The order of the conditions under which each subject performed the task was determined at random. The results of the experiment are given in the following table.

		Normal conditions	
		Successful	*Unsuccessful*
Under stress	*Successful*	52	9
	Unsuccessful	20	9

a) Apply McNemar's test to the above results.

(8 Marks)

b) Use the conventional 2×2 chi-squared test on the above results, without using Yates' correction. How does any difference in the outcome of the two tests (a) and (b) arise?

(12 Marks)

Question 3

In a trial of anti-inflammatory drugs in the treatment of eczema, each member of a sample of 500 adults suffering from eczema was allocated at random to receive one of two treatments. After one month, the patients were asked to state whether their eczema improved. They replied as follows.

	Improved	*Not Improved*
Treatment A	205	45
Treatment B	180	70

Test the statistical significance of the saturated log-linear model for the data given in the above table.

Question 4

A marketing research firm was engaged by an automobile manufacturer to conduct a pilot study to examine the feasibility of using logistic regression for ascertaining the likelihood that a family will purchase a new car during the next year. A random sample of 33 suburban families was selected. Data on annual family income and the current age of the oldest family automobile were obtained. A follow-up interview conducted 12 months later was used to determine whether the family actually purchased a new car or did not purchase a new car. The model below was fitted;

```
> data <- read.table("car_table.txt", header=T)
> attach(data)
> glm_ <- glm(purchase~income+age, family="binomial")
> summary(glm_)

Call:
glm(formula = purchase ~ income + age, family = "binomial")

Deviance Residuals:
    Min       1Q   Median       3Q      Max
-1.6119  -1.0141  -0.7480    1.2611   1.9644

Coefficients:
             Estimate Std. Error z value Pr(>|z|)
(Intercept) -4.78938    2.10195  -2.279   0.0242 *
income       0.06773    0.02906   2.414   0.0158 *
age          0.02163    0.01907   1.135   0.1249
```

a) State the response function.

(3 Marks)

b) Using the logistic regression model output above (coefficients) advise appropriately.

(10 marks)

c) What is the estimated probability that a family with annual income of E50,000 and an oldest car of 3 years will purchase a new car next year?

(3 marks)

d) Using the output below, state whether the two-factor interaction effect between annual family income and age of oldest automobile should be added to the regression model containing family income and age of oldest automobile as first-order terms; use $\alpha = 0.05$. What is the approximate p-value?

(4 marks)

```
> glm3 <- update(glm2, .~.+age:income)
> summary(glm3)

Call:
glm(formula = purchase ~ income + age + income:age, family = "binomial")

Deviance Residuals:
    Min       1Q   Median       3Q      Max
-1.4196  -1.0022  -1.0334   1.0731   1.3924

Coefficients:
             Estimate Std. Error z value Pr(>|z|)
(Intercept) -2.371291   2.862477  -1.828    0.417
income       0.011316   0.064770   0.175    1.984
age          0.313160   0.899512   0.348    0.718
income:age   0.011161   1.026493   1.087    0.174
```

Question 5

A cohort of subjects, some non-smokers and others smokers, was observed for several years. The number of cases of cancer of the lung diagnosed among the different categories was recorded. Data regarding the number of years of smoking were also obtained from each individual. For each category the person-years of observation were calculated. The investigators wish to address the question of the relative risks of smoking. In the observed data the average number of cigarettes smoked per day represents the daily dose, and the years of smoking together with the average number of cigarettes smoked daily represents the total dose inhaled over time. The results of the analysis are given below:

```
 Response variate: CASES
     Distribution: Poisson
    Link function: Log
     Fitted terms: Constant, PERSONYR, CIGS_DAY, SMOKING_


*** Summary of analysis ***

                                                mean  deviance
              d.f.      deviance      deviance      ratio
Regression       3 63.168816931 21.056272310      21.06
Residual        31 74.122027311  2.391033139
Total           34 137.290844242  4.037966007

Change          -3 -63.168816931 21.056272310      21.06
* MESSAGE: ratios are based on dispersion parameter with value 1


*** Estimates of regression coefficients ***

                  estimate          s.e.       t(*)
Constant            -4.669         0.988      -4.72
PERSONYR          0.000410      0.000104       3.94
CIGS_DAY            0.0559        0.0100       5.58
SMOKING_            0.0868        0.0166       5.34
*    MESSAGE: s.e.s are based on dispersion parameter with value 1
```

Justify the method of analysis, state the model, interpret all relevant estimates and write a short report. (20 Marks)

Question 6

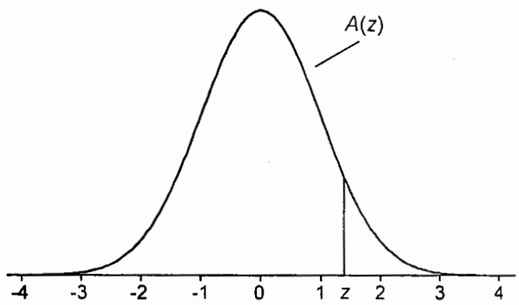
```
> fitl=glm(Freq ~ sex*agegrp+polviews*sex+polviews*agegrp,
+           family=poisson, data=IP)
> anova(fitl,test='Chisq')
Analysis of Deviance Table

Model: poisson, link: log

Response: Freq

Terms added sequentially (first to last)


                Df Deviance Resid. Df Resid. Dev P(>|Chi|)
NULL                              53     366.00
sex              1     2.44        54     363.56     0.12
agegrp           3     1.05        51     363.52     1.00
polviews         6   466.16        45      96.27 1.536e-97
sex:agegrp       3    11.31        42      87.45     0.01
sex:polviews     6     3.66        36      83.79     0.72
agegrp:polviews 18    62.17        18      21.61 9.045e-07
```

State model and justify the method used in the above analysis. Also comment on each of the variables in the model.

(20 marks)

TABLE A.1

Cumulative Standardized Normal Distribution



$A(z)$ is the integral of the standardized normal distribution from $-\infty$ to z (in other words, the area under the curve to the left of z). It gives the probability of a normal random variable not being more than z standard deviations above its mean. Values of z of particular importance:

z	$A(z)$	
1.645	0.9500	Lower limit of right 5% tail
1.960	0.9750	Lower limit of right 2.5% tail
2.326	0.9900	Lower limit of right 1% tail
2.576	0.9950	Lower limit of right 0.5% tail
3.090	0.9990	Lower limit of right 0.1% tail
3.291	0.9995	Lower limit of right 0.05% tail

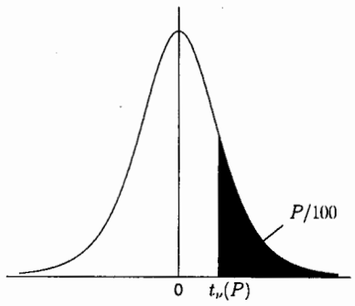
z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998
3.5	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998
3.6	0.9998	0.9998	0.9999							

Percentage Points of the t-Distribution

This table gives the percentage points $t_\nu(P)$ for various values of P and degrees of freedom ν, as indicated by the figure to the right.

The lower percentage points are given by symmetry as $-t_\nu(P)$, and the probability that $|t| \geq t_\nu(P)$ is $2P/100$.

The limiting distribution of t as $\nu \to \infty$ is the normal distribution with zero mean and unit variance.



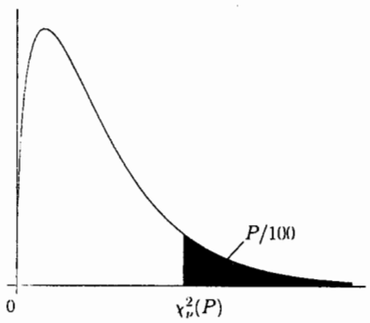
	Percentage points P						
ν	**10**	**5**	**2.5**	**1**	**0.5**	**0.1**	**0.05**
1	3.078	6.314	12.706	31.821	63.657	318.309	636.619
2	1.886	2.920	4.303	6.965	9.925	22.327	31.599
3	1.638	2.353	3.182	4.541	5.841	10.215	12.924
4	1.533	2.132	2.776	3.747	4.604	7.173	8.610
5	1.476	2.015	2.571	3.365	4.032	5.893	6.869
6	1.440	1.943	2.447	3.143	3.707	5.208	5.959
7	1.415	1.895	2.365	2.998	3.499	4.785	5.408
8	1.397	1.860	2.306	2.896	3.355	4.501	5.041
9	1.383	1.833	2.262	2.821	3.250	4.297	4.781
10	1.372	1.812	2.228	2.764	3.169	4.144	4.587
11	1.363	1.796	2.201	2.718	3.106	4.025	4.437
12	1.356	1.782	2.179	2.681	3.055	3.930	4.318
13	1.350	1.771	2.160	2.650	3.012	3.852	4.221
14	1.345	1.761	2.145	2.624	2.977	3.787	4.140
15	1.341	1.753	2.131	2.602	2.947	3.733	4.073
16	1.337	1.746	2.120	2.583	2.921	3.686	4.015
18	1.330	1.734	2.101	2.552	2.878	3.610	3.922
21	1.323	1.721	2.080	2.518	2.831	3.527	3.819
25	1.316	1.708	2.060	2.485	2.787	3.450	3.725
30	1.310	1.697	2.042	2.457	2.750	3.385	3.646
40	1.303	1.684	2.021	2.423	2.704	3.307	3.551
50	1.299	1.676	2.009	2.403	2.678	3.261	3.496
70	1.294	1.667	1.994	2.381	2.648	3.211	3.435
100	1.290	1.660	1.984	2.364	2.626	3.174	3.390
∞	1.282	1.645	1.960	2.326	2.576	3.090	3.291

Percentage Points of the χ^2-Distribution

This table gives the percentage points $\chi^2_\nu(P)$ for various values of P and degrees of freedom ν, as indicated by the figure to the right.

If X is a variable distributed as χ^2 with ν degrees of freedom, $P/100$ is the probability that $X \geq \chi^2_\nu(P)$.

For $\nu > 100$, $\sqrt{2X}$ is approximately normally distributed with mean $\sqrt{2\nu - 1}$ and unit variance.



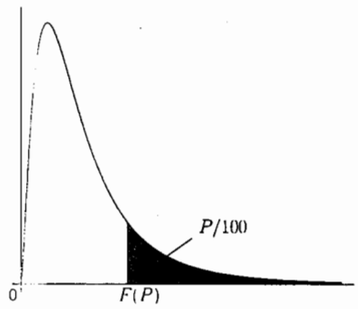
	Percentage points P						
ν	**10**	**5**	**2.5**	**1**	**0.5**	**0.1**	**0.05**
1	2.706	3.841	5.024	6.635	7.879	10.828	12.116
2	4.605	5.991	7.378	9.210	10.597	13.816	15.202
3	6.251	7.815	9.348	11.345	12.838	16.266	17.730
4	7.779	9.488	11.143	13.277	14.860	18.467	19.997
5	9.236	11.070	12.833	15.086	16.750	20.515	22.105
6	10.645	12.592	14.449	16.812	18.548	22.458	24.103
7	12.017	14.067	16.013	18.475	20.278	24.322	26.018
8	13.362	15.507	17.535	20.090	21.955	26.124	27.868
9	14.684	16.919	19.023	21.666	23.589	27.877	29.666
10	15.987	18.307	20.483	23.209	25.188	29.588	31.420
11	17.275	19.675	21.920	24.725	26.757	31.264	33.137
12	18.549	21.026	23.337	26.217	28.300	32.909	34.821
13	19.812	22.362	24.736	27.688	29.819	34.528	36.478
14	21.064	23.685	26.119	29.141	31.319	36.123	38.109
15	22.307	24.996	27.488	30.578	32.801	37.697	39.719
16	23.542	26.296	28.845	32.000	34.267	39.252	41.308
17	24.769	27.587	30.191	33.409	35.718	40.790	42.879
18	25.989	28.869	31.526	34.805	37.156	42.312	44.434
19	27.204	30.144	32.852	36.191	38.582	43.820	45.973
20	28.412	31.410	34.170	37.566	39.997	45.315	47.498
25	34.382	37.652	40.646	44.314	46.928	52.620	54.947
30	40.256	43.773	46.979	50.892	53.672	59.703	62.162
40	51.805	55.758	59.342	63.691	66.766	73.402	76.095
50	63.167	67.505	71.420	76.154	79.490	86.661	89.561
80	96.578	101.879	106.629	112.329	116.321	124.839	128.261

5 Percent Points of the F-Distribution

This table gives the percentage points $F_{\nu_1,\nu_2}(P)$ for $P = 0.05$ and degrees of freedom ν_1, ν_2, as indicated by the figure to the right.

The lower percentage points, that is the values $F'_{\nu_1,\nu_2}(P)$ such that the probability that $F \leq F'_{\nu_1,\nu_2}(P)$ is equal to $P/100$, may be found using the formula

$$F'_{\nu_1,\nu_2}(P) = 1/F_{\nu_1,\nu_2}(P)$$



	ν_1								
ν_2	**1**	**2**	**3**	**4**	**5**	**6**	**12**	**24**	∞
2	18.513	19.000	19.164	19.247	19.296	19.330	19.413	19.454	19.496
3	10.128	9.552	9.277	9.117	9.013	8.941	8.745	8.639	8.526
4	7.709	6.944	6.591	6.388	6.256	6.163	5.912	5.774	5.628
5	6.608	5.786	5.409	5.192	5.050	4.950	4.678	4.527	4.365
6	5.987	5.143	4.757	4.534	4.387	4.284	4.000	3.841	3.669
7	5.591	4.737	4.347	4.120	3.972	3.866	3.575	3.410	3.230
8	5.318	4.459	4.066	3.838	3.687	3.581	3.284	3.115	2.928
9	5.117	4.256	3.863	3.633	3.482	3.374	3.073	2.900	2.707
10	4.965	4.103	3.708	3.478	3.326	3.217	2.913	2.737	2.538
11	4.844	3.982	3.587	3.357	3.204	3.095	2.788	2.609	2.404
12	4.747	3.885	3.490	3.259	3.106	2.996	2.687	2.505	2.296
13	4.667	3.806	3.411	3.179	3.025	2.915	2.604	2.420	2.206
14	4.600	3.739	3.344	3.112	2.958	2.848	2.534	2.349	2.131
15	4.543	3.682	3.287	3.056	2.901	2.790	2.475	2.288	2.066
16	4.494	3.634	3.239	3.007	2.852	2.741	2.425	2.235	2.010
17	4.451	3.592	3.197	2.965	2.810	2.699	2.381	2.190	1.960
18	4.414	3.555	3.160	2.928	2.773	2.661	2.342	2.150	1.917
19	4.381	3.522	3.127	2.895	2.740	2.628	2.308	2.114	1.878
20	4.351	3.493	3.098	2.866	2.711	2.599	2.278	2.082	1.843
25	4.242	3.385	2.991	2.759	2.603	2.490	2.165	1.964	1.711
30	4.171	3.316	2.922	2.690	2.534	2.421	2.092	1.887	1.622
40	4.085	3.232	2.839	2.606	2.449	2.336	2.003	1.793	1.509
50	4.034	3.183	2.790	2.557	2.400	2.286	1.952	1.737	1.438
100	3.936	3.087	2.696	2.463	2.305	2.191	1.850	1.627	1.283
∞	3.841	2.996	2.605	2.372	2.214	2.099	1.752	1.517	1.002