**UNIVERSITY OF SWAZILAND**

**FINAL EXAMINATION PAPER 2011**

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|---|---|---|
| **TITLE OF PAPER** | **:** | **MULTIVARIATE ANALYSIS** |
| **COURSE CODE** | **:** | **ST410** |
| **TIME ALLOWED** | **:** | **2 (TWO) HOURS** |
| **REQUIRMENTS** | **:** | **STATISTICAL TABLES AND CALCULATOR** |
| **INSTRUCTIONS** | **:** | **ANSWER ANY 4 (FOUR) QUESTIONS. ALL QUESTIONS CARRY EQUAL MARKS.** |

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## QUESTION ONE.

[ 4 + 10 + 6 + 5 marks ]

1.1 Explain why factor analysis is important in multivariate data analysis.

1.2 Discuss Factor analysis and principal component analysis with respect to the similarities and differences.

1.3 The procedure for a factor analysis can be described in terms of three stages. State these three stages.

1.4 The table below was found by running factor analysis using SPSS. Find the first equation of the un-rotated factor model. Assume $\lambda_1 = 3.112$.

**Component Matrix[a]**

| | Component | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| X1 | -.902 | .032 | -.341 | .017 | -.020 | .024 | -.089 | .244 | .002 |
| X2 | -.662 | -.001 | .630 | .117 | .292 | -.111 | .115 | -.203 | .001 |
| X3 | .434 | -.581 | -.614 | .060 | -.197 | .017 | .128 | -.195 | .001 |
| X4 | .558 | -.147 | -.359 | .024 | .720 | -.115 | -.033 | .070 | .000 |
| X5 | .391 | .326 | .087 | .807 | .052 | .280 | -.011 | .014 | .000 |
| X6 | .673 | .549 | .080 | .174 | -.225 | -.375 | .095 | .091 | .001 |
| X7 | .231 | .744 | -.117 | -.504 | .111 | .227 | .248 | -.012 | .001 |
| X8 | .755 | -.074 | .441 | -.327 | -.039 | .088 | -.337 | -.019 | .001 |
| X9 | .362 | -.695 | .505 | -.043 | -.019 | .079 | .267 | .227 | .000 |

Extraction Method: Principal Component Analysis.

a. 9 components extracted.

## QUESTION TWO.

[ 10 + 3 + 7 + 5 marks ]

2.1 What is Principal component analysis (PCA) and states all important properties of PCA.

2.2 It is not necessary to standardise the variables while doing PCA. But in certain situation, we need to do standardisation of variables for PCA. Explain that situation.

2.3 Show that the covariance matrix becomes correlation matrix if we standardise the variables.

2.4 The following table represent the correlation matrix of seven variables. Examine the table and discuss the suitability of principal component analysis. How many principal components will you have?

**Correlation Matrix**

| | | X1 | X2 | X3 | X4 | X5 | X6 | X7 |
|---|---|---|---|---|---|---|---|---|
| Correlation | X1 | 1.000 | .650 | .095 | .094 | .107 | .009 | .082 |
| | X2 | .650 | 1.000 | -.128 | -.076 | .009 | -.064 | -.026 |
| | X3 | .095 | -.128 | 1.000 | .639 | .683 | .568 | .363 |
| | X4 | .094 | -.076 | .639 | 1.000 | .907 | .598 | .879 |
| | X5 | .107 | .009 | .683 | .907 | 1.000 | .788 | .722 |
| | X6 | .009 | -.064 | .568 | .598 | .788 | 1.000 | .255 |
| | X7 | .082 | -.026 | .363 | .879 | .722 | .255 | 1.000 |

**QUESTION THREE.** [ 5 + 12 + 6 + 2 marks]

Suppose we have three variables in each of the 3 groups with sample sizes $n_A$=3, $n_B$=4 and $n_C$=5. Consider the followings:

$$\bar{x} = \begin{bmatrix} 4 & 13 & 5 \\ 6 & 10 & 6 \\ 3 & 12 & 7 \end{bmatrix}, \quad \bar{X} = \begin{pmatrix} 4.25 \\ 11.58 \\ 6.17 \end{pmatrix} \quad S^2 = \begin{pmatrix} 5.7 \\ 8.6 \\ 1.6 \end{pmatrix}, \quad C^{-1} = \begin{bmatrix} 2.170 & -0.258 & -2.435 \\ -0.258 & 0.156 & 0.148 \\ -2.435 & 0.148 & 3.643 \end{bmatrix}$$

$$\mathbf{W} = \begin{bmatrix} 42.0 & 39.0 & 10.0 \\ 39.0 & 78.0 & 11.0 \\ 10.0 & 11.0 & 10.0 \end{bmatrix}, \quad \& \quad \mathbf{T} = \begin{bmatrix} 62.25 & 24.25 & 8.5 \\ 24.25 & 94.92 & 8.83 \\ 8.5 & 8.83 & 17.67 \end{bmatrix}$$

where $\bar{x}$ is the matrix of means, the first row represents the means of the three variables in group A, etc.; $\bar{X}$ is the vector of means of the three variables; $S^2$ is the vector of variances of the three variables; $C^{-1}$ is the inverse of the pooled covariance matrix of group B and C; W is the within sum of squire matrix and T is the total sum of squire matrix.

3.1 Perform Hotellings' $\mathbf{T}^2$ test considering groups B and C.

3.2 Compute Wilk's $\Lambda$ statistic and $\varphi$. Use $\chi^2$ and $F$ approximation to test the equality of population mean vectors. Specify the null and alternative hypotheses.

3.3 Complete the ANOVA tables, with one extra column for the conclusion, of the one-way analysis of variance procedure to test the equality of variable means for each of those three variables.

3.4 Comment on the results found in part (3.3) and (3.4).

**QUESTION FOUR.** [ 3 + 4 + 3 + 5 + 10 marks ]

4.1 Define the discriminant function analysis and discuss why we do this analysis.

4.2 State the steps to obtain canonical discriminant functions.

4.3 The following table shows the eigenvalues and corresponding eigenvectors of $W^{-1}B$:

| Component | Eigenvalue | Eigenvectors | | | | |
|---|---|---|---|---|---|---|
| | | $X_1$ | $X_2$ | $X_3$ | $X_4$ | $X_5$ |
| 1 | 2.616 | 0.452 | 0.462 | 0.451 | 0.471 | 0.398 |
| 2 | 1.532 | −0.051 | 0.300 | 0.325 | 0.185 | −0.377 |
| 3 | 0.386 | 0.691 | 0.341 | −0.455 | −0.411 | −0.179 |
| 4 | 0.302 | −0.420 | 0.548 | −0.606 | 0.388 | 0.069 |
| 5 | 0.165 | 0.374 | −0.530 | −0.343 | 0.652 | −0.192 |

a. How many groups and variables were considered in this problem?

b. List all the canonical discriminant functions.

c. Assuming that the $i^{th}$ sample size, $n_i = 30$ for all i; test whether each of these functions varies significantly from group to group.

**QUESTION FIVE.** [ 4 + 4 + 4 + 4 + 9 marks ]

The following tables are part of the complete output from running SPSS for a set of variables; not necessarily from the same set of variables. Tables 1-5 are obtained running Factor Analysis and Tables 6-8 are obtained from Discriminant Function Analysis:

**Table 1:**

| Component | Eigenvalues |
|---|---|
| 1 | 3.724 |
| 2 | 1.857 |
| 3 | 0.663 |
| 4 | 0.391 |
| 5 | 0.293 |
| 6 | 0.044 |
| 7 | 0.028 |

**Table 2:**

**Component Matrix[a]**

| | Component | | | | | |
|---|---|---|---|---|---|---|
| | 1 | 2 | 3 | 4 | 5 | 6 |
| X1 | .114 | .902 | .060 | -.246 | -.332 | -.010 |
| X2 | -.059 | .912 | .036 | .232 | .331 | .022 |
| X3 | .763 | -.063 | .349 | -.489 | .228 | .009 |
| X4 | .958 | -.006 | -.241 | .003 | -.023 | .114 |
| X5 | .973 | .035 | .043 | .147 | .009 | -.165 |
| X6 | .752 | -.079 | .533 | .341 | -.141 | .060 |
| X7 | .772 | .041 | -.626 | .017 | .003 | .001 |

Extraction Method: Principal Component Analysis.

a. 6 components extracted.

**Table 3:**

**Component Matrix[a]**

| | Component | |
|---|---|---|
| | 1 | 2 |
| X1 | .114 | .902 |
| X2 | -.059 | .912 |
| X3 | .763 | -.063 |
| X4 | .958 | -.006 |
| X5 | .973 | .035 |
| X6 | .752 | -.079 |
| X7 | .772 | .041 |

Extraction Method: Principal Component Analysis.

a. 2 components extracted.

**Table 4:**

**Rotated Component Matrix[a]**

| | Component | |
|---|---|---|
| | 1 | 2 |
| X1 | .091 | .904 |
| X2 | -.083 | .910 |
| X3 | .764 | -.044 |
| X4 | .958 | .018 |
| X5 | .971 | .060 |
| X6 | .754 | -.060 |
| X7 | .770 | .060 |

Extraction Method: Principal Component Analysis.
Rotation Method: Varimax with Kaiser Normalizatio

a. Rotation converged in 3 iterations.

**Table 5:**

**Component Score Coefficient Matrix**

| | Component | |
|---|---|---|
| | 1 | 2 |
| X1 | .017 | .545 |
| X2 | -.030 | .550 |
| X3 | .211 | -.033 |
| X4 | .264 | .003 |
| X5 | .268 | .028 |
| X6 | .209 | -.043 |
| X7 | .212 | .030 |

Extraction Method: Principal Component Analysis.
Rotation Method: Varimax with Kaiser Normalization.

**Table 6:**

**Wilks' Lambda**

| Test of Function(s) | Wilks' Lambda | Chi-square | df | Sig. |
|---|---|---|---|---|
| 1 through 3 | .031 | 109.114 | 21 | .000 |
| 2 through 3 | .483 | 22.894 | 12 | .029 |
| 3 | .799 | 7.055 | 5 | .217 |

**Table 7:**

**Standardized Canonical Discriminant Function Coefficients**

| | Function | | |
|---|---|---|---|
| | 1 | 2 | 3 |
| X1 | 1.061 | .138 | .026 |
| X2 | -.165 | -.215 | -.153 |
| X3 | -.161 | .304 | 1.481 |
| X4 | .036 | -1.449 | -1.390 |
| X5 | .904 | -2.608 | -1.039 |
| X6 | -.473 | 1.913 | .510 |
| X7 | -.371 | 2.190 | 1.599 |

**Table 8:**

**Canonical Discriminant Function Coefficients**

| | Function | | |
|---|---|---|---|
| | 1 | 2 | 3 |
| X1 | .151 | .020 | .004 |
| X2 | -.010 | -.013 | -.009 |
| X3 | -.009 | .017 | .082 |
| X4 | .003 | -.131 | -.125 |
| X5 | .076 | -.219 | -.087 |
| X6 | -.040 | .161 | .043 |
| X7 | -.020 | .116 | .085 |
| (Constant) | -5.498 | .573 | -.220 |

Unstandardized coefficients

5.1 How many factors will you choose if you wish to use factor analysis method? Explain your answer.

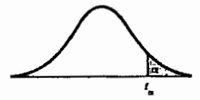
5.2 How many factors will you get in your factor model from Table 2? List the last two equations of your model and compute their communalities.

5.3 Suppose the same data were analyzed using with a restriction on the number of factors. How many factors were chosen in Table 3? List the first two equations of your model and compute their communalities.

5.4 List all equations needed to compute factor scores.

5.5 Write all the discriminant functions and test whether each of those are significant at 5% level of significance.

**Table 5**
Percentage points of the t distributions

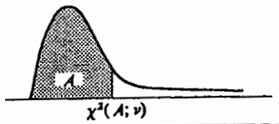



| $t_{.100}$ | $t_{.050}$ | $t_{.025}$ | $t_{.010}$ | $t_{.005}$ | d.f. |
|---|---|---|---|---|---|
| 3.078 | 6.314 | 12.706 | 31.821 | 63.657 | 1 |
| 1.886 | 2.920 | 4.303 | 6.965 | 9.925 | 2 |
| 1.638 | 2.353 | 3.182 | 4.541 | 5.841 | 3 |
| 1.533 | 2.132 | 2.776 | 3.747 | 4.604 | 4 |
| 1.476 | 2.015 | 2.571 | 3.365 | 4.032 | 5 |
| 1.440 | 1.943 | 2.447 | 3.143 | 3.707 | 6 |
| 1.415 | 1.895 | 2.365 | 2.998 | 3.499 | 7 |
| 1.397 | 1.860 | 2.306 | 2.896 | 3.355 | 8 |
| 1.383 | 1.833 | 2.262 | 2.821 | 3.250 | 9 |
| 1.372 | 1.812 | 2.228 | 2.764 | 3.169 | 10 |
| 1.363 | 1.796 | 2.201 | 2.718 | 3.106 | 11 |
| 1.356 | 1.782 | 2.179 | 2.681 | 3.055 | 12 |
| 1.350 | 1.771 | 2.160 | 2.650 | 3.012 | 13 |
| 1.345 | 1.761 | 2.145 | 2.624 | 2.977 | 14 |
| 1.341 | 1.753 | 2.131 | 2.602 | 2.947 | 15 |
| 1.337 | 1.746 | 2.120 | 2.583 | 2.921 | 16 |
| 1.333 | 1.740 | 2.110 | 2.567 | 2.898 | 17 |
| 1.330 | 1.734 | 2.101 | 2.552 | 2.878 | 18 |
| 1.328 | 1.729 | 2.093 | 2.539 | 2.861 | 19 |
| 1.325 | 1.725 | 2.086 | 2.528 | 2.845 | 20 |
| 1.323 | 1.721 | 2.080 | 2.518 | 2.831 | 21 |
| 1.321 | 1.717 | 2.074 | 2.508 | 2.819 | 22 |
| 1.319 | 1.714 | 2.069 | 2.500 | 2.807 | 23 |
| 1.318 | 1.711 | 2.064 | 2.492 | 2.797 | 24 |
| 1.316 | 1.708 | 2.060 | 2.485 | 2.787 | 25 |
| 1.315 | 1.706 | 2.056 | 2.479 | 2.779 | 26 |
| 1.314 | 1.703 | 2.052 | 2.473 | 2.771 | 27 |
| 1.313 | 1.701 | 2.048 | 2.467 | 2.763 | 28 |
| 1.311 | 1.699 | 2.045 | 2.462 | 2.756 | 29 |
| 1.282 | 1.645 | 1.960 | 2.326 | 2.576 | inf. |

From "Table of Percentage Points of the *t*-Distribution." Computed by Maxine Merrington, *Biometrika*, Vol. 32 (1941), p. 300. Reproduced by permission of Professor E. S. Pearson.

**TABLE A.3** Percentiles of the $\chi^2$ Distribution

Entry is $\chi^2(A; \nu)$ where $P\{\chi^2(\nu) \le \chi^2(A; \nu)\} = A$



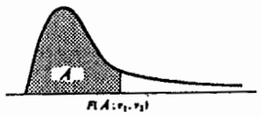

| | $A$ | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| $\nu$ | .005 | .010 | .025 | .050 | .100 | .900 | .950 | .975 | .990 | .995 |
| 1 | $0.0^4393$ | $0.0^3157$ | $0.0^3982$ | $0.0^2393$ | 0.0158 | 2.71 | 3.84 | 5.02 | 6.63 | 7.88 |
| 2 | 0.0100 | 0.0201 | 0.0506 | 0.103 | 0.211 | 4.61 | 5.99 | 7.38 | 9.21 | 10.60 |
| 3 | 0.072 | 0.115 | 0.216 | 0.352 | 0.584 | 6.25 | 7.81 | 9.35 | 11.34 | 12.84 |
| 4 | 0.207 | 0.297 | 0.484 | 0.711 | 1.064 | 7.78 | 9.49 | 11.14 | 13.28 | 14.86 |
| 5 | 0.412 | 0.554 | 0.831 | 1.145 | 1.61 | 9.24 | 11.07 | 12.83 | 15.09 | 16.75 |
| 6 | 0.676 | 0.872 | 1.24 | 1.64 | 2.20 | 10.64 | 12.59 | 14.45 | 16.81 | 18.55 |
| 7 | 0.989 | 1.24 | 1.69 | 2.17 | 2.83 | 12.02 | 14.07 | 16.01 | 18.48 | 20.28 |
| 8 | 1.34 | 1.65 | 2.18 | 2.73 | 3.49 | 13.36 | 15.51 | 17.53 | 20.09 | 21.96 |
| 9 | 1.73 | 2.09 | 2.70 | 3.33 | 4.17 | 14.68 | 16.92 | 19.02 | 21.67 | 23.59 |
| 10 | 2.16 | 2.56 | 3.25 | 3.94 | 4.87 | 15.99 | 18.31 | 20.48 | 23.21 | 25.19 |
| 11 | 2.60 | 3.05 | 3.82 | 4.57 | 5.58 | 17.28 | 19.68 | 21.92 | 24.73 | 26.76 |
| 12 | 3.07 | 3.57 | 4.40 | 5.23 | 6.30 | 18.55 | 21.03 | 23.34 | 26.22 | 28.30 |
| 13 | 3.57 | 4.11 | 5.01 | 5.89 | 7.04 | 19.81 | 22.36 | 24.74 | 27.69 | 29.82 |
| 14 | 4.07 | 4.66 | 5.63 | 6.57 | 7.79 | 21.06 | 23.68 | 26.12 | 29.14 | 31.32 |
| 15 | 4.60 | 5.23 | 6.26 | 7.26 | 8.55 | 22.31 | 25.00 | 27.49 | 30.58 | 32.80 |
| 16 | 5.14 | 5.81 | 6.91 | 7.96 | 9.31 | 23.54 | 26.30 | 28.85 | 32.00 | 34.27 |
| 17 | 5.70 | 6.41 | 7.56 | 8.67 | 10.09 | 24.77 | 27.59 | 30.19 | 33.41 | 35.72 |
| 18 | 6.26 | 7.01 | 8.23 | 9.39 | 10.86 | 25.99 | 28.87 | 31.53 | 34.81 | 37.16 |
| 19 | 6.84 | 7.63 | 8.91 | 10.12 | 11.65 | 27.20 | 30.14 | 32.85 | 36.19 | 38.58 |
| 20 | 7.43 | 8.26 | 9.59 | 10.85 | 12.44 | 28.41 | 31.41 | 34.17 | 37.57 | 40.00 |
| 21 | 8.03 | 8.90 | 10.28 | 11.59 | 13.24 | 29.62 | 32.67 | 35.48 | 38.93 | 41.40 |
| 22 | 8.64 | 9.54 | 10.98 | 12.34 | 14.04 | 30.81 | 33.92 | 36.78 | 40.29 | 42.80 |
| 23 | 9.26 | 10.20 | 11.69 | 13.09 | 14.85 | 32.01 | 35.17 | 38.08 | 41.64 | 44.18 |
| 24 | 9.89 | 10.86 | 12.40 | 13.85 | 15.66 | 33.20 | 36.42 | 39.36 | 42.98 | 45.56 |
| 25 | 10.52 | 11.52 | 13.12 | 14.61 | 16.47 | 34.38 | 37.65 | 40.65 | 44.31 | 46.93 |
| 26 | 11.16 | 12.20 | 13.84 | 15.38 | 17.29 | 35.56 | 38.89 | 41.92 | 45.64 | 48.29 |
| 27 | 11.81 | 12.88 | 14.57 | 16.15 | 18.11 | 36.74 | 40.11 | 43.19 | 46.96 | 49.64 |
| 28 | 12.46 | 13.56 | 15.31 | 16.93 | 18.94 | 37.92 | 41.34 | 44.46 | 48.28 | 50.99 |
| 29 | 13.12 | 14.26 | 16.05 | 17.71 | 19.77 | 39.09 | 42.56 | 45.72 | 49.59 | 52.34 |
| 30 | 13.79 | 14.95 | 16.79 | 18.49 | 20.60 | 40.26 | 43.77 | 46.98 | 50.89 | 53.67 |
| 40 | 20.71 | 22.16 | 24.43 | 26.51 | 29.05 | 51.81 | 55.76 | 59.34 | 63.69 | 66.77 |
| 50 | 27.99 | 29.71 | 32.36 | 34.76 | 37.69 | 63.17 | 67.50 | 71.42 | 76.15 | 79.49 |
| 60 | 35.53 | 37.48 | 40.48 | 43.19 | 46.46 | 74.40 | 79.08 | 83.30 | 88.38 | 91.95 |
| 70 | 43.28 | 45.44 | 48.76 | 51.74 | 55.33 | 85.53 | 90.53 | 95.02 | 100.4 | 104.2 |
| 80 | 51.17 | 53.54 | 57.15 | 60.39 | 64.28 | 96.58 | 101.9 | 106.6 | 112.3 | 116.3 |
| 90 | 59.20 | 61.75 | 65.65 | 69.13 | 73.29 | 107.6 | 113.1 | 118.1 | 124.1 | 128.3 |
| 100 | 67.33 | 70.06 | 74.22 | 77.93 | 82.36 | 118.5 | 124.3 | 129.6 | 135.8 | 140.2 |

Source: Reprinted, with permission, from C. M. Thompson, "Table of Percentage Points of the Chi-Square Distribution," *Biometrika* 32 (1941), pp. 188–89.



**TABLE A.4** Percentiles of the $F$ Distribution

Entry is $F(A; \nu_1, \nu_2)$ where $P\{F(\nu_1, \nu_2) \le F(A; \nu_1, \nu_2)\} = A$




$$F(A; \nu_1, \nu_2) = \frac{1}{F(1-A; \nu_2, \nu_1)}$$

**TABLE A.4 *(continued)*** Percentiles of the *F* Distribution

| Den. df | *A* | Numerator df 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|---|
| 1 | .50 | 1.00 | 1.50 | 1.71 | 1.82 | 1.89 | 1.94 | 1.98 | 2.00 | 2.03 |
| | .90 | 39.9 | 49.5 | 53.6 | 55.8 | 57.2 | 58.2 | 58.9 | 59.4 | 59.9 |
| | .95 | 161 | 200 | 216 | 225 | 230 | 234 | 237 | 239 | 241 |
| | .975 | 648 | 800 | 864 | 900 | 922 | 937 | 948 | 957 | 963 |
| | .99 | 4,052 | 5,000 | 5,403 | 5,625 | 5,764 | 5,859 | 5,928 | 5,981 | 6,022 |
| | .995 | 16,211 | 20,000 | 21,615 | 22,500 | 23,056 | 23,437 | 23,715 | 23,925 | 24,091 |
| | .999 | 405,280 | 500,000 | 540,380 | 562,500 | 576,400 | 585,940 | 592,870 | 598,140 | 602,280 |
| 2 | .50 | 0.667 | 1.00 | 1.13 | 1.21 | 1.25 | 1.28 | 1.30 | 1.32 | 1.33 |
| | .90 | 8.53 | 9.00 | 9.16 | 9.24 | 9.29 | 9.33 | 9.35 | 9.37 | 9.38 |
| | .95 | 18.5 | 19.0 | 19.2 | 19.2 | 19.3 | 19.3 | 19.4 | 19.4 | 19.4 |
| | .975 | 38.5 | 39.0 | 39.2 | 39.2 | 39.3 | 39.3 | 39.4 | 39.4 | 39.4 |
| | .99 | 98.5 | 99.0 | 99.2 | 99.2 | 99.3 | 99.3 | 99.4 | 99.4 | 99.4 |
| | .995 | 199 | 199 | 199 | 199 | 199 | 199 | 199 | 199 | 199 |
| | .999 | 998.5 | 999.0 | 999.2 | 999.2 | 999.3 | 999.3 | 999.4 | 999.4 | 999.4 |
| 3 | .50 | 0.585 | 0.881 | 1.00 | 1.06 | 1.10 | 1.13 | 1.15 | 1.16 | 1.17 |
| | .90 | 5.54 | 5.46 | 5.39 | 5.34 | 5.31 | 5.28 | 5.27 | 5.25 | 5.24 |
| | .95 | 10.1 | 9.55 | 9.28 | 9.12 | 9.01 | 8.94 | 8.89 | 8.85 | 8.81 |
| | .975 | 17.4 | 16.0 | 15.4 | 15.1 | 14.9 | 14.7 | 14.6 | 14.5 | 14.5 |
| | .99 | 34.1 | 30.8 | 29.5 | 28.7 | 28.2 | 27.9 | 27.7 | 27.5 | 27.3 |
| | .995 | 55.6 | 49.8 | 47.5 | 46.2 | 45.4 | 44.8 | 44.4 | 44.1 | 43.9 |
| | .999 | 167.0 | 148.5 | 141.1 | 137.1 | 134.6 | 132.8 | 131.6 | 130.6 | 129.9 |
| 4 | .50 | 0.549 | 0.828 | 0.941 | 1.00 | 1.04 | 1.06 | 1.08 | 1.09 | 1.10 |
| | .90 | 4.54 | 4.32 | 4.19 | 4.11 | 4.05 | 4.01 | 3.98 | 3.95 | 3.94 |
| | .95 | 7.71 | 6.94 | 6.59 | 6.39 | 6.26 | 6.16 | 6.09 | 6.04 | 6.00 |
| | .975 | 12.2 | 10.6 | 9.98 | 9.60 | 9.36 | 9.20 | 9.07 | 8.98 | 8.90 |
| | .99 | 21.2 | 18.0 | 16.7 | 16.0 | 15.5 | 15.2 | 15.0 | 14.8 | 14.7 |
| | .995 | 31.3 | 26.3 | 24.3 | 23.2 | 22.5 | 22.0 | 21.6 | 21.4 | 21.1 |
| | .999 | 74.1 | 61.2 | 56.2 | 53.4 | 51.7 | 50.5 | 49.7 | 49.0 | 48.5 |
| 5 | .50 | 0.528 | 0.799 | 0.907 | 0.965 | 1.00 | 1.02 | 1.04 | 1.05 | 1.06 |
| | .90 | 4.06 | 3.78 | 3.62 | 3.52 | 3.45 | 3.40 | 3.37 | 3.34 | 3.32 |
| | .95 | 6.61 | 5.79 | 5.41 | 5.19 | 5.05 | 4.95 | 4.88 | 4.82 | 4.77 |
| | .975 | 10.0 | 8.43 | 7.76 | 7.39 | 7.15 | 6.98 | 6.85 | 6.76 | 6.68 |
| | .99 | 16.3 | 13.3 | 12.1 | 11.4 | 11.0 | 10.7 | 10.5 | 10.3 | 10.2 |
| | .995 | 22.8 | 18.3 | 16.5 | 15.6 | 14.9 | 14.5 | 14.2 | 14.0 | 13.8 |
| | .999 | 47.2 | 37.1 | 33.2 | 31.1 | 29.8 | 28.8 | 28.2 | 27.6 | 27.2 |
| 6 | .50 | 0.515 | 0.780 | 0.886 | 0.942 | 0.977 | 1.00 | 1.02 | 1.03 | 1.04 |
| | .90 | 3.78 | 3.46 | 3.29 | 3.18 | 3.11 | 3.05 | 3.01 | 2.98 | 2.96 |
| | .95 | 5.99 | 5.14 | 4.76 | 4.53 | 4.39 | 4.28 | 4.21 | 4.15 | 4.10 |
| | .975 | 8.81 | 7.26 | 6.60 | 6.23 | 5.99 | 5.82 | 5.70 | 5.60 | 5.52 |
| | .99 | 13.7 | 10.9 | 9.78 | 9.15 | 8.75 | 8.47 | 8.26 | 8.10 | 7.98 |
| | .995 | 18.6 | 14.5 | 12.9 | 12.0 | 11.5 | 11.1 | 10.8 | 10.6 | 10.4 |
| | .999 | 35.5 | 27.0 | 23.7 | 21.9 | 20.8 | 20.0 | 19.5 | 19.0 | 18.7 |
| 7 | .50 | 0.506 | 0.767 | 0.871 | 0.926 | 0.960 | 0.983 | 1.00 | 1.01 | 1.02 |
| | .90 | 3.59 | 3.26 | 3.07 | 2.96 | 2.88 | 2.83 | 2.78 | 2.75 | 2.72 |
| | .95 | 5.59 | 4.74 | 4.35 | 4.12 | 3.97 | 3.87 | 3.79 | 3.73 | 3.68 |
| | .975 | 8.07 | 6.54 | 5.89 | 5.52 | 5.29 | 5.12 | 4.99 | 4.90 | 4.82 |
| | .99 | 12.2 | 9.55 | 8.45 | 7.85 | 7.46 | 7.19 | 6.99 | 6.84 | 6.72 |
| | .995 | 16.2 | 12.4 | 10.9 | 10.1 | 9.52 | 9.16 | 8.89 | 8.68 | 8.51 |
| | .999 | 29.2 | 21.7 | 18.8 | 17.2 | 16.2 | 15.5 | 15.0 | 14.6 | 14.3 |



**TABLE A.4 *(continued)*** Percentiles of the *F* Distribution

| Den. df | *A* | Numerator df 10 | 12 | 15 | 20 | 24 | 30 | 60 | 120 | ∞ |
|---|---|---|---|---|---|---|---|---|---|---|
| 1 | .50 | 2.04 | 2.07 | 2.09 | 2.12 | 2.13 | 2.15 | 2.17 | 2.18 | 2.20 |
| | .90 | 60.2 | 60.7 | 61.2 | 61.7 | 62.0 | 62.3 | 62.8 | 63.1 | 63.3 |
| | .95 | 242 | 244 | 246 | 248 | 249 | 250 | 252 | 253 | 254 |
| | .975 | 969 | 977 | 985 | 993 | 997 | 1,001 | 1,010 | 1,014 | 1,018 |
| | .99 | 6,056 | 6,106 | 6,157 | 6,209 | 6,235 | 6,261 | 6,313 | 6,339 | 6,366 |
| | .995 | 24,224 | 24,426 | 24,630 | 24,836 | 24,940 | 25,044 | 25,253 | 25,359 | 25,464 |
| | .999 | 605,620 | 610,670 | 615,760 | 620,910 | 623,500 | 626,100 | 631,340 | 633,970 | 636,620 |
| 2 | .50 | 1.34 | 1.36 | 1.38 | 1.39 | 1.40 | 1.41 | 1.43 | 1.43 | 1.44 |
| | .90 | 9.39 | 9.41 | 9.42 | 9.44 | 9.45 | 9.46 | 9.47 | 9.48 | 9.49 |
| | .95 | 19.4 | 19.4 | 19.4 | 19.4 | 19.5 | 19.5 | 19.5 | 19.5 | 19.5 |
| | .975 | 39.4 | 39.4 | 39.4 | 39.4 | 39.5 | 39.5 | 39.5 | 39.5 | 39.5 |
| | .99 | 99.4 | 99.4 | 99.4 | 99.4 | 99.5 | 99.5 | 99.5 | 99.5 | 99.5 |
| | .995 | 199 | 199 | 199 | 199 | 199 | 199 | 199 | 199 | 200 |
| | .999 | 999.4 | 999.4 | 999.4 | 999.4 | 999.5 | 999.5 | 999.5 | 999.5 | 999.5 |
| 3 | .50 | 1.18 | 1.20 | 1.21 | 1.23 | 1.23 | 1.24 | 1.25 | 1.26 | 1.27 |
| | .90 | 5.23 | 5.22 | 5.20 | 5.18 | 5.18 | 5.17 | 5.15 | 5.14 | 5.13 |
| | .95 | 8.79 | 8.74 | 8.70 | 8.66 | 8.64 | 8.62 | 8.57 | 8.55 | 8.53 |
| | .975 | 14.4 | 14.3 | 14.3 | 14.2 | 14.1 | 14.1 | 14.0 | 13.9 | 13.9 |
| | .99 | 27.2 | 27.1 | 26.9 | 26.7 | 26.6 | 26.5 | 26.3 | 26.2 | 26.1 |
| | .995 | 43.7 | 43.4 | 43.1 | 42.8 | 42.6 | 42.5 | 42.1 | 42.0 | 41.8 |
| | .999 | 129.2 | 128.3 | 127.4 | 126.4 | 125.9 | 125.4 | 124.5 | 124.0 | 123.5 |
| 4 | .50 | 1.11 | 1.13 | 1.14 | 1.15 | 1.16 | 1.16 | 1.18 | 1.18 | 1.19 |
| | .90 | 3.92 | 3.90 | 3.87 | 3.84 | 3.83 | 3.82 | 3.79 | 3.78 | 3.76 |
| | .95 | 5.96 | 5.91 | 5.86 | 5.80 | 5.77 | 5.75 | 5.69 | 5.66 | 5.63 |
| | .975 | 8.84 | 8.75 | 8.66 | 8.56 | 8.51 | 8.46 | 8.36 | 8.31 | 8.26 |
| | .99 | 14.5 | 14.4 | 14.2 | 14.0 | 13.9 | 13.8 | 13.7 | 13.6 | 13.5 |
| | .995 | 21.0 | 20.7 | 20.4 | 20.2 | 20.0 | 19.9 | 19.6 | 19.5 | 19.3 |
| | .999 | 48.1 | 47.4 | 46.8 | 46.1 | 45.8 | 45.4 | 44.7 | 44.4 | 44.1 |
| 5 | .50 | 1.07 | 1.09 | 1.10 | 1.11 | 1.12 | 1.12 | 1.14 | 1.14 | 1.15 |
| | .90 | 3.30 | 3.27 | 3.24 | 3.21 | 3.19 | 3.17 | 3.14 | 3.12 | 3.11 |
| | .95 | 4.74 | 4.68 | 4.62 | 4.56 | 4.53 | 4.50 | 4.43 | 4.40 | 4.37 |
| | .975 | 6.62 | 6.52 | 6.43 | 6.33 | 6.28 | 6.23 | 6.12 | 6.07 | 6.02 |
| | .99 | 10.1 | 9.89 | 9.72 | 9.55 | 9.47 | 9.38 | 9.20 | 9.11 | 9.02 |
| | .995 | 13.6 | 13.4 | 13.1 | 12.9 | 12.8 | 12.7 | 12.4 | 12.3 | 12.1 |
| | .999 | 26.9 | 26.4 | 25.9 | 25.4 | 25.1 | 24.9 | 24.3 | 24.1 | 23.8 |
| 6 | .50 | 1.05 | 1.06 | 1.07 | 1.08 | 1.09 | 1.10 | 1.11 | 1.12 | 1.12 |
| | .90 | 2.94 | 2.90 | 2.87 | 2.84 | 2.82 | 2.80 | 2.76 | 2.74 | 2.72 |
| | .95 | 4.06 | 4.00 | 3.94 | 3.87 | 3.84 | 3.81 | 3.74 | 3.70 | 3.67 |
| | .975 | 5.46 | 5.37 | 5.27 | 5.17 | 5.12 | 5.07 | 4.96 | 4.90 | 4.85 |
| | .99 | 7.87 | 7.72 | 7.56 | 7.40 | 7.31 | 7.23 | 7.06 | 6.97 | 6.88 |
| | .995 | 10.2 | 10.0 | 9.81 | 9.59 | 9.47 | 9.36 | 9.12 | 9.00 | 8.88 |
| | .999 | 18.4 | 18.0 | 17.6 | 17.1 | 16.9 | 16.7 | 16.2 | 16.0 | 15.7 |
| 7 | .50 | 1.03 | 1.04 | 1.05 | 1.07 | 1.07 | 1.08 | 1.09 | 1.10 | 1.10 |
| | .90 | 2.70 | 2.67 | 2.63 | 2.59 | 2.58 | 2.56 | 2.51 | 2.49 | 2.47 |
| | .95 | 3.64 | 3.57 | 3.51 | 3.44 | 3.41 | 3.38 | 3.30 | 3.27 | 3.23 |
| | .975 | 4.76 | 4.67 | 4.57 | 4.47 | 4.42 | 4.36 | 4.25 | 4.20 | 4.14 |
| | .99 | 6.62 | 6.47 | 6.31 | 6.16 | 6.07 | 5.99 | 5.82 | 5.74 | 5.65 |
| | .995 | 8.38 | 8.18 | 7.97 | 7.75 | 7.65 | 7.53 | 7.31 | 7.19 | 7.08 |
| | .999 | 14.1 | 13.7 | 13.3 | 12.9 | 12.7 | 12.5 | 12.1 | 11.9 | 11.7 |

**TABLE A.4 *(continued)*** Percentiles of the $F$ Distribution

| Den. df | $\alpha$ | Numerator df 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|---|
| 8 | .50 | 0.499 | 0.757 | 0.860 | 0.915 | 0.948 | 0.971 | 0.988 | 1.00 | 1.01 |
| | .90 | 3.46 | 3.11 | 2.92 | 2.81 | 2.73 | 2.67 | 2.62 | 2.59 | 2.56 |
| | .95 | 5.32 | 4.46 | 4.07 | 3.84 | 3.69 | 3.58 | 3.50 | 3.44 | 3.39 |
| | .975 | 7.57 | 6.06 | 5.42 | 5.05 | 4.82 | 4.65 | 4.53 | 4.43 | 4.36 |
| | .99 | 11.3 | 8.65 | 7.59 | 7.01 | 6.63 | 6.37 | 6.18 | 6.03 | 5.91 |
| | .995 | 14.7 | 11.0 | 9.60 | 8.81 | 8.30 | 7.95 | 7.69 | 7.50 | 7.34 |
| | .999 | 25.4 | 18.5 | 15.8 | 14.4 | 13.5 | 12.9 | 12.4 | 12.0 | 11.8 |
| 9 | .50 | 0.494 | 0.749 | 0.852 | 0.906 | 0.939 | 0.962 | 0.978 | 0.990 | 1.00 |
| | .90 | 3.36 | 3.01 | 2.81 | 2.69 | 2.61 | 2.55 | 2.51 | 2.47 | 2.44 |
| | .95 | 5.12 | 4.26 | 3.86 | 3.63 | 3.48 | 3.37 | 3.29 | 3.23 | 3.18 |
| | .975 | 7.21 | 5.71 | 5.08 | 4.72 | 4.48 | 4.32 | 4.20 | 4.10 | 4.03 |
| | .99 | 10.6 | 8.02 | 6.99 | 6.42 | 6.06 | 5.80 | 5.61 | 5.47 | 5.35 |
| | .995 | 13.6 | 10.1 | 8.72 | 7.96 | 7.47 | 7.13 | 6.88 | 6.69 | 6.54 |
| | .999 | 22.9 | 16.4 | 13.9 | 12.6 | 11.7 | 11.1 | 10.7 | 10.4 | 10.1 |
| 10 | .50 | 0.490 | 0.743 | 0.845 | 0.899 | 0.932 | 0.954 | 0.971 | 0.983 | 0.992 |
| | .90 | 3.29 | 2.92 | 2.73 | 2.61 | 2.52 | 2.46 | 2.41 | 2.38 | 2.35 |
| | .95 | 4.96 | 4.10 | 3.71 | 3.48 | 3.33 | 3.22 | 3.14 | 3.07 | 3.02 |
| | .975 | 6.94 | 5.46 | 4.83 | 4.47 | 4.24 | 4.07 | 3.95 | 3.85 | 3.78 |
| | .99 | 10.0 | 7.56 | 6.55 | 5.99 | 5.64 | 5.39 | 5.20 | 5.06 | 4.94 |
| | .995 | 12.8 | 9.43 | 8.08 | 7.34 | 6.87 | 6.54 | 6.30 | 6.12 | 5.97 |
| | .999 | 21.0 | 14.9 | 12.6 | 11.3 | 10.5 | 9.93 | 9.52 | 9.20 | 8.96 |
| 12 | .50 | 0.484 | 0.735 | 0.835 | 0.888 | 0.921 | 0.943 | 0.959 | 0.972 | 0.981 |
| | .90 | 3.18 | 2.81 | 2.61 | 2.48 | 2.39 | 2.33 | 2.28 | 2.24 | 2.21 |
| | .95 | 4.75 | 3.89 | 3.49 | 3.26 | 3.11 | 3.00 | 2.91 | 2.85 | 2.80 |
| | .975 | 6.55 | 5.10 | 4.47 | 4.12 | 3.89 | 3.73 | 3.61 | 3.51 | 3.44 |
| | .99 | 9.33 | 6.93 | 5.95 | 5.41 | 5.06 | 4.82 | 4.64 | 4.50 | 4.39 |
| | .995 | 11.8 | 8.51 | 7.23 | 6.52 | 6.07 | 5.76 | 5.52 | 5.35 | 5.20 |
| | .999 | 18.6 | 13.0 | 10.8 | 9.63 | 8.89 | 8.38 | 8.00 | 7.71 | 7.48 |
| 15 | .50 | 0.478 | 0.726 | 0.826 | 0.878 | 0.911 | 0.933 | 0.949 | 0.960 | 0.970 |
| | .90 | 3.07 | 2.70 | 2.49 | 2.36 | 2.27 | 2.21 | 2.16 | 2.12 | 2.09 |
| | .95 | 4.54 | 3.68 | 3.29 | 3.06 | 2.90 | 2.79 | 2.71 | 2.64 | 2.59 |
| | .975 | 6.20 | 4.77 | 4.15 | 3.80 | 3.58 | 3.41 | 3.29 | 3.20 | 3.12 |
| | .99 | 8.68 | 6.36 | 5.42 | 4.89 | 4.56 | 4.32 | 4.14 | 4.00 | 3.89 |
| | .995 | 10.8 | 7.70 | 6.48 | 5.80 | 5.37 | 5.07 | 4.85 | 4.67 | 4.54 |
| | .999 | 16.6 | 11.3 | 9.34 | 8.25 | 7.57 | 7.09 | 6.74 | 6.47 | 6.26 |
| 20 | .50 | 0.472 | 0.718 | 0.816 | 0.868 | 0.900 | 0.922 | 0.938 | 0.950 | 0.959 |
| | .90 | 2.97 | 2.59 | 2.38 | 2.25 | 2.16 | 2.09 | 2.04 | 2.00 | 1.96 |
| | .95 | 4.35 | 3.49 | 3.10 | 2.87 | 2.71 | 2.60 | 2.51 | 2.45 | 2.39 |
| | .975 | 5.87 | 4.46 | 3.86 | 3.51 | 3.29 | 3.13 | 3.01 | 2.91 | 2.84 |
| | .99 | 8.10 | 5.85 | 4.94 | 4.43 | 4.10 | 3.87 | 3.70 | 3.56 | 3.46 |
| | .995 | 9.94 | 6.99 | 5.82 | 5.17 | 4.76 | 4.47 | 4.26 | 4.09 | 3.96 |
| | .999 | 14.8 | 9.95 | 8.10 | 7.10 | 6.46 | 6.02 | 5.69 | 5.44 | 5.24 |
| 24 | .50 | 0.469 | 0.714 | 0.812 | 0.863 | 0.895 | 0.917 | 0.932 | 0.944 | 0.953 |
| | .90 | 2.93 | 2.54 | 2.33 | 2.19 | 2.10 | 2.04 | 1.98 | 1.94 | 1.91 |
| | .95 | 4.26 | 3.40 | 3.01 | 2.78 | 2.62 | 2.51 | 2.42 | 2.36 | 2.30 |
| | .975 | 5.72 | 4.32 | 3.72 | 3.38 | 3.15 | 2.99 | 2.87 | 2.78 | 2.70 |
| | .99 | 7.82 | 5.61 | 4.72 | 4.22 | 3.90 | 3.67 | 3.50 | 3.36 | 3.26 |
| | .995 | 9.55 | 6.66 | 5.52 | 4.89 | 4.49 | 4.20 | 3.99 | 3.83 | 3.69 |
| | .999 | 14.0 | 9.34 | 7.55 | 6.59 | 5.98 | 5.55 | 5.23 | 4.99 | 4.80 |



**TABLE A.4 *(continued)*** Percentiles of the $F$ Distribution

| Den. df | $\alpha$ | Numerator df 10 | 12 | 15 | 20 | 24 | 30 | 60 | 120 | $\infty$ |
|---|---|---|---|---|---|---|---|---|---|---|
| 8 | .50 | 1.02 | 1.03 | 1.04 | 1.05 | 1.06 | 1.07 | 1.08 | 1.08 | 1.09 |
| | .90 | 2.54 | 2.50 | 2.46 | 2.42 | 2.40 | 2.38 | 2.34 | 2.32 | 2.29 |
| | .95 | 3.35 | 3.28 | 3.22 | 3.15 | 3.12 | 3.08 | 3.01 | 2.97 | 2.93 |
| | .975 | 4.30 | 4.20 | 4.10 | 4.00 | 3.95 | 3.89 | 3.78 | 3.73 | 3.67 |
| | .99 | 5.81 | 5.67 | 5.52 | 5.36 | 5.28 | 5.20 | 5.03 | 4.95 | 4.86 |
| | .995 | 7.21 | 7.01 | 6.81 | 6.61 | 6.50 | 6.40 | 6.18 | 6.06 | 5.95 |
| | .999 | 11.5 | 11.2 | 10.8 | 10.5 | 10.3 | 10.1 | 9.73 | 9.53 | 9.33 |
| 9 | .50 | 1.01 | 1.02 | 1.03 | 1.04 | 1.05 | 1.05 | 1.07 | 1.07 | 1.08 |
| | .90 | 2.42 | 2.38 | 2.34 | 2.30 | 2.28 | 2.25 | 2.21 | 2.18 | 2.16 |
| | .95 | 3.14 | 3.07 | 3.01 | 2.94 | 2.90 | 2.86 | 2.79 | 2.75 | 2.71 |
| | .975 | 3.96 | 3.87 | 3.77 | 3.67 | 3.61 | 3.56 | 3.45 | 3.39 | 3.33 |
| | .99 | 5.26 | 5.11 | 4.96 | 4.81 | 4.73 | 4.65 | 4.48 | 4.40 | 4.31 |
| | .995 | 6.42 | 6.23 | 6.03 | 5.83 | 5.73 | 5.62 | 5.41 | 5.30 | 5.19 |
| | .999 | 9.89 | 9.57 | 9.24 | 8.90 | 8.72 | 8.55 | 8.19 | 8.00 | 7.81 |
| 10 | .50 | 1.00 | 1.01 | 1.02 | 1.03 | 1.04 | 1.05 | 1.06 | 1.06 | 1.07 |
| | .90 | 2.32 | 2.28 | 2.24 | 2.20 | 2.18 | 2.16 | 2.11 | 2.08 | 2.06 |
| | .95 | 2.98 | 2.91 | 2.84 | 2.77 | 2.74 | 2.70 | 2.62 | 2.58 | 2.54 |
| | .975 | 3.72 | 3.62 | 3.52 | 3.42 | 3.37 | 3.31 | 3.20 | 3.14 | 3.08 |
| | .99 | 4.85 | 4.71 | 4.56 | 4.41 | 4.33 | 4.25 | 4.08 | 4.00 | 3.91 |
| | .995 | 5.85 | 5.66 | 5.47 | 5.27 | 5.17 | 5.07 | 4.86 | 4.75 | 4.64 |
| | .999 | 8.75 | 8.45 | 8.13 | 7.80 | 7.64 | 7.47 | 7.12 | 6.94 | 6.76 |
| 12 | .50 | 0.989 | 1.00 | 1.01 | 1.02 | 1.03 | 1.03 | 1.05 | 1.05 | 1.06 |
| | .90 | 2.19 | 2.15 | 2.10 | 2.06 | 2.04 | 2.01 | 1.96 | 1.93 | 1.90 |
| | .95 | 2.75 | 2.69 | 2.62 | 2.54 | 2.51 | 2.47 | 2.38 | 2.34 | 2.30 |
| | .975 | 3.37 | 3.28 | 3.18 | 3.07 | 3.02 | 2.96 | 2.85 | 2.79 | 2.72 |
| | .99 | 4.30 | 4.16 | 4.01 | 3.86 | 3.78 | 3.70 | 3.54 | 3.45 | 3.36 |
| | .995 | 5.09 | 4.91 | 4.72 | 4.53 | 4.43 | 4.33 | 4.12 | 4.01 | 3.90 |
| | .999 | 7.29 | 7.00 | 6.71 | 6.40 | 6.25 | 6.09 | 5.76 | 5.59 | 5.42 |
| 15 | .50 | 0.977 | 0.989 | 1.00 | 1.01 | 1.02 | 1.02 | 1.03 | 1.04 | 1.05 |
| | .90 | 2.06 | 2.02 | 1.97 | 1.92 | 1.90 | 1.87 | 1.82 | 1.79 | 1.76 |
| | .95 | 2.54 | 2.48 | 2.40 | 2.33 | 2.29 | 2.25 | 2.16 | 2.11 | 2.07 |
| | .975 | 3.06 | 2.96 | 2.86 | 2.76 | 2.70 | 2.64 | 2.52 | 2.46 | 2.40 |
| | .99 | 3.80 | 3.67 | 3.52 | 3.37 | 3.29 | 3.21 | 3.05 | 2.96 | 2.87 |
| | .995 | 4.42 | 4.25 | 4.07 | 3.88 | 3.79 | 3.69 | 3.48 | 3.37 | 3.26 |
| | .999 | 6.08 | 5.81 | 5.54 | 5.25 | 5.10 | 4.95 | 4.64 | 4.48 | 4.31 |
| 20 | .50 | 0.966 | 0.977 | 0.989 | 1.00 | 1.01 | 1.01 | 1.02 | 1.03 | 1.03 |
| | .90 | 1.94 | 1.89 | 1.84 | 1.79 | 1.77 | 1.74 | 1.68 | 1.64 | 1.61 |
| | .95 | 2.35 | 2.28 | 2.20 | 2.12 | 2.08 | 2.04 | 1.95 | 1.90 | 1.84 |
| | .975 | 2.77 | 2.68 | 2.57 | 2.46 | 2.41 | 2.35 | 2.22 | 2.16 | 2.09 |
| | .99 | 3.37 | 3.23 | 3.09 | 2.94 | 2.86 | 2.78 | 2.61 | 2.52 | 2.42 |
| | .995 | 3.85 | 3.68 | 3.50 | 3.32 | 3.22 | 3.12 | 2.92 | 2.81 | 2.69 |
| | .999 | 5.08 | 4.82 | 4.56 | 4.29 | 4.15 | 4.00 | 3.70 | 3.54 | 3.38 |
| 24 | .50 | 0.961 | 0.972 | 0.983 | 0.994 | 1.00 | 1.01 | 1.02 | 1.02 | 1.03 |
| | .90 | 1.88 | 1.83 | 1.78 | 1.73 | 1.70 | 1.67 | 1.61 | 1.57 | 1.53 |
| | .95 | 2.25 | 2.18 | 2.11 | 2.03 | 1.98 | 1.94 | 1.84 | 1.79 | 1.73 |
| | .975 | 2.64 | 2.54 | 2.44 | 2.33 | 2.27 | 2.21 | 2.08 | 2.01 | 1.94 |
| | .99 | 3.17 | 3.03 | 2.89 | 2.74 | 2.66 | 2.58 | 2.40 | 2.31 | 2.21 |
| | .995 | 3.59 | 3.42 | 3.25 | 3.06 | 2.97 | 2.87 | 2.66 | 2.55 | 2.43 |
| | .999 | 4.64 | 4.39 | 4.14 | 3.87 | 3.74 | 3.59 | 3.29 | 3.14 | 2.97 |

**TABLE A.4 *(continued)*** Percentiles of the $F$ Distribution

| Den. df | $A$ | Numerator df 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|---|
| 30 | .50 | 0.466 | 0.709 | 0.807 | 0.858 | 0.890 | 0.912 | 0.927 | 0.939 | 0.948 |
| | .90 | 2.88 | 2.49 | 2.28 | 2.14 | 2.05 | 1.98 | 1.93 | 1.88 | 1.85 |
| | .95 | 4.17 | 3.32 | 2.92 | 2.69 | 2.53 | 2.42 | 2.33 | 2.27 | 2.21 |
| | .975 | 5.57 | 4.18 | 3.59 | 3.25 | 3.03 | 2.87 | 2.75 | 2.65 | 2.57 |
| | .99 | 7.56 | 5.39 | 4.51 | 4.02 | 3.70 | 3.47 | 3.30 | 3.17 | 3.07 |
| | .995 | 9.18 | 6.35 | 5.24 | 4.62 | 4.23 | 3.95 | 3.74 | 3.58 | 3.45 |
| | .999 | 13.3 | 8.77 | 7.05 | 6.12 | 5.53 | 5.12 | 4.82 | 4.58 | 4.39 |
| 60 | .50 | 0.461 | 0.701 | 0.798 | 0.849 | 0.880 | 0.901 | 0.917 | 0.928 | 0.937 |
| | .90 | 2.79 | 2.39 | 2.18 | 2.04 | 1.95 | 1.87 | 1.82 | 1.77 | 1.74 |
| | .95 | 4.00 | 3.15 | 2.76 | 2.53 | 2.37 | 2.25 | 2.17 | 2.10 | 2.04 |
| | .975 | 5.29 | 3.93 | 3.34 | 3.01 | 2.79 | 2.63 | 2.51 | 2.41 | 2.33 |
| | .99 | 7.08 | 4.98 | 4.13 | 3.65 | 3.34 | 3.12 | 2.95 | 2.82 | 2.72 |
| | .995 | 8.49 | 5.80 | 4.73 | 4.14 | 3.76 | 3.49 | 3.29 | 3.13 | 3.01 |
| | .999 | 12.0 | 7.77 | 6.17 | 5.31 | 4.76 | 4.37 | 4.09 | 3.86 | 3.69 |
| 120 | .50 | 0.458 | 0.697 | 0.793 | 0.844 | 0.875 | 0.896 | 0.912 | 0.923 | 0.932 |
| | .90 | 2.75 | 2.35 | 2.13 | 1.99 | 1.90 | 1.82 | 1.77 | 1.72 | 1.68 |
| | .95 | 3.92 | 3.07 | 2.68 | 2.45 | 2.29 | 2.18 | 2.09 | 2.02 | 1.96 |
| | .975 | 5.15 | 3.80 | 3.23 | 2.89 | 2.67 | 2.52 | 2.39 | 2.30 | 2.22 |
| | .99 | 6.85 | 4.79 | 3.95 | 3.48 | 3.17 | 2.96 | 2.79 | 2.66 | 2.56 |
| | .995 | 8.18 | 5.54 | 4.50 | 3.92 | 3.55 | 3.28 | 3.09 | 2.93 | 2.81 |
| | .999 | 11.4 | 7.32 | 5.78 | 4.95 | 4.42 | 4.04 | 3.77 | 3.55 | 3.38 |
| ∞ | .50 | 0.455 | 0.693 | 0.789 | 0.839 | 0.870 | 0.891 | 0.907 | 0.918 | 0.927 |
| | .90 | 2.71 | 2.30 | 2.08 | 1.94 | 1.85 | 1.77 | 1.72 | 1.67 | 1.63 |
| | .95 | 3.84 | 3.00 | 2.60 | 2.37 | 2.21 | 2.10 | 2.01 | 1.94 | 1.88 |
| | .975 | 5.02 | 3.69 | 3.12 | 2.79 | 2.57 | 2.41 | 2.29 | 2.19 | 2.11 |
| | .99 | 6.63 | 4.61 | 3.78 | 3.32 | 3.02 | 2.80 | 2.64 | 2.51 | 2.41 |
| | .995 | 7.88 | 5.30 | 4.28 | 3.72 | 3.35 | 3.09 | 2.90 | 2.74 | 2.62 |
| | .999 | 10.8 | 6.91 | 5.42 | 4.62 | 4.10 | 3.74 | 3.47 | 3.27 | 3.10 |



**TABLE A.4 *(concluded)*** Percentiles of the $F$ Distribution

| Den. df | $A$ | Numerator df 10 | 12 | 15 | 20 | 24 | 30 | 60 | 120 | ∞ |
|---|---|---|---|---|---|---|---|---|---|---|
| 30 | .50 | 0.955 | 0.966 | 0.978 | 0.989 | 0.994 | 1.00 | 1.01 | 1.02 | 1.02 |
| | .90 | 1.82 | 1.77 | 1.72 | 1.67 | 1.64 | 1.61 | 1.54 | 1.50 | 1.46 |
| | .95 | 2.16 | 2.09 | 2.01 | 1.93 | 1.89 | 1.84 | 1.74 | 1.68 | 1.62 |
| | .975 | 2.51 | 2.41 | 2.31 | 2.20 | 2.14 | 2.07 | 1.94 | 1.87 | 1.79 |
| | .99 | 2.98 | 2.84 | 2.70 | 2.55 | 2.47 | 2.39 | 2.21 | 2.11 | 2.01 |
| | .995 | 3.34 | 3.18 | 3.01 | 2.82 | 2.73 | 2.63 | 2.42 | 2.30 | 2.18 |
| | .999 | 4.24 | 4.00 | 3.75 | 3.49 | 3.36 | 3.22 | 2.92 | 2.76 | 2.59 |
| 60 | .50 | 0.945 | 0.956 | 0.967 | 0.978 | 0.983 | 0.989 | 1.00 | 1.01 | 1.01 |
| | .90 | 1.71 | 1.66 | 1.60 | 1.54 | 1.51 | 1.48 | 1.40 | 1.35 | 1.29 |
| | .95 | 1.99 | 1.92 | 1.84 | 1.75 | 1.70 | 1.65 | 1.53 | 1.47 | 1.39 |
| | .975 | 2.27 | 2.17 | 2.06 | 1.94 | 1.88 | 1.82 | 1.67 | 1.58 | 1.48 |
| | .99 | 2.63 | 2.50 | 2.35 | 2.20 | 2.12 | 2.03 | 1.84 | 1.73 | 1.60 |
| | .995 | 2.90 | 2.74 | 2.57 | 2.39 | 2.29 | 2.19 | 1.96 | 1.83 | 1.69 |
| | .999 | 3.54 | 3.32 | 3.08 | 2.83 | 2.69 | 2.55 | 2.25 | 2.08 | 1.89 |
| 120 | .50 | 0.939 | 0.950 | 0.961 | 0.972 | 0.978 | 0.983 | 0.994 | 1.00 | 1.01 |
| | .90 | 1.65 | 1.60 | 1.55 | 1.48 | 1.45 | 1.41 | 1.32 | 1.26 | 1.19 |
| | .95 | 1.91 | 1.83 | 1.75 | 1.66 | 1.61 | 1.55 | 1.43 | 1.35 | 1.25 |
| | .975 | 2.16 | 2.05 | 1.95 | 1.82 | 1.76 | 1.69 | 1.53 | 1.43 | 1.31 |
| | .99 | 2.47 | 2.34 | 2.19 | 2.03 | 1.95 | 1.86 | 1.66 | 1.53 | 1.38 |
| | .995 | 2.71 | 2.54 | 2.37 | 2.19 | 2.09 | 1.98 | 1.75 | 1.61 | 1.43 |
| | .999 | 3.24 | 3.02 | 2.78 | 2.53 | 2.40 | 2.26 | 1.95 | 1.77 | 1.54 |
| ∞ | .50 | 0.934 | 0.945 | 0.956 | 0.967 | 0.972 | 0.978 | 0.989 | 0.994 | 1.00 |
| | .90 | 1.60 | 1.55 | 1.49 | 1.42 | 1.38 | 1.34 | 1.24 | 1.17 | 1.00 |
| | .95 | 1.83 | 1.75 | 1.67 | 1.57 | 1.52 | 1.46 | 1.32 | 1.22 | 1.00 |
| | .975 | 2.05 | 1.94 | 1.83 | 1.71 | 1.64 | 1.57 | 1.39 | 1.27 | 1.00 |
| | .99 | 2.32 | 2.18 | 2.04 | 1.88 | 1.79 | 1.70 | 1.47 | 1.32 | 1.00 |
| | .995 | 2.52 | 2.36 | 2.19 | 2.00 | 1.90 | 1.79 | 1.53 | 1.36 | 1.00 |
| | .999 | 2.96 | 2.74 | 2.51 | 2.27 | 2.13 | 1.99 | 1.66 | 1.45 | 1.00 |

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