UNIVERISTY OF SWAZILAND

MAIN EXAMINATION PAPER, DECEMBER 2011

TITLE OF PAPER	:	PROBABILITY THEORY
COURSE CODE	:	ST201
TIME ALLOWED	:	THREE (3) HOURS
INSTRUCTIONS	:	ANSWER ANY FIVE (5) QUESTIONS
REQUIREMENTS	:	SCIENTIFIC CALCULATOR

Question 1

(a) There are 24 elephants in a game reserve. The warden tags six of the elephants with small radio transmitters and returns them to the reserve. The next month, he randomly selects five elephants from the reserve. He counts how many of these elephants are tagged. Assume that no elephants leave or enter the reserve, or die or give birth, between the tagging and the selection; and that all outcomes of the selection are equally likely. Find the probability that exactly two of the selected elephants are tagged, giving the answer correct to 3 decimal places.

(6 Marks)

(b) A couple are planning to have a family. They decide to stop having children either when they have two boys or when they have four children. Suppose that they are successful in their plan.

(i) Write down the sample space.

(4Marks)

(ii) Assume that, each time that they have a child, the probability that it is a boy is 1/2, independent of all other times. Find P(E) and P(F) where E = "there are at least two girls", F = "there are more girls than boys".

(10Marks)

Question 2

(a) Suppose that X and Y are independent random variables with the same probability density function (pdf) $f(x)$. Write down, without proof, a formula for the pdf of X + Y.

(2 Marks)

(b) Suppose that f (x) = x/2 for $0 < x < 2$ (and f (x) = 0 elsewhere).

(i) Find the pdf of W = X + Y for $0 < w < 2$ and for $2 < w < 4$.

(12 Marks)

(ii) Find the pdf of $V = (X - 1)^2$.

(6 Marks)

Question 3

The random variable X has the binomial distribution with probability mass function

$$P(X = x) = \binom{2}{x} p^x (1-p)^{2-x}, \quad x = 0, 1, 2; \quad 0 < p < 1.$$

(a) Write down E(X), Var(X) and P(X = 2) in terms of the parameter p. Also find P(X = 0 | X < 2) and P(X = 1 | X < 2), simplifying your answers as far as possible.

(7 Marks)

(b) Let $Y = X_1 + X_2 + \ldots + X_{100}$ be the sum of 100 independent random variables, each distributed as X.

(i) Explain why Y has the B(200, p) distribution.

(2 Marks)

(ii) Use a suitable approximation to find $P(Y > 140)$ when $p = ⅔$.

(4 Marks)

(iii)Use a suitable approximation to find $P(Y > 2)$ when $p = 0.02$.

(3 Marks)

(iv)Use a suitable approximation to find $P(Y \leq 197)$ when $p = 0.98$.

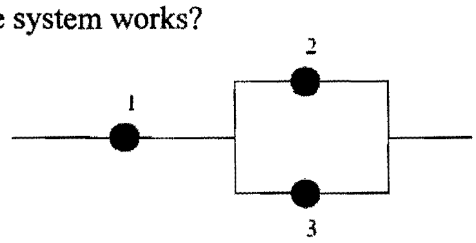
(4 Marks)

Question 4

(a) In a binary transmission channel, a 1 is transmitted with probability 2/3 and a 0 with probability 1/3. The conditional probability of receiving a 1 when a 1 was sent is 0.95; the conditional probability of receiving a 0 when a 0 was sent is 0.90. Given that a 1 is received, what is the probability that a 1 was transmitted?

(10 Marks)

(b) Consider the following system. Each component has a probability 0.1 of failing. What is the probability that the system works?



(10 Marks)

Question 5

(c) A radioactive source of material emits a radioactive particle with probability 1/100 in each second. Let X be the number of particles emitted in one hour. What is the distribution of X and its parameter? Draw or sketch the pmf of X.

(10 Marks)

(d) An electrical component has a lifetime X that is exponentially distributed with parameter $\lambda = 1/10$ per year. What is the probability the component is still alive after 5 years?

(10 Marks)

Question 6

A random vector (X, Y) has joint pdf f, given by

$$f(x, y) = 2\,e^{-x-2y}, \quad x > 0, y > 0 \,.$$

Calculate E[XY].

(6Marks)

Calculate the covariance of X + Y and X − Y .

(14 Marks)

Question 7

You are visiting the Lubombo region, but regrettably your insect repellent has run out.

(a) As a consequence, at each second, a mosquito lands on your neck with probability 0.2.

(i) What's the *pmf* for the time until the first mosquito lands on you?

(4 Marks)

(ii) What's the expected time until the first mosquito lands on you?

(4 Marks)

(iii) What if you weren't bitten for the first ten seconds -what would be the expected time until the first mosquito lands on you (from time t=10)?

(4 Marks)

(b) Instead, imagine the rainforest had only one mosquito, which arrived in the following way: the time of arrival is $\lambda = 0.2$.

(i) What's the expected time until the first mosquito lands on you?

(4 Marks)

(ii) What if you weren't bitten for the first ten seconds -what would be the expected time until the first mosquito lands on you (from time t=10)?

(4 Marks)