UNIVERSITY OF SWAZILAND

SUPPLEMENTARY EXAMINATION PAPER 2012

TITLE OF PAPER : MATHEMATICS FOR STATISTICIANS

COURSE CODE : ST 202

TIME ALLOWED : TWO (2) HOURS

REQUIREMENTS : CALCULATOR

INSTRUCTIONS : THIS PAPER HAS FIVE (5). ANSWER ANY THREE (3) QUESTIONS.

Question 1 [20 marks, 12+8]

(a) Ascertain the LU decomposition of the following matrix (B).

$$B = \begin{pmatrix} 1 & 3 & 5 & 2 \\ 0 & -1 & 3 & 4 \\ 2 & 1 & 9 & 6 \\ 3 & 2 & 4 & 8 \end{pmatrix}$$

Also find the determinant of B.

(b) An investor saves money in a bank account paying interest at a fixed rate of 100r%, where the interest is paid once per year, at the end of the year. She deposits an amount D at the beginning of each of the next N years. Show that she will then have saved an amount equal to

$$\frac{D}{r} \left([1+r]^N - 1 \right),$$

just after the last of these deposits.

Question 2 [20 marks, 8+8+4]

(a) Solve the following system of equations using the Gauss-Jordan elimination method.

$$4x - y - z = 4$$
$$2x - 3y - z = 4$$
$$2x - 5y + z = 8$$

(b) Determine the integrals.

$$\int \frac{x}{\sqrt{e^x}} dx, \qquad\qquad \int \frac{x}{x^2 + 5x + 4} dx.$$

(c) Consider

$$A = \begin{pmatrix} 2 & 0 & 0 \\ 0 & 3 & -2 \\ 0 & -2 & 3 \end{pmatrix}$$

Find the rank of A.

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Question 3 [20 marks, 8+4+8]

(a) Functions f and g are as follows

$$f(x) = x^4 + 2x^3 + 2x^2 + 2, \qquad g(x) = -x^4 + 2x^3 + 18x + 20.$$

Show that the curves $y = f(x)$ and $y = g(x)$ intersect for exactly two values of x. Find these values of x. (Do not attempt to sketch the curves.)

(b) Use L'Hôpital's Rule to determine

$$\lim_{x \to +\infty} x(\pi/2 - \tan^{-1} x)$$

(c) Find the eigenvalues and eigenvectors of the matrix

$$B = \begin{pmatrix} 2 & 1 & 0 \\ 0 & 1 & 0 \\ -1 & 0 & 3 \end{pmatrix}$$

Question 4 [20 marks, 4+6+2+4+4]

(a) Use Newton's Method to determine an approximation to the solution to $\cos x = x$ that lies in the interval $[0,2]$. Find the approximation to six decimal places.

(b) An apartment complex has 250 apartments to rent. If they rent x apartments then their monthly profit, in dollars, is given by,

$$P(x) = -8x^2 + 3200x - 80,000$$

How many apartments should they rent in order to maximize their profit?

(c) It has been determined that the probability density function for the wait in line at a counter is given by,

$$f(t) = \begin{cases} 0 & \text{if } t < 0 \\ 0.1\exp(-\frac{t}{10}) & \text{if } t \geq 0 \end{cases}$$

where t is the number of minutes spent waiting in line. Answer each of the following questions about this probability density function.

   (i) Verify that this is in fact a probability density function.

   (ii) Determine the probability that a person will wait in line for at least 6 minutes.

   (iii) Determine the mean wait in line.

Question 5 [20 marks, 8+6+6]

(a) Determine the center of mass for the region bounded by $y = x^3$ and $y = \sqrt{x}$.

(b) Find the geometric series that has second term equal to 5 and sum to infinity equal to 20.

(c) The function f is defined for positive y and all x by

$$f(x,y) = x^2 \ln y - y \ln y.$$

Find the critical (or stationary) points of f and determine, for each, whether it is a local maximum, a local minimum, or a saddle point.

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