UNIVERSITY OF SWAZILAND

SUPPLEMENTARY EXAMINATION PAPER 2012

TITLE OF PAPER : SAMPLE SURVEY THEORY

COURSE CODE : ST306

TIME ALLOWED : TWO (2) HOURS

REQUIREMENTS : CALCULATOR AND STATISTICAL TABLES

INSTRUCTIONS : ANSWER ANY THREE (3) QUESTIONS

Question 1   [20 marks, 8+8+2+1+1]

(a) Someone else has taken a small survey, using an SRS, of energy usage in houses. On the basis of the survey, each house is categorized as having electric heating or some other kind of heating. The January electricity consumption in kilowatt-hours for each house is recorded (y_i) and the results are given below:

Type of Heating	Number of Houses	Sample Mean	Sample Variance
Electric	24	972	202,396
Nonelectric	36	463	96,721
Total	60		

From other records, it is known that 16,450 of the 35,000 houses have electric heating, and 18,550 have nonelectric heating.

   (i) Using the sample, give an estimate and its standard error of the proportion of houses with electric heating. Does your 95% CI include the true proportion?

   (ii) Give an estimate and its standard error of the average number of kilowatt-hours used by houses in the city. What type of estimator did you use, and why did you choose that estimator?

(b) A relatively new idea in survey design is to administer the survey via an online website but invite participants via paper invitation. One example of this is a customer satisfaction survey conducted by the Subway restaurant at Nationwide Children's Hospital. For this survey, the restaurant takes a systematic sample of purchases and prints a receipt to complete the questionnaire at the bottom of the sampled receipt. The cashier verbally points out the request as she hands the customers the receipt. The request looks like this:

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Iake our 1-minute Survey at
www.tellsubway.com and receive a free
cookie. Keep your receipt and write
your unique coupon code
here_____.
*****Come
back and visit SUBWAY(R) within 7 days
and receive double points on your
purchase! Offer available for a limited
time at participating locations.*****
Host Order ID: 05N.g3hu
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   (i) What do you think is the biggest **advantage** of including a paper invitation?

   (ii) What do you think is the biggest **advantage** of including an online portion of the survey?

   (iii) What do you think is the biggest **disadvantage** of including an online portion of the survey?

Question 2   [20 marks, 6+6+8]

(a) At one university there were 807 faculty members and research specialists in the College of Liberal Arts and Science in 1993; the list of faculty and their reported publications for 1992-1993 were available on the computer system. For each faculty member, the number of refereed publications was recorded. This number is not directly available on the database, so the investigator is required to examine each record separately. A frequency table for number of refereed publications is given for an SRS of 50 faculty members.

Refereed publications	0	1	2	3	4	5	6	7	8	9	10
Faculty members	28	4	3	4	4	2	1	0	2	1	1

(i) Estimate the mean number of publications per faculty member and give a standard error for your estimate.

(ii) Estimate the proportion of faculty members with no publications and give a 95% CI for your estimate.

(b) A public opinion researcher has a budget of E20,000 for taking a survey. She knows that 90% of all households have telephones. Telephone interviews cost E10 per household; in-person interviews cost E30 each if all interviews are conducted in person and E40 each if only nonphone households are interviewed in person (because there will be extra travel costs). Assume that the variances in the phone and nonphone strata are similar and that the fixed costs are $c_0 = $ E5000. How many households should be interviewed in each stratum if households with a phone are contacted by telephone and households without a phone are contacted in person.

Question 3 [20 marks, 10+2+4+4]

(a) A sampling experiment is conducted by sampling 100 individuals at random out of a target population of 2000 and tabulating their monthly incomes (y_k) and educational levels (x_k = highest grade attained, including college as 13 to 16). The results of the study can be summarized as follows:

$$\bar{x} = 12.8 \qquad \bar{y} = 3072 \qquad s_y^2 = 1994^2 \qquad s_x^2 = 3.5^2$$

$$s_{xy} = \frac{1}{99}\sum_{i=1}^{n}(x_i - \bar{x})(y_i - \bar{y}) = 2805$$

Assume that it is known that the populationwide average educational level $\mu_X = 13.4$.

Give an approximately unbiased 95% two-sided confidence interval for average income based instead upon the parameters determined in the whole population from the linear regression model:

$$E(y_k) = \beta_0 + \beta_1 x_k \quad ; \quad \text{var}(y_k) = \sigma$$

(b) The Columbus Police Department (CPD) has been installing traffic light camera systems to identify and ticket auto drivers who fail to stop at red traffic lights. When the system senses that an automobile has entered the intersection while the traffic light was red, the cameras take a photo of the front and back of the car, as well as record a short video of the potential violation. At a later time, a police officer looks at the photos and watches the video. If the officer believes there was a traffic violation, the owner of the car (as identified by the license plate) is fined $50.

The CPD annually evaluates the officer assessment of the photos. For this evaluation, a supervisor reviews a simple random sample of all the photos taken that year and either confirms the original officer's opinion, or finds an error. The goal of this study is to estimate the proportion of errors made that year.

3

In 2009, the supervisor examined 200 of the 35385 camera-recorded incidents. Of these, she disagreed with 8 of the original officers' decisions. For this problem, assume there is no non-sampling error.

(i) Estimate the true proportion of mistakes made by the original officers.

(ii) Estimate the standard error for your estimate in part (a).

(iii) Are you confident that the officers made mistakes on more than 1% of the recorded incidents? Justify your answer.

Question 4 [20 marks, 7+7+6]

A manufacturer of band saws wants to estimate the average repair cost per month for the saws he has sold to certain industries. He cannot obtain a repair cost for each saw, but he can obtain the total amount spent for saw repairs and the total number of saws owned by each industry. Thus he decides to use cluster sampling, with each industry as an experimental unit. The manufacturer selects a simple random sample of size $n = 20$ from the $N = 82$ industries he services. The data on total cost of repairs per industry and the number of saws per industry are as given in the accompanying table.

Industry	Number of Saws	Total Repair Cost for Past Month (SZL)	Industry	Number of Saws	Total Repair Cost for Past Month (SZL)
1	3	50	11	8	140
2	7	110	12	6	120
3	11	230	13	3	70
4	9	140	14	2	50
5	2	50	15	1	10
6	12	260	16	4	60
7	14	240	17	12	280
8	3	45	18	6	150
9	5	60	19	5	110
10	9	230	20	8	120

(a) Estimate the total amount spent by the 82 industries on band saw repairs and the associated 95% confidence interval.

(b) After checking his sales records, the manufacturer finds that he sold a total of 690 band saws to these industries. Using this additional information, estimate the total amount spent on saw repairs by these industries, and the associated 95% confidence interval.

(c) The manufacturer wants to estimate the average repair cost per saw for next month. How many industries should he select for his sample if he wants to estimate this average cost to within SZL2.00 with 95% confidence?

Useful formulas

$$s^2 = \frac{\sum_{i=1}^{n}(y_i - \bar{y})^2}{n-1} \qquad \sum_{i=1}^{n}(y_i - \bar{y})^2 = \sum_{i=1}^{n} y_i^2 - \frac{\sum_{i=1}^{n} y_i}{n}$$

$$\hat{\mu}_{srs} = \bar{y} \qquad \hat{V}(\hat{\mu}_{srs}) = \left(\frac{N-n}{N}\right)\frac{s^2}{n}$$

$$\hat{\tau}_{srs} = N\hat{\mu}_{srs} \qquad \hat{V}(\hat{\tau}_{srs}) = N^2\hat{V}(\hat{\mu}_{srs})$$

$$\hat{p}_{srs} = \sum_{i=1}^{n}\frac{y_i}{n} \qquad \left(\frac{N-n}{N}\right)\frac{\hat{p}(1-\hat{p})}{n-1}\left(\frac{N-n}{N}\right)$$

$$\hat{\tau}_{hh} = \frac{1}{n}\sum_{i=1}^{n}\frac{y_i}{p_i} \qquad \hat{V}(\hat{\mu}_{hh}) = \frac{1}{n(n-1)}\sum_{i=1}^{n}\left(\frac{y_i}{p_i} - \hat{\tau}_{hh}\right)^2$$

$$\hat{\mu}_{hh} = \frac{\hat{\tau}_{hh}}{N} \qquad \hat{V}(\hat{\mu}_{hh}) = \frac{1}{N^2}\hat{V}(\hat{\tau}_{hh})$$

$$\hat{\tau}_{ht} = \sum_{i=1}^{\nu}\frac{y_i}{\pi_i} \qquad \hat{V}(\hat{\tau}_{ht}) = \sum_{i=1}^{\nu}\left(\frac{1}{\pi_i^2} - \frac{1}{\pi_i}\right)y_i^2 +$$

$$2\sum_{i=1}^{\nu}\sum_{j>i}^{\nu}\left(\frac{1}{\pi_i\pi_j} - \frac{1}{\pi_{ij}}\right)y_iy_j$$

$$\hat{\mu}_{ht} = \frac{\hat{\tau}_{ht}}{N} \qquad \hat{V}(\hat{\mu}_{ht}) = \frac{1}{N^2}\hat{V}(\hat{\tau}_{ht})$$

$$\hat{r} = \frac{\sum_{i=1}^{n} y_i}{\sum_{i=1}^{n} x_i} \qquad \hat{V}(\hat{r}) = \left(\frac{N-n}{Nn\mu_x^2}\right)\frac{\sum_{i=1}^{n}(y_i - rx_i)^2}{n-1}$$

$$\hat{\mu}_r = r\mu_x \qquad \hat{V}(\hat{\mu}_r) = \left(\frac{N-n}{Nn}\right)\frac{\sum_{i=1}^{n}(y_i - rx_i)^2}{n-1}$$

$$\hat{\tau}_r = Nr\mu_x = r\tau_x \qquad \hat{V}(\hat{\tau}_r) = \frac{N(N-n)}{n}\frac{\sum_{i=1}^{n}(y_i - rx_i)^2}{n-1}$$

$$\hat{\mu}_L = a + b\mu_x \qquad \hat{V}(\hat{\mu}_L) = \frac{N-n}{Nn(n-1)}\sum_{i=1}^{n}(y_i - a - bx_i)^2$$

$$\hat{\tau}_L = N\mu_L \qquad \hat{V}(\hat{\tau}_L) = \frac{N(N-n)}{n(n-1)}\sum_{i=1}^{n}(y_i - a - bx_i)^2$$

$$\hat{\mu}_{str} = \sum_{h=1}^{L}\frac{N_h}{N}\bar{y}_h \qquad \hat{V}(\hat{\mu}_{str}) = \frac{1}{N^2}\sum_{h=1}^{L}N_h^2\left(\frac{N_h - n_h}{N_h}\right)\frac{s_h^2}{n_h}$$

$$\hat{\tau}_{str} = N\hat{\mu}_{str} \qquad \hat{V}(\hat{\tau}_{str}) = N^2\hat{V}(\hat{\mu}_{str})$$

$$\hat{p}_{str} = \sum_{h=1}^{L}\frac{N_h}{N}\hat{p}_h \qquad \hat{V}(\hat{p}_{str}) = \frac{1}{N^2}\sum_{h=1}^{L}N_h^2\left(\frac{N_h - n_h}{N_h}\right)\left(\frac{\hat{p}_h(1-\hat{p}_h)}{n_h - 1}\right)$$

$$\hat{\mu}_{pstr} = \sum_{h=1}^{L}w_h\bar{y}_h \qquad \hat{V}(\hat{\mu}_{pstr}) = \frac{1}{n}\left(\frac{N-n}{N}\right)\sum_{h=1}^{L}w_h s_h^2 + \frac{1}{n^2}\sum_{h=1}^{L}(1-w_h)s_h^2$$

$$\hat{\tau}_{cl} = \frac{M}{nL} \sum_{i=1}^{n} \sum_{j=1}^{L} y_{ij} = \frac{N}{n} \sum_{i=1}^{n} \sum_{j=1}^{L} y_{ij} = \frac{N}{n} \sum_{i=1}^{n} y_i = N\bar{y}$$

$$\hat{\mu}_{cl} = \frac{1}{nL} \sum_{i=1}^{n} \sum_{j=1}^{L} y_{ij} = \frac{1}{nL} \sum_{i=1}^{n} y_i = \frac{\bar{y}}{L} = \frac{\hat{\tau}_{cl}}{M}$$

where $\bar{y} = \frac{1}{n} \sum_{i=1}^{n} y_i = \frac{\hat{\tau}_{cl}}{N}$

$$\hat{V}(\hat{\tau}_{cl}) = N(N-n)\frac{s_u^2}{n} \qquad\qquad \hat{V}(\hat{\mu}_{cl}) = \frac{N(N-n)}{M^2}\frac{s_u^2}{n}$$

where $s_u^2 = \frac{\sum_{i=1}^{n}(y_i - \bar{y})^2}{n-1}$.

$$\hat{\mu}_1 = \bar{y} = \frac{\hat{\tau}_{cl}}{N} \qquad\qquad \hat{V}(\hat{\mu}_1 = \frac{N-n}{N}\frac{s_u^2}{n}$$

The formulas for systematic sampling are the same as those used for one-stage cluster sampling. Change the subscript *cl* to *sys* to denote the fact that data were collected under systematic sampling.

$$\hat{\mu}_{c(a)} = \frac{\sum_{i=1}^{n} y_i}{\sum_{i=1}^{n} M_i} = \frac{\sum_{i=1}^{n} y_i}{m} \qquad\qquad \hat{V}(\hat{\mu}_{c(a)}) = \frac{(N-n)N}{n(n-1)M^2} \sum_{i=1}^{n} M_i^2(\bar{y} - \hat{\mu}_{c(a)})^2$$

$$\hat{\mu}_{c(b)} = \frac{N}{M}\frac{\sum_{i=1}^{n} y_i}{n} = \frac{N}{nM} \sum_{i=1}^{n} y_i \qquad \hat{V}(\hat{\mu}_{c(b)}) = \frac{(N-n)N}{n(n-1)M^2} \sum_{i=1}^{n}(y_i - \bar{y})^2 = \frac{(N-n)N}{nM^2}s_u^2$$

$$\hat{p}_c = \frac{\sum_{i=1}^{n} p_i}{n} \quad \hat{V}(\hat{p}_c) = \left(\frac{N-Nn}{nN}\right) \sum_{i=1}^{n} \frac{(p_i - \hat{p}_c)^2}{n-1} = \left(\frac{1-f}{n}\right) \sum_{i=1}^{n} \frac{(p_i - \hat{p}_c)^2}{n-1}$$

$$\hat{p}_c = \frac{\sum_{i=1}^{n} y_i}{\sum_{i=1}^{n} M_i} \qquad\qquad \hat{V}(\hat{p}_c) = \left(\frac{1-f}{n\bar{m}^2}\right)\frac{\sum_{i=1}^{n}(y_i - \hat{p}_c M_i)^2}{n-1}$$

To estimate τ, multiply $\hat{\mu}_{c(\cdot)}$ by M. To get the estimated variances, multiply $\hat{V}(\hat{\mu}_{c(\cdot)})$ by M^2. If M is not known, substitute M with Nm/n. $\bar{m} = \sum_{i=1}^{n} M_i/n$.

$$n \text{ for } \mu \text{ SRS} \qquad n = \frac{N\sigma^2}{(N-1)(d^2/z^2) + \sigma^2}$$

$$n \text{ for } \tau \text{ SRS} \qquad n = \frac{N\sigma^2}{(N-1)(d^2/z^2 N^2) + \sigma^2}$$

$$n \text{ for } p \text{ SRS} \qquad n = \frac{Np(1-p)}{(N-1)(d^2/z^2) + p(1-p)}$$

$$n \text{ for } \mu \text{ SYS} \qquad n = \frac{N\sigma^2}{(N-1)(d^2/z^2) + \sigma^2}$$

$$n \text{ for } \tau \text{ SYS} \qquad n = \frac{N\sigma^2}{(N-1)(d^2/z^2 N^2) + \sigma^2}$$

$$n \text{ for } \mu \text{ STR} \qquad n = \frac{\sum_{h=1}^{L} N_h^2(\sigma_h^2/w_h)}{N^2(d^2/z^2) + \sum_{h=1}^{L} N_h\sigma_h^2}$$

$$n \text{ for } \tau \text{ STR} \qquad n = \frac{\sum_{h=1}^{L} N_h^2(\sigma_h^2/w_h)}{N^2(d^2/z^2 N^2) + \sum_{h=1}^{L} N_h\sigma_h^2}$$

where $w_h = \frac{n_h}{n}$.

Allocations for STR μ:

$$n_h = (c - c_0) \left(\frac{N_h \sigma_h / \sqrt{c_h}}{\sum_{k=1}^{L} N_k \sigma_k \sqrt{c_k}} \right) \qquad n = \frac{\left(\sum_{k=1}^{L} N_k \sigma_k \sqrt{c_k} \right) \left(\sum_{k=1}^{L} N_k \sigma_k / \sqrt{c_k} \right)}{N^2 (d^2/z^2) + \sum_{k=1}^{L} N_k \sigma_k^2}$$

$$n_h = n \left(\frac{N_h}{N} \right) \qquad n = \frac{\sum_{k=1}^{L} N_k \sigma_k}{N^2 (d^2/z^2) + \frac{1}{N} \sum_{k=1}^{L} N_k \sigma_k^2}$$

$$n_h = n \left(\frac{N_h \sigma_h}{\sum_{k=1}^{L} N_k \sigma_k} \right) \qquad n = \frac{\left(\sum_{k=1}^{L} N_k \sigma_k \right)^2}{N^2 (d^2/z^2) + \sum_{k=1}^{L} N_k \sigma_k^2}$$

Allocations for STR τ:

$$\text{change } N^2 (d^2/z^2) \text{ to } N^2 (d^2/z^2 N^2)$$

Allocations for STR p:

$$n_h = n \left(\frac{N_i \sqrt{p_h(1 - p_h)/c_h}}{\sum_{k=1}^{L} N_k \sqrt{p_k(1 - p_k)/c_k}} \right) \qquad n = \frac{\sum_{k=1}^{L} N_k p_k (1 - p_k)/w_k}{N^2 (d^2/z^2) + \sum_{k=1}^{L} N_k p_k (1 - p_k)}$$

7

TABLE A.1

Cumulative Standardized Normal Distribution

$A(z)$ is the integral of the standardized normal distribution from $-\infty$ to z (in other words, the area under the curve to the left of z). It gives the probability of a normal random variable not being more than z standard deviations above its mean. Values of z of particular importance:
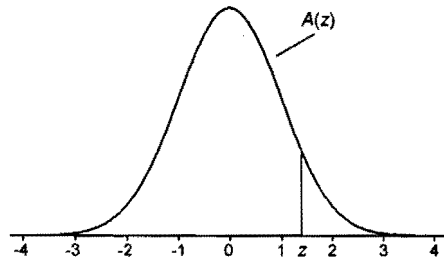
z	$A(z)$	
1.645	0.9500	Lower limit of right 5% tail
1.960	0.9750	Lower limit of right 2.5% tail
2.326	0.9900	Lower limit of right 1% tail
2.576	0.9950	Lower limit of right 0.5% tail
3.090	0.9990	Lower limit of right 0.1% tail
3.291	0.9995	Lower limit of right 0.05% tail

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998
3.5	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998
3.6	0.9998	0.9998	0.9999							

TABLE A.2

t Distribution: Critical Values of t

Degrees of freedom	Two-tailed test: One-tailed test:	10% 5%	5% 2.5%	2% 1%	1% 0.5%	0.2% 0.1%	0.1% 0.05%
		Significance level					
1		6.314	12.706	31.821	63.657	318.309	636.619
2		2.920	4.303	6.965	9.925	22.327	31.599
3		2.353	3.182	4.541	5.841	10.215	12.924
4		2.132	2.776	3.747	4.604	7.173	8.610
5		2.015	2.571	3.365	4.032	5.893	6.869
6		1.943	2.447	3.143	3.707	5.208	5.959
7		1.894	2.365	2.998	3.499	4.785	5.408
8		1.860	2.306	2.896	3.355	4.501	5.041
9		1.833	2.262	2.821	3.250	4.297	4.781
10		1.812	2.228	2.764	3.169	4.144	4.587
11		1.796	2.201	2.718	3.106	4.025	4.437
12		1.782	2.179	2.681	3.055	3.930	4.318
13		1.771	2.160	2.650	3.012	3.852	4.221
14		1.761	2.145	2.624	2.977	3.787	4.140
15		1.753	2.131	2.602	2.947	3.733	4.073
16		1.746	2.120	2.583	2.921	3.686	4.015
17		1.740	2.110	2.567	2.898	3.646	3.965
18		1.734	2.101	2.552	2.878	3.610	3.922
19		1.729	2.093	2.539	2.861	3.579	3.883
20		1.725	2.086	2.528	2.845	3.552	3.850
21		1.721	2.080	2.518	2.831	3.527	3.819
22		1.717	2.074	2.508	2.819	3.505	3.792
23		1.714	2.069	2.500	2.807	3.485	3.768
24		1.711	2.064	2.492	2.797	3.467	3.745
25		1.708	2.060	2.485	2.787	3.450	3.725
26		1.706	2.056	2.479	2.779	3.435	3.707
27		1.703	2.052	2.473	2.771	3.421	3.690
28		1.701	2.048	2.467	2.763	3.408	3.674
29		1.699	2.045	2.462	2.756	3.396	3.659
30		1.697	2.042	2.457	2.750	3.385	3.646
32		1.694	2.037	2.449	2.738	3.365	3.622
34		1.691	2.032	2.441	2.728	3.348	3.601
36		1.688	2.028	2.434	2.719	3.333	3.582
38		1.686	2.024	2.429	2.712	3.319	3.566
40		1.684	2.021	2.423	2.704	3.307	3.551
42		1.682	2.018	2.418	2.698	3.296	3.538
44		1.680	2.015	2.414	2.692	3.286	3.526
46		1.679	2.013	2.410	2.687	3.277	3.515
48		1.677	2.011	2.407	2.682	3.269	3.505
50		1.676	2.009	2.403	2.678	3.261	3.496
60		1.671	2.000	2.390	2.660	3.232	3.460
70		1.667	1.994	2.381	2.648	3.211	3.435
80		1.664	1.990	2.374	2.639	3.195	3.416
90		1.662	1.987	2.368	2.632	3.183	3.402
100		1.660	1.984	2.364	2.626	3.174	3.390
120		1.658	1.980	2.358	2.617	3.160	3.373
150		1.655	1.976	2.351	2.609	3.145	3.357
200		1.653	1.972	2.345	2.601	3.131	3.340
300		1.650	1.968	2.339	2.592	3.118	3.323
400		1.649	1.966	2.336	2.588	3.111	3.315
500		1.648	1.965	2.334	2.586	3.107	3.310
600		1.647	1.964	2.333	2.584	3.104	3.307
∞		1.645	1.960	2.326	2.576	3.090	3.291