DEPARTMENT OF STATISTICS AND DEMOGRAPHY

MAIN EXAMINATION, 2011/12

| COURSE TITLE: | OPERATIONS RESEARCH I |
| :---: | :---: |
| COURSE CODE: | ST 408 |
| TIME ALLOWED: | TWO (2) HOURS |
| INSTRUCTION: | ANSWER ANY THREE QUESTIONS <br> ALL QUESTIONS CARRY EQUAL MARKS (20 MARKS) |
| SPECIAL REQUIREMENTS: | SCIENTIFIC CALCULATORS AND GRAPH PAPER |

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## Question 1

Customer's arrivals at Bubu's gas station follow a Poisson process with mean arrival rate of one customer every 4 minutes and the service time follow an exponential distribution with mean service time of 2.5 minutes per customer.
(a) What is the probability that anytime the gas station will be idle?
(b) What is the average number of customers in the system?
(c) What is the average queue length?
(d) What is the average time a customer spends in the system?
(e) What is the average time a customer waits in the line to be served?
(20 marks)

## Question 2

The Wilson Publishing Company produces books for the retail market. Demand for a current book is expected top occur at a constant annual rate of 7200 copies. The cost of one copy of the book is E14.50. The holding cost is based on an $18 \%$ annual rate, and production setup costs are E150 per setup. The equipment on which the book is produced has an annual production volume of 25000 copies. There are 250 working days per year and the lead time for a production run is 15 days. Use the production lot size model to compute the following values:
(a) Minimum-cost production lot size;
(b) Number of production runs per year;
(c) Cycle time;
(d) Length of production run;
(e) Total annual cost;
(f) Re-order point.
(20 marks)

## Question 3

(a)Vuka Utentele Construction is in the process of installing power lines to a large housing development. Sipho Ndlovu, the project manager wants to minimise the total length of wire used, which will minimise his costs. The housing development is as shown below. Each house has been numbered, and the distances between houses are given in hundreds of metres. What do you recommend?

(b) Nhlangano City Council is considering making several of its streets one-way. What is the maximum number of cars per hour that can travel from east to west ?


Insika Production was in the final design phases of its new film, Killer Worms, to be released next summer. Market Wise, a firm hired to coordinate the release Killer Worms toys, identified 16 critical tasks to be completed before the release of the film. The tasks are as follows :

| Activity | Immediate Predecessor | Optimistic Time | Most Likely Time | Pessimistic Time |
| :--- | :--- | :--- | :--- | :--- |
| Task 1 | - | 1 | 2 | 4 |
| Task 2 | - | 3 | 3.5 | 5 |
| Task 3 | - | 10 | 12 | 13 |
| Task 4 | - | 4 | 5 | 7 |
| Task 5 | - | 2 | 4 | 5 |
| Task 6 | Task 1 | 6 | 7 | 8 |
| Task 7 | Task 2 | 2 | 4 | 5.5 |
| Task 8 | Task 3 | 5 | 7.7 | 9 |
| Task 9 | Task 3 | 9.9 | 10 | 12 |
| Task 10 | Task 3 | 2 | 4 | 5 |
| Task 11 | Task 4 | 2 | 4 | 6 |
| Task 12 | Task 5 | 2 | 4 | 6 |
| Task 13 | Task 6,7,8 | 5 | 6 | 6.5 |
| Task 14 | Task 10,11,12 | 1 | 1.1 | 2 |
| Task 15 | Task 9,13 | 5 | 7 | 8 |
| Task 16 | Task 14 | 5 | 7 | 9 |

(a) How many weeks in advance of the film releases should Market Wise start its marketing campaign? What are the critical activities?
(b) If tasks 9 and 10 were not necessary, what impact would this have on the critical path and the number of weeks needed to complete the marketing compaign ?

## Question 5

An oil company must decide whether or not to drill an oil well in a particular area. The decision maker believes that the area could be dry, reasonably good or a bonanza, with the respective probabilities 0.35 , 039 and 0.26 . If the well is dry, no revenue is generated. If the well is reasonably good, the expected revenue is $\mathrm{E} 86,752.00$. If the well is a bonanza, the expected revenue is $\mathrm{E} 250,050.00$. In any case, the cost of drilling the well is E43, 500.00. At a cost of E19, 500.00, the company can take a series of seismic surveys that usually help determine the underlying geological structure of the site. These experiments will disclose whether there is no structure, open structure or closed structure, denoted by $\mathrm{I}_{1}, \mathrm{I}_{2}$ and $\mathrm{I}_{3}$ respectively. Let $\mathrm{S} 1=$ dry well, $\mathrm{S} 2=$ reasonably good well, and $\mathrm{S} 3=$ bonanza. Past experience has shown the following conditional probabilities:

$$
\begin{array}{lll}
P\left(I_{1} \mid S_{1}\right)=0.45 & P\left(I_{2} \mid S_{1}\right)=0.31 & P\left(I_{3} \mid S_{1}\right)=0.24 \\
P\left(I_{1} \mid S_{2}\right)=0.30 ; & P\left(I_{2} \mid S_{2}\right)=0.49 & P\left(I_{3} \mid S_{2}\right)=0.21 \\
P\left(I_{1} \mid S_{3}\right)=0.25 & P\left(I_{2} \mid S_{3}\right)=0.15 & P\left(I_{3} \mid S_{3}\right)=0.60
\end{array}
$$

(a) Draw and label the decision tree for the above problem
(b) Evaluate the decision tree
(c) What is the best decision to be made under the above conditions?

$$
\begin{array}{lll}
\text { [Hint: } \mathrm{P}\left(\mathrm{I}_{1}\right)=0.34 . \mathrm{P}\left(\mathrm{I}_{2}\right)=0.35, \mathrm{P}\left(\mathrm{I}_{3}\right)=0.31 \\
P\left(S_{1} \mid I_{1}\right)=0.4632 & P\left(S_{2} \mid I_{1}\right)=0.3088 & P\left(S_{3} \mid I_{1}\right)=0.2571 \\
P\left(S_{1} \mid I_{2}\right)=0.3454 & P\left(S_{2} \mid I_{2}\right)=0.5460 & P\left(S_{3} \mid I_{2}\right)=0.1671 \\
P\left(S_{1} \mid I_{3}\right)=0.2013 & P\left(S_{2} \mid I_{3}\right)=0.1761 & \left.P\left(S_{3} \mid I_{3}\right)=0.5032\right]
\end{array}
$$

## END OF EXAM!!

Appendix B Tables
Table B
(concluded)
2. Areas under the standardized normal curve, from $-\infty$ to $+z$


| $z$ | . 00 | . 01 | . 02 | . 03 | . 04 | . 05 | . 06 | . 07 | . 08 | . 09 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | . . . . . 5000 | . 5040 | . 5080 | . 5120 | . 5160 | . 5199 | . 5239 | . 5279 | . 5319 | . 5359 |
|  | ..... . 5398 | . 5438 | . 5478 | . 5517 | . 5557 | . 5596 | . 5636 | . 5675 | . 5714 | . 5753 |
|  | . .... . 5793 | . 5832 | . 5871 | . 5910 | . 5948 | . 5987 | . 6026 | . 6064 | . 6103 | . 6141 |
|  | .... . 6179 | . 6217 | . 6255 | . 6293 | . 6331 | . 6368 | . 6406 | . 6443 | . 6480 | . 6517 |
|  | . . . . 6554 | . 6591 | . 6628 | . 6664 | . 6700 | . 6736 | . 6772 | . 6808 | . 6844 | . 6879 |
|  | .... . 6915 | . 6950 | . 6985 | . 7019 | . 7054 | . 7088 | . 7123 | . 7157 | . 7190 | . 7224 |
|  | 6.... . 7257 | . 7291 | . 7324 | . 7357 | . 7389 | . 7422 | . 7454 | . 7486 | . 7517 | . 7549 |
|  | . . . . . 7580 | . 7611 | . 7642 | . 7673 | . 7703 | . 7734 | . 7764 | . 7794 | . 7823 | . 7852 |
|  | . . . . . 7881 | . 7910 | . 7939 | . 7967 | . 7995 | . 8023 | . 8051 | . 8078 | . 8106 | . 8133 |
|  | . . . . 8159 | . 8186 | . 8212 | . 8238 | . 8264 | . 8289 | . 8315 | . 8340 | . 8365 | . 8389 |
|  | . ... . 8413 | . 8438 | . 8461 | . 8485 | . 8508 | . 8531 | . 8554 | . 8577 | . 8599 | . 8621 |
|  | . . . . . 8643 | . 8665 | . 8686 | . 8708 | . 8729 | . 8749 | . 8770 | . 8790 | . 8810 | . 8830 |
|  | . . . . 8849 | . 8869 | . 8888 | . 8907 | . 8925 | . 8944 | . 8962 | . 8980 | . 8997 | . 9015 |
|  | . . . . 9032 | . 9049 | . 9066 | . 9082 | . 9099 | . 9115 | . 9131 | . 9147 | . 9162 | . 9177 |
|  | . . . . 9192 | . 9207 | . 9222 | . 9236 | . 9251 | . 9265 | . 9279 | . 9292 | . 9306 | . 9319 |
|  | . . . . . 9332 | . 9345 | . 9357 | . 9370 | . 9382 | . 9394 | . 9406 | . 9418 | . 9429 | . 9441 |
|  | . . . . . 9452 | . 9463 | . 9474 | . 9484 | . 9495 | . 9505 | . 9515 | . 9525 | . 9535 | . 9545 |
|  | . . . . . 9554 | . 9564 | . 9573 | . 9582 | . 9591 | . 9599 | . 9608 | . 9616 | . 9625 | . 9633 |
| 1.8 | . . . . . 9641 | . 9649 | . 9656 | . 9664 | . 9671 | . 9678 | . 9686 | . 9693 | . 9699 | . 9706 |
|  | . . . . 9713 | . 8719 | . 9726 | . 9732 | . 9738 | . 9744 | . 9750 | . 9756 | . 9761 | . 9767 |
|  | . . . . 9772 | . 9778 | . 9783 | . 9788 | . 9793 | . 9798 | . 9803 | . 9808 | . 9812 | . 9817 |
|  | 1.... . 9821 | . 9826 | . 9830 | . 9834 | . 9838 | . 9842 | . 9846 | . 9850 | . 9854 | . 9857 |
|  | 2.... . 9861 | . 9864 | . 9868 | . 9871 | . 9875 | . 9878 | . 9881 | . 9884 | . 9887 | . 9890 |
|  | . . . . . 9893 | . 9896 | . 9898 | . 9901 | . 9904 | . 9906 | . 9909 | . 9911 | . 9913 | . 9916 |
|  | 4.... . 9918 | . 9920 | . 9922 | . 9925 | . 9927 | . 9929 | . 9931 | . 9932 | . 9934 | . 9936 |
|  | 5.... 9938 | . 9940 | . 9941 | . 9943 | . 9945 | . 9946 | . 9948 | . 9949 | . 9951 | . 9952 |
|  | . . . . . 9953 | . 9955 | . 9956 | . 9957 | . 9859 | . 9960 | . 9961 | . 9962 | . 9963 | . 9964 |
|  | $7 . .$. . 9965 | . 9966 | . 9967 | . 9968 | . 9969 | . 9970 | . 9971 | . 9972 | . 9973 | . 9974 |
|  | 8.... . 9974 | . 9975 | . 9976 | . 9977 | . 9977 | . 9978 | . 9979 | . 9979 | . 9980 | . 9981 |
|  | . . . . 9981 | . 9982 | . 9982 | . 9983 | . 9984 | . 9984 | . 9985 | . 9985 | . 9986 | . 9986 |
|  | 0.... . 9987 | . 9987 | . 9987 | . 9988 | . 9988 | . 9989 | . 9989 | . 9989 | . 9990 | . 9990 |
|  | 1 . . . . 9990 | . 9991 | . 9991 | . 9991 | . 9991 | . 9992 | . 9992 | . 9992 | . 9993 | . 9993 |
|  | $2 . . .$. . 9993 | . 9993 | . 9994 | . 9994 | . 9994 | . 9994 | . 9994 | . 9995 | . 9995 | . 9995 |
|  | 3 . . . . 9995 | . 9995 | . 9995 | . 9996 | . 9996 | . 9996 | . 9996 | . 9996 | . 9996 | . 9997 |
|  | $4 . .$. . 9997 | . 9997 | . 9997 | . 9997 | . 9997 | . 9997 | . 9997 | . 9997 | . 9997 | . 9998 |

