

**UNIVERSITY OF SWAZILAND**

**FINAL EXAMINATION PAPER 2012**

**TITLE OF PAPER : MULTIVARIATE ANALYSIS**

**COURSE CODE : ST410**

**TIME ALLOWED : 2 (TWO) HOURS**

**REQUIRMENTS : STATISTICAL TABLES  
AND CALCULATOR**

**INSTRUCTIONS : ANSWER ANY 4 (FOUR) QUESTIONS.  
ALL QUESTIONS CARRY EQUAL MARKS.**

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**QUESTION ONE.**

[ 7 + 8 + 5 + 5 marks ]

- 1.1 Discuss factor analysis and principal component analysis with respect to their properties.
- 1.2 Discuss all steps of the methods of principal component factor analysis to obtain the factors.
- 1.3 The following table represents the correlation matrix of seven variables. Examine the table and discuss the suitability of factor analysis.

**Correlation Matrix**

	X1	X2	X3	X4	X5	X6	X7
Correlation X1	1.000	.650	.095	.094	.107	.009	.082
X2	.650	1.000	-.128	-.076	.009	-.064	-.026
X3	.095	-.128	1.000	.639	.683	.568	.363
X4	.094	-.076	.639	1.000	.907	.598	.879
X5	.107	.009	.683	.907	1.000	.788	.722
X6	.009	-.064	.568	.598	.788	1.000	.255
X7	.082	-.026	.363	.879	.722	.255	1.000

- 1.4 Suppose you applied the factor analysis method to the above seven variables which produces two factors: Factor A and Factor B. Predict, using the above correlation matrix, the possible variables that will contribute significantly to Factor A and to Factor B. Explain why?

**QUESTION TWO.**

[ 4 + 5 + 4 + 4 + 8 marks ]

- 2.1 Define principal component analysis (PCA).
- 2.2 State at least five properties of PCA.
- 2.3 Explain the reason why it is important to standardise the variables while doing PCA in some special situation.
- 2.4 Consider the following table:

Component	Eigenvalue	Eigenvectors				
		X <sub>1</sub>	X <sub>2</sub>	X <sub>3</sub>	X <sub>4</sub>	X <sub>5</sub>
1	2.616	0.452	0.462	0.451	0.471	0.398
2	1.532	-0.051	0.300	0.325	0.185	-0.377
3	0.386	0.691	0.341	-0.455	-0.411	-0.179
4	0.302	-0.420	0.548	-0.606	0.388	0.069
5	0.165	0.374	-0.530	-0.343	0.652	-0.192

- a. How many components will you choose? Explain why.
- b. List those selected components and interpret those in terms of original variables, X<sub>i</sub>'s.

**QUESTION THREE.**

[ 6 + 12 + 6 + 1 marks ]

- 3.1 Suppose we have two groups, with 10 subjects in each group. The means for the two variables ( $X_1$  and  $X_2$ ) measures in group A are 10 and 7.5, while the means in group B are 9 and 9.5. The respective pooled sample variances are 9 and 4 for variables  $X_1$  and  $X_2$ , while the pooled covariance is 4.2. Perform Hotellings'  $T^2$  test.
- 3.2 Suppose we have three variables in each of the 3 groups, with 10 subjects per group. Let the sum of squares matrices be as follows:

$$\mathbf{B} = \begin{bmatrix} 1.68 & 1.38 & 1.26 \\ 1.38 & 1.14 & 1.08 \\ 1.26 & 1.08 & 1.26 \end{bmatrix}, \quad \mathbf{W} = \begin{bmatrix} 1.24 & 0.06 & 0.56 \\ 0.06 & 1.08 & 0.18 \\ 0.56 & 0.18 & 2.74 \end{bmatrix}, \quad \& \quad \mathbf{T} = \begin{bmatrix} 2.92 & 1.44 & 1.82 \\ 1.44 & 2.22 & 1.26 \\ 1.82 & 1.26 & 4.00 \end{bmatrix}$$

- Compute Wilk's  $\Lambda$  statistic and  $\phi$ . Use  $\chi^2$  and  $F$  approximation to test the equality of population mean vectors. You must state the null and alternative hypotheses and the final conclusion.
- Perform the one-way analysis of variance procedure to test the equality of variable means for each of those three variables.
- Comment on the results found in part (a) and (b).

**QUESTION FOUR.**

[ 3 + 3 + 1 + 3 + 5 + 10 marks ]

- Discuss the purpose of the discriminant function analysis.
- State the criteria for selecting canonical discriminant functions.
- How many discriminant functions can one get while using discriminant function analysis?
- The following table shows the eigenvalues and corresponding eigenvectors of  $\mathbf{W}^{-1}\mathbf{B}$ :

Component	Eigenvalue	Eigenvectors			
		$X_1$	$X_2$	$X_3$	$X_4$
1	0.425	-0.127	0.037	0.145	-0.083
2	0.039	0.039	0.210	-0.068	-0.077
3	0.016	-0.093	0.025	0.015	0.295
4	0.002	0.149	-0.000	0.133	0.067

- How many groups and variables were considered in this problem?
- List all the canonical discriminant functions.
- Assuming that the  $i^{\text{th}}$  sample size,  $n_i = 25$  for all  $i = 1, 2, 3, 4, 5$ ; test whether the first two of these functions vary significantly from group to group.

**QUESTION FIVE.**

[ 3 + 3 + 4 + 4 + 3 + 8 marks ]

The following tables are part of the full output running SPSS for a set of multivariate variables; not necessarily from the same set of variables. Tables 1-6 were obtained running Factor Analysis and Tables 7-9 were from Discriminant Function Analysis:

**Table 1:**

**Correlation Matrix**

	X1	X2	X3	X4	X5	X6	X7
Correlation X1	1.000	.036	-.671	-.400	-.538	-.645	-.764
X2	.036	1.000	.445	.405	-.026	-.495	-.221
X3	-.671	.445	1.000	.385	.494	.080	.200
X4	-.400	.405	.385	1.000	.060	.199	.185
X5	-.538	-.026	.494	.060	1.000	.271	.210
X6	-.645	-.495	.080	.199	.271	1.000	.424
X7	-.764	-.221	.200	.185	.210	.424	1.000

**Table 2:**

Component	Eigenvalues
1	2.965
2	1.850
3	0.971
4	0.643
5	0.359
6	0.211
7	0.000

**Table 3:**

**Component Matrix<sup>a</sup>**

	Component					
	1	2	3	4	5	6
X1	-.986	5.070E-02	-2.47E-02	.113	.102	-2.99E-02
X2	-1.24E-02	.936	7.822E-02	-.109	3.541E-04	.325
X3	.683	.566	-.258	-3.93E-02	-.333	-.186
X4	.483	.481	.590	.327	.241	-.153
X5	.621	4.503E-02	-.677	.186	.344	4.014E-02
X6	.641	-.562	.179	.402	-.186	.212
X7	.705	-.319	.246	-.560	.166	2.041E-02

Extraction Method: Principal Component Analysis.  
a. 6 components extracted.

**Table 4:**

**Component Matrix<sup>a</sup>**

	Component		
	1	2	3
X1	-.986	5.070E-02	-2.47E-02
X2	-1.24E-02	.936	7.822E-02
X3	.683	.566	-.258
X4	.483	.481	.590
X5	.621	4.503E-02	-.677
X6	.641	-.562	.179
X7	.705	-.319	.246

Extraction Method: Principal Component Analysis.  
a. 3 components extracted.

**Table 5:**

**Rotated Component Matrix<sup>a</sup>**

	Component		
	1	2	3
X1	-.764	-.550	-.301
X2	-.515	.195	.761
X3	8.552E-02	.756	.523
X4	.295	-2.72E-02	.851
X5	.185	.892	-.127
X6	.858	8.626E-02	-.124
X7	.790	.141	.120

Extraction Method: Principal Component Analysis.  
Rotation Method: Varimax with Kaiser Normalization.  
a. Rotation converged in 6 iterations.

**Table 6:**

**Component Score Coefficient Matrix**

	Component		
	1	2	3
X1	-.270	-.170	-.103
X2	-.263	.078	.433
X3	-.102	.422	.169
X4	.192	-.291	.584
X5	-.113	.658	-.290
X6	.400	-.097	-.063
X7	.365	-.099	.083

Extraction Method: Principal Component Analysis.  
 Rotation Method: Varimax with Kaiser Normalization.

**Table 7:**

**Wilks' Lambda**

Test of Function(s)	Wilks' Lambda	Chi-square	df	Sig.
1 through 2	.147	39.244	12	.000
2	.656	8.655	5	.124

**Table 8:**

**Canonical Discriminant Function Coefficients**

	Function	
	1	2
X1	.019	.083
X2	.305	-1.603
X3	.059	.135
X4	1.512	1.263
X5	.076	.057
X6	-.268	-.064
(Constant)	.217	-3.732

Unstandardized coefficients

**Table 9:**

**Standardized Canonical Discriminant Function Coefficient**

	Function	
	1	2
X1	.268	1.193
X2	.202	-1.065
X3	.412	.938
X4	.573	.478
X5	.129	.097
X6	-1.040	-.249

- Examine Table 1 and discuss the suitability of principal component analysis. How many principal components will you have?
- How many factors will you choose if you wish to use factor analysis method? Explain your answer.
- How many factors will you get in your factor model from Table 3? List the last two equations of your model and compute their communalities.
- Suppose the same data were analyzed using the restriction on the number of factors. How many factors were chosen in Table 4? List the first two equations of your model and compute their communalities.
- List all equations needed to compute factor scores.
- Write all the discriminant functions and test whether each of those is significant at 10% level of significance.

TABLE A.2 Percentiles of the  $t$  Distribution

Entry is  $t(A; \nu)$  where  $P\{t(\nu) \leq t(A; \nu)\} = A$



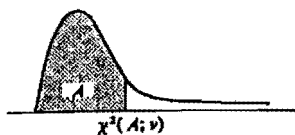
$\nu$	A						
	.60	.70	.80	.85	.90	.95	.975
1	0.325	0.727	1.376	1.963	3.078	6.314	12.706
2	0.289	0.617	1.061	1.386	1.886	2.920	4.303
3	0.277	0.584	0.978	1.250	1.638	2.353	3.182
4	0.271	0.569	0.941	1.190	1.533	2.132	2.776
5	0.267	0.559	0.920	1.156	1.476	2.015	2.571
6	0.265	0.553	0.906	1.134	1.440	1.943	2.447
7	0.263	0.549	0.896	1.119	1.415	1.895	2.365
8	0.262	0.546	0.889	1.108	1.397	1.860	2.306
9	0.261	0.543	0.883	1.100	1.383	1.833	2.262
10	0.260	0.542	0.879	1.093	1.372	1.812	2.228
11	0.260	0.540	0.876	1.088	1.363	1.796	2.201
12	0.259	0.539	0.873	1.083	1.356	1.782	2.179
13	0.259	0.537	0.870	1.079	1.350	1.771	2.160
14	0.258	0.537	0.868	1.076	1.345	1.761	2.145
15	0.258	0.536	0.866	1.074	1.341	1.753	2.131
16	0.258	0.535	0.865	1.071	1.337	1.746	2.120
17	0.257	0.534	0.863	1.069	1.333	1.740	2.110
18	0.257	0.534	0.862	1.067	1.330	1.734	2.101
19	0.257	0.533	0.861	1.066	1.328	1.729	2.093
20	0.257	0.533	0.860	1.064	1.325	1.725	2.086
21	0.257	0.532	0.859	1.063	1.323	1.721	2.080
22	0.256	0.532	0.858	1.061	1.321	1.717	2.074
23	0.256	0.532	0.858	1.060	1.319	1.714	2.069
24	0.256	0.531	0.857	1.059	1.318	1.711	2.064
25	0.256	0.531	0.856	1.058	1.316	1.708	2.060
26	0.256	0.531	0.856	1.058	1.315	1.706	2.056
27	0.256	0.531	0.855	1.057	1.314	1.703	2.052
28	0.256	0.530	0.855	1.056	1.313	1.701	2.048
29	0.256	0.530	0.854	1.055	1.311	1.699	2.045
30	0.256	0.530	0.854	1.055	1.310	1.697	2.042
40	0.255	0.529	0.851	1.050	1.303	1.684	2.021
60	0.254	0.527	0.848	1.045	1.296	1.671	2.000
120	0.254	0.526	0.845	1.041	1.289	1.658	1.980
$\infty$	0.253	0.524	0.842	1.036	1.282	1.645	1.960

TABLE A.2 (concluded) Percentiles of the  $t$  Distribution

$\nu$	A						
	.98	.985	.99	.9925	.995	.9975	.9995
1	15.895	21.205	31.821	42.434	63.657	127.322	636.590
2	4.849	5.643	6.965	8.073	9.923	14.089	31.598
3	3.482	3.896	4.541	5.047	5.841	7.453	12.924
4	2.999	3.298	3.747	4.088	4.604	5.598	8.610
5	2.757	3.003	3.365	3.634	4.032	4.773	6.869
6	2.612	2.829	3.143	3.372	3.707	4.317	5.959
7	2.517	2.715	2.998	3.203	3.499	4.029	5.408
8	2.449	2.634	2.896	3.085	3.355	3.833	5.041
9	2.398	2.574	2.821	2.998	3.250	3.690	4.781
10	2.359	2.527	2.764	2.932	3.169	3.581	4.587
11	2.328	2.491	2.718	2.879	3.106	3.497	4.437
12	2.303	2.461	2.681	2.836	3.055	3.428	4.318
13	2.282	2.436	2.650	2.801	3.012	3.372	4.221
14	2.264	2.415	2.624	2.771	2.977	3.326	4.140
15	2.249	2.397	2.602	2.746	2.947	3.286	4.073
16	2.235	2.382	2.583	2.724	2.921	3.252	4.015
17	2.224	2.368	2.567	2.706	2.898	3.222	3.965
18	2.214	2.356	2.552	2.689	2.878	3.197	3.922
19	2.205	2.346	2.539	2.674	2.861	3.174	3.883
20	2.197	2.336	2.528	2.661	2.845	3.153	3.849
21	2.189	2.328	2.518	2.649	2.831	3.135	3.819
22	2.183	2.320	2.508	2.639	2.819	3.119	3.792
23	2.177	2.313	2.500	2.629	2.807	3.104	3.768
24	2.172	2.307	2.492	2.620	2.797	3.091	3.745
25	2.167	2.301	2.485	2.612	2.787	3.078	3.725
26	2.162	2.296	2.479	2.605	2.779	3.067	3.707
27	2.158	2.291	2.473	2.598	2.771	3.057	3.690
28	2.154	2.286	2.467	2.592	2.763	3.047	3.674
29	2.150	2.282	2.462	2.586	2.756	3.038	3.659
30	2.147	2.278	2.457	2.581	2.750	3.030	3.646
40	2.123	2.250	2.423	2.542	2.704	2.971	3.551
60	2.099	2.223	2.390	2.504	2.660	2.915	3.460
120	2.076	2.196	2.358	2.468	2.617	2.860	3.373
$\infty$	2.054	2.170	2.326	2.432	2.576	2.807	3.291

TABLE A.3 Percentiles of the  $\chi^2$  Distribution

Entry is  $\chi^2(A; \nu)$  where  $P\{\chi^2(\nu) \leq \chi^2(A; \nu)\} = A$

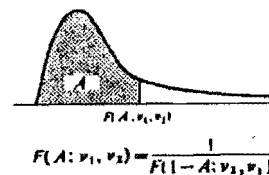


$\nu$	A									
	.005	.010	.025	.050	.100	.900	.950	.975	.990	.995
1	0.0 <sup>+</sup> 393	0.0 <sup>+</sup> 157	0.0 <sup>+</sup> 982	0.0 <sup>+</sup> 393	0.0158	2.71	3.84	5.02	6.63	7.88
2	0.0100	0.0201	0.0506	0.103	0.211	4.61	5.99	7.38	9.21	10.60
3	0.072	0.115	0.216	0.352	0.584	6.25	7.81	9.35	11.34	12.84
4	0.207	0.297	0.484	0.711	1.064	7.78	9.49	11.14	13.28	14.86
5	0.412	0.534	0.831	1.145	1.61	9.24	11.07	12.83	15.09	16.75
6	0.676	0.872	1.24	1.64	2.20	10.64	12.59	14.45	16.81	18.55
7	0.989	1.24	1.69	2.17	2.83	12.02	14.07	16.01	18.48	20.28
8	1.34	1.65	2.18	2.73	3.49	13.36	15.51	17.53	20.09	21.96
9	1.73	2.09	2.70	3.33	4.17	14.68	16.92	19.02	21.67	23.59
10	2.16	2.56	3.25	3.94	4.87	15.99	18.31	20.48	23.21	25.19
11	2.60	3.05	3.82	4.57	5.58	17.28	19.68	21.92	24.73	26.76
12	3.07	3.57	4.40	5.23	6.30	18.55	21.03	23.34	26.22	28.30
13	3.57	4.11	5.01	5.89	7.04	19.81	22.36	24.74	27.69	29.82
14	4.07	4.66	5.63	6.57	7.79	21.06	23.68	26.12	29.14	31.32
15	4.60	5.23	6.26	7.26	8.55	22.31	25.00	27.49	30.58	32.80
16	5.14	5.81	6.91	7.96	9.31	23.54	26.30	28.85	32.00	34.27
17	5.70	6.41	7.56	8.67	10.09	24.77	27.59	30.19	33.41	35.72
18	6.26	7.01	8.23	9.39	10.86	25.99	28.87	31.53	34.81	37.16
19	6.84	7.63	8.91	10.12	11.63	27.20	30.14	32.85	36.19	38.58
20	7.43	8.26	9.59	10.85	12.44	28.41	31.41	34.17	37.57	40.00
21	8.03	8.90	10.28	11.59	13.24	29.62	32.67	35.48	38.93	41.40
22	8.64	9.54	10.98	12.34	14.04	30.81	33.92	36.78	40.29	42.80
23	9.26	10.20	11.69	13.09	14.85	32.01	35.17	38.08	41.64	44.18
24	9.89	10.86	12.40	13.85	15.66	33.20	36.42	39.36	42.98	45.56
25	10.52	11.52	13.12	14.61	16.47	34.38	37.65	40.65	44.31	46.93
26	11.16	12.20	13.84	15.38	17.29	35.56	38.89	41.92	45.64	48.29
27	11.81	12.88	14.57	16.15	18.11	36.74	40.11	43.19	46.96	49.64
28	12.46	13.56	15.31	16.93	18.94	37.92	41.34	44.46	48.28	50.99
29	13.12	14.26	16.05	17.71	19.77	39.09	42.56	45.72	49.59	52.34
30	13.79	14.95	16.79	18.49	20.60	40.26	43.77	46.98	50.89	53.67
40	20.71	22.16	24.43	26.51	29.05	51.81	55.76	59.34	63.69	66.77
50	27.99	29.71	32.36	34.76	37.69	63.17	67.50	71.42	76.15	79.49
60	35.53	37.48	40.48	43.19	46.46	74.40	79.08	83.30	88.38	91.95
70	43.28	45.44	48.76	51.74	55.33	85.53	90.53	95.02	100.4	104.2
80	51.17	53.54	57.15	60.39	64.28	96.58	101.9	106.6	112.3	116.3
90	59.20	61.75	65.65	69.13	73.29	107.6	113.1	118.1	124.1	128.3
100	67.33	70.06	74.22	77.93	82.36	118.5	124.3	129.6	135.8	140.2

Source: Reprinted, with permission, from C. M. Thompson, "Table of Percentage Points of the Chi-Square Distribution," *Biometrika* 32 (1941), pp. 188-89.

TABLE A.4 Percentiles of the  $F$  Distribution

Entry is  $F(A; \nu_1, \nu_2)$  where  $P\{F(\nu_1, \nu_2) \leq F(A; \nu_1, \nu_2)\} = A$



$$F(A; \nu_1, \nu_2) = \frac{1}{F(1-A; \nu_2, \nu_1)}$$

Den. df	A	Numerator df								
		1	2	3	4	5	6	7	8	9
1	.50	1.00	1.50	1.71	1.82	1.89	1.94	1.98	2.00	2.03
	.90	39.9	49.5	53.6	55.8	57.2	58.2	58.9	59.4	59.9
	.95	161	200	216	225	230	234	237	239	241
	.975	648	800	864	900	922	937	948	957	963
	.99	4,052	5,000	5,403	5,625	5,764	5,859	5,928	5,981	6,022
	.995	16,211	20,000	21,615	22,500	23,056	23,437	23,715	23,925	24,091
	.999	405,280	500,000	540,380	562,500	576,400	585,940	592,870	598,140	602,280
2	.50	0.667	1.00	1.13	1.21	1.25	1.28	1.30	1.32	1.33
	.90	8.53	9.00	9.16	9.24	9.29	9.33	9.35	9.37	9.38
	.95	18.5	19.0	19.2	19.2	19.3	19.3	19.4	19.4	19.4
	.975	38.5	39.0	39.2	39.2	39.3	39.3	39.4	39.4	39.4
	.99	98.5	99.0	99.2	99.2	99.3	99.3	99.4	99.4	99.4
	.995	199	199	199	199	199	199	199	199	199
	.999	998.5	999.0	999.2	999.2	999.3	999.3	999.4	999.4	999.4
3	.50	0.585	0.881	1.00	1.06	1.10	1.13	1.15	1.16	1.17
	.90	5.54	5.46	5.39	5.34	5.31	5.28	5.27	5.25	5.24
	.95	10.1	9.55	9.28	9.12	9.01	8.94	8.89	8.85	8.81
	.975	17.4	16.0	15.4	15.1	14.9	14.7	14.6	14.5	14.5
	.99	34.1	30.8	29.5	28.7	28.2	27.9	27.7	27.5	27.3
	.995	55.6	49.8	47.5	46.2	45.4	44.8	44.4	44.1	43.9
	.999	167.0	148.5	141.1	137.1	134.6	132.8	131.6	130.6	129.9
4	.50	0.549	0.828	0.941	1.00	1.04	1.06	1.08	1.09	1.10
	.90	4.54	4.32	4.19	4.11	4.05	4.01	3.98	3.95	3.94
	.95	7.71	6.94	6.59	6.39	6.26	6.16	6.09	6.04	6.00
	.975	12.2	10.6	9.98	9.60	9.36	9.20	9.07	8.98	8.90
	.99	21.2	18.0	16.7	16.0	15.5	15.2	15.0	14.8	14.7
	.995	31.3	26.3	24.3	23.2	22.5	22.0	21.6	21.4	21.1
	.999	74.1	61.2	56.2	53.4	51.7	50.5	49.7	49.0	48.5
5	.50	0.528	0.799	0.907	0.965	1.00	1.02	1.04	1.05	1.06
	.90	4.06	3.78	3.62	3.52	3.45	3.40	3.37	3.34	3.32
	.95	6.61	5.79	5.41	5.19	5.05	4.95	4.88	4.82	4.77
	.975	10.0	8.43	7.76	7.39	7.15	6.98	6.85	6.76	6.68
	.99	16.3	13.3	12.1	11.4	11.0	10.7	10.5	10.3	10.2
	.995	22.8	18.3	16.5	15.6	14.9	14.5	14.2	14.0	13.8
	.999	47.2	37.1	33.2	31.1	29.8	28.8	28.2	27.6	27.2
6	.50	0.515	0.780	0.886	0.942	0.977	1.00	1.02	1.03	1.04
	.90	3.78	3.46	3.29	3.18	3.11	3.05	3.01	2.98	2.96
	.95	5.99	5.14	4.76	4.53	4.39	4.28	4.21	4.15	4.10
	.975	8.81	7.26	6.60	6.23	5.99	5.82	5.70	5.60	5.52
	.99	13.7	10.9	9.78	9.15	8.75	8.47	8.26	8.10	7.98
	.995	18.6	14.5	12.9	12.0	11.5	11.1	10.8	10.6	10.4
	.999	35.5	27.0	23.7	21.9	20.8	20.0	19.5	19.0	18.7
7	.50	0.506	0.767	0.871	0.926	0.960	0.983	1.00	1.01	1.02
	.90	3.59	3.26	3.07	2.96	2.88	2.83	2.78	2.75	2.72
	.95	5.59	4.74	4.35	4.12	3.97	3.87	3.79	3.73	3.68
	.975	8.07	6.54	5.89	5.52	5.29	5.12	4.99	4.90	4.82
	.99	12.2	9.55	8.45	7.85	7.46	7.19	6.99	6.84	6.72
	.995	16.2	12.4	10.9	10.1	9.52	9.16	8.89	8.68	8.51
	.999	29.2	21.7	18.8	17.2	16.2	15.5	15.0	14.6	14.3

Den. df	A	Numerator df								
		10	12	15	20	24	30	60	120	∞
1	.50	2.04	2.07	2.09	2.12	2.13	2.15	2.17	2.18	2.20
	.90	60.2	60.7	61.2	61.7	62.0	62.3	62.8	63.1	63.3
	.95	242	244	246	248	249	250	252	253	254
	.975	969	977	985	993	997	1,001	1,010	1,014	1,018
	.99	6,056	6,106	6,157	6,209	6,235	6,261	6,313	6,339	6,366
	.995	24,224	24,426	24,630	24,836	24,940	25,044	25,253	25,359	25,464
	.999	603,620	610,670	615,760	620,910	623,500	626,100	631,340	633,970	636,620
2	.50	1.34	1.36	1.38	1.39	1.40	1.41	1.43	1.43	1.44
	.90	9.39	9.41	9.42	9.44	9.45	9.46	9.47	9.48	9.49
	.95	19.4	19.4	19.4	19.4	19.5	19.5	19.5	19.5	19.5
	.975	39.4	39.4	39.4	39.4	39.5	39.5	39.5	39.5	39.5
	.99	99.4	99.4	99.4	99.4	99.5	99.5	99.5	99.5	99.5
	.995	199	199	199	199	199	199	199	199	200
	.999	999.4	999.4	999.4	999.4	999.5	999.5	999.5	999.5	999.5
3	.50	1.18	1.20	1.21	1.23	1.23	1.24	1.25	1.26	1.27
	.90	5.23	5.22	5.20	5.18	5.18	5.17	5.15	5.14	5.13
	.95	8.79	8.74	8.70	8.66	8.64	8.62	8.57	8.55	8.53
	.975	14.4	14.3	14.3	14.2	14.1	14.1	14.0	13.9	13.9
	.99	27.2	27.1	26.9	26.7	26.6	26.5	26.3	26.2	26.1
	.995	43.7	43.4	43.1	42.8	42.6	42.5	42.1	42.0	41.8
	.999	129.2	128.3	127.4	126.4	125.9	125.4	124.5	124.0	123.5
4	.50	1.11	1.13	1.14	1.15	1.16	1.16	1.18	1.18	1.19
	.90	3.92	3.90	3.87	3.84	3.83	3.82	3.79	3.78	3.76
	.95	5.96	5.91	5.86	5.80	5.77	5.75	5.69	5.66	5.63
	.975	8.84	8.75	8.66	8.56	8.51	8.46	8.36	8.31	8.26
	.99	14.5	14.4	14.2	14.0	13.9	13.8	13.7	13.6	13.5
	.995	21.0	20.7	20.4	20.2	20.0	19.9	19.6	19.5	19.3
	.999	48.1	47.4	46.8	46.1	45.8	45.4	44.7	44.4	44.1
5	.50	1.07	1.09	1.10	1.11	1.12	1.12	1.14	1.14	1.15
	.90	3.30	3.27	3.24	3.21	3.19	3.17	3.14	3.12	3.11
	.95	4.74	4.68	4.62	4.56	4.53	4.50	4.43	4.40	4.37
	.975	6.62	6.52	6.43	6.33	6.28	6.23	6.12	6.07	6.02
	.99	10.1	9.89	9.72	9.55	9.47	9.38	9.20	9.11	9.02
	.995	13.6	13.4	13.1	12.9	12.8	12.7	12.4	12.3	12.1
	.999	26.9	26.4	25.9	25.4	25.1	24.9	24.3	24.1	23.8
6	.50	1.05	1.06	1.07	1.08	1.09	1.10	1.11	1.12	1.12
	.90	2.94	2.90	2.87	2.84	2.82	2.80	2.76	2.74	2.72
	.95	4.06	4.00	3.94	3.87	3.84	3.81	3.74	3.70	3.67
	.975	5.46	5.37	5.27	5.17	5.12	5.07	4.96	4.90	4.85
	.99	7.87	7.72	7.56	7.40	7.31	7.23	7.06	6.97	6.88
	.995	10.2	10.0	9.81	9.59	9.47	9.36	9.12	9.00	8.88
	.999	18.4	18.0	17.6	17.1	16.9	16.7	16.2	16.0	15.7
7	.50	1.03	1.04	1.05	1.07	1.07	1.08	1.09	1.10	1.10
	.90	2.70	2.67	2.63	2.59	2.58	2.56	2.51	2.49	2.47
	.95	3.64	3.57	3.51	3.44	3.41	3.38	3.30	3.27	3.23
	.975	4.76	4.67	4.57	4.47	4.42	4.36	4.25	4.20	4.14
	.99	6.62	6.47	6.31	6.16	6.07	5.99	5.82	5.74	5.65
	.995	8.38	8.18	7.97	7.75	7.65	7.53	7.31	7.19	7.08
	.999	14.1	13.7	13.3	12.9	12.7	12.5	12.1	11.9	11.7



Den. df	Numerator df								
	1	2	3	4	5	6	7	8	9
8 .50	0.499	0.757	0.860	0.915	0.948	0.971	0.988	1.00	1.01
.90	3.46	3.11	2.92	2.81	2.73	2.67	2.62	2.59	2.56
.95	5.32	4.46	4.07	3.84	3.69	3.58	3.50	3.44	3.39
.975	7.57	6.06	5.42	5.05	4.82	4.65	4.53	4.43	4.36
.99	11.3	8.65	7.59	7.01	6.63	6.37	6.18	6.03	5.91
.995	14.7	11.0	9.60	8.81	8.30	7.95	7.69	7.50	7.34
.999	25.4	18.5	15.8	14.4	13.5	12.9	12.4	12.0	11.8
9 .50	0.494	0.749	0.852	0.906	0.939	0.962	0.978	0.990	1.00
.90	3.36	3.01	2.81	2.69	2.61	2.55	2.51	2.47	2.44
.95	5.12	4.26	3.86	3.63	3.48	3.37	3.29	3.23	3.18
.975	7.21	5.71	5.08	4.72	4.48	4.32	4.20	4.10	4.03
.99	10.6	8.02	6.99	6.42	6.06	5.80	5.61	5.47	5.35
.995	13.6	10.1	8.72	7.96	7.47	7.13	6.88	6.69	6.54
.999	22.9	16.4	13.9	12.6	11.7	11.1	10.7	10.4	10.1
10 .50	0.490	0.743	0.845	0.899	0.932	0.954	0.971	0.983	0.992
.90	3.29	2.92	2.73	2.61	2.52	2.46	2.41	2.38	2.35
.95	4.96	4.10	3.71	3.48	3.33	3.22	3.14	3.07	3.02
.975	6.94	5.46	4.83	4.47	4.24	4.07	3.95	3.85	3.78
.99	10.0	7.36	6.55	5.99	5.64	5.39	5.20	5.06	4.94
.995	12.8	9.43	8.08	7.34	6.87	6.54	6.30	6.12	5.97
.999	21.0	14.9	12.6	11.3	10.5	9.93	9.52	9.20	8.96
12 .50	0.484	0.735	0.835	0.888	0.921	0.943	0.959	0.972	0.981
.90	3.18	2.81	2.61	2.48	2.39	2.33	2.28	2.24	2.21
.95	4.75	3.89	3.49	3.26	3.11	3.00	2.91	2.85	2.80
.975	6.55	5.10	4.47	4.12	3.89	3.73	3.61	3.51	3.44
.99	9.33	6.93	5.95	5.41	5.06	4.82	4.64	4.50	4.39
.995	11.8	8.51	7.23	6.52	6.07	5.76	5.52	5.35	5.20
.999	18.6	13.0	10.8	9.63	8.89	8.38	8.00	7.71	7.48
15 .50	0.478	0.726	0.826	0.878	0.911	0.933	0.949	0.960	0.970
.90	3.07	2.70	2.49	2.36	2.27	2.21	2.16	2.12	2.09
.95	4.54	3.68	3.29	3.06	2.90	2.79	2.71	2.64	2.59
.975	6.20	4.77	4.15	3.80	3.58	3.41	3.29	3.20	3.12
.99	8.68	6.36	5.42	4.89	4.56	4.32	4.14	4.00	3.89
.995	10.8	7.70	6.48	5.80	5.37	5.07	4.83	4.67	4.54
.999	16.6	11.3	9.34	8.25	7.57	7.09	6.74	6.47	6.26
20 .50	0.472	0.718	0.816	0.868	0.900	0.922	0.938	0.950	0.959
.90	2.97	2.59	2.38	2.25	2.16	2.09	2.04	2.00	1.96
.95	4.35	3.49	3.10	2.87	2.71	2.60	2.51	2.45	2.39
.975	5.87	4.46	3.86	3.51	3.29	3.13	3.01	2.91	2.84
.99	8.10	5.85	4.94	4.43	4.10	3.87	3.70	3.56	3.46
.995	9.94	6.99	5.82	5.17	4.76	4.47	4.26	4.09	3.96
.999	14.8	9.95	8.10	7.10	6.46	6.02	5.69	5.44	5.24
24 .50	0.469	0.714	0.812	0.863	0.895	0.917	0.932	0.944	0.953
.90	2.93	2.54	2.33	2.19	2.10	2.04	1.98	1.94	1.91
.95	4.26	3.40	3.01	2.78	2.62	2.51	2.42	2.36	2.30
.975	5.72	4.32	3.72	3.38	3.15	2.99	2.87	2.78	2.70
.99	7.82	5.61	4.72	4.22	3.90	3.67	3.50	3.36	3.26
.995	9.55	6.66	5.52	4.89	4.49	4.20	3.99	3.83	3.69
.999	14.0	9.34	7.55	6.59	5.98	5.55	5.23	4.99	4.80

Den. df	Numerator df								
	10	12	15	20	24	30	60	120	∞
8 .50	1.02	1.03	1.04	1.05	1.06	1.07	1.08	1.08	1.09
.90	2.54	2.50	2.46	2.42	2.40	2.38	2.34	2.32	2.29
.95	3.35	3.28	3.22	3.15	3.12	3.08	3.01	2.97	2.93
.975	4.30	4.20	4.10	4.00	3.95	3.89	3.78	3.73	3.67
.99	5.81	5.67	5.52	5.36	5.28	5.20	5.01	4.95	4.86
.995	7.21	7.01	6.81	6.61	6.50	6.40	6.18	6.06	5.95
.999	11.5	11.2	10.8	10.5	10.3	10.1	9.73	9.53	9.33
9 .50	1.01	1.02	1.03	1.04	1.05	1.05	1.07	1.07	1.08
.90	2.42	2.38	2.34	2.30	2.28	2.25	2.21	2.18	2.16
.95	3.14	3.07	3.01	2.94	2.90	2.86	2.79	2.75	2.71
.975	3.96	3.87	3.77	3.67	3.61	3.56	3.45	3.39	3.33
.99	5.26	5.11	4.96	4.81	4.73	4.65	4.48	4.40	4.31
.995	6.42	6.23	6.03	5.83	5.73	5.62	5.41	5.30	5.19
.999	9.89	9.57	9.24	8.90	8.72	8.55	8.19	8.00	7.81
10 .50	1.00	1.01	1.02	1.03	1.04	1.05	1.06	1.06	1.07
.90	2.32	2.28	2.24	2.20	2.18	2.16	2.11	2.08	2.06
.95	2.98	2.91	2.84	2.77	2.74	2.70	2.62	2.58	2.54
.975	3.72	3.62	3.52	3.42	3.37	3.31	3.20	3.14	3.08
.99	4.85	4.71	4.56	4.41	4.33	4.25	4.08	4.00	3.91
.995	5.85	5.66	5.47	5.27	5.17	5.07	4.86	4.75	4.64
.999	8.75	8.45	8.13	7.80	7.64	7.47	7.12	6.94	6.76
12 .50	0.989	1.00	1.01	1.02	1.03	1.03	1.05	1.05	1.06
.90	2.19	2.15	2.10	2.06	2.04	2.01	1.96	1.93	1.90
.95	2.75	2.69	2.62	2.54	2.51	2.47	2.38	2.34	2.30
.975	3.37	3.28	3.18	3.07	3.02	2.96	2.85	2.79	2.72
.99	4.30	4.16	4.01	3.86	3.78	3.70	3.54	3.45	3.36
.995	5.09	4.91	4.72	4.53	4.43	4.33	4.12	4.01	3.90
.999	7.29	7.00	6.71	6.40	6.25	6.09	5.76	5.59	5.42
15 .50	0.977	0.989	1.00	1.01	1.02	1.02	1.03	1.04	1.05
.90	2.06	2.02	1.97	1.92	1.90	1.87	1.82	1.79	1.76
.95	2.54	2.48	2.40	2.33	2.29	2.25	2.16	2.11	2.07
.975	3.06	2.96	2.86	2.76	2.70	2.64	2.52	2.46	2.40
.99	3.80	3.67	3.52	3.37	3.29	3.21	3.05	2.96	2.87
.995	4.42	4.25	4.07	3.88	3.79	3.69	3.48	3.37	3.26
.999	6.08	5.81	5.54	5.25	5.10	4.93	4.64	4.48	4.31
20 .50	0.966	0.977	0.989	1.00	1.01	1.01	1.02	1.03	1.03
.90	1.94	1.89	1.84	1.79	1.77	1.74	1.68	1.64	1.61
.95	2.35	2.28	2.20	2.12	2.08	2.04	1.95	1.90	1.84
.975	2.77	2.68	2.57	2.46	2.41	2.35	2.22	2.16	2.09
.99	3.37	3.23	3.09	2.94	2.86	2.78	2.61	2.52	2.42
.995	3.85	3.68	3.50	3.32	3.22	3.12	2.92	2.81	2.69
.999	5.08	4.82	4.56	4.29	4.15	4.00	3.70	3.54	3.38
24 .50	0.961	0.972	0.983	0.994	1.00	1.01	1.02	1.02	1.03
.90	1.88	1.83	1.78	1.73	1.70	1.67	1.61	1.57	1.53
.95	2.25	2.18	2.11	2.03	1.98	1.94	1.84	1.79	1.73
.975	2.64	2.54	2.44	2.33	2.27	2.21	2.08	2.01	1.94
.99	3.17	3.03	2.89	2.74	2.66	2.58	2.40	2.31	2.21
.995	3.59	3.42	3.25	3.06	2.97	2.87	2.66	2.55	2.43
.999	4.64	4.39	4.14	3.87	3.74	3.59	3.29	3.14	2.97

