

UNIVERSITY OF SWAZILAND

FINAL EXAMINATION PAPER 2013

TITLE OF PAPER : LINEAR STATISTICAL METHODS
COURSE CODE : ST204
TIME ALLOWED : 2 (TWO) HOURS
**REQUIRMENTS : STATISTICAL TABLES
AND CALCULATOR**
**INSTRUCTIONS : ANSWER ANY 4 (FOUR) QUESTIONS.
ALL QUESTIONS CARRY EQUAL MARKS.**

**THIS PAPER IS NOT TO BE OPENED UNTIL PERMISSION HAS BEEN
GRANTED BY THE INVIGILATOR**

QUESTION ONE.

[6 + 4 + 3 + 6 + 6 marks]

- 1.1 Define Simple Linear Regression Model with distribution of error terms unspecified and also define when the distribution is specified.
- 1.2 Discuss the important features of the model.
- 1.3 Assume that $X = 0$ is within the scope of the model defined in part 1.1. What is the implication for the regression function if $\beta_0 = 0$? How would the regression function plot on a graph?
- 1.4 Show that the least square estimate of the regression coefficient, β_1 as follows:

$$\hat{\beta}_1 = \frac{n \sum X_i Y_i - \sum X_i \sum Y_i}{n \sum X_i^2 - (\sum X_i)^2}$$

- 1.5 Define residuals. State some of the departures from the simple linear regression model that the residuals can detect.

QUESTION TWO.

[2 + 5 + 2 + 1 + 5 + 5 + 5 marks]

A substance used in biological and medical research is shipped by airfreight to users in cartons of 1000 ampules. The data below, involving 10 shipments, were collected on the number of times the carton was transferred from one aircraft to another over the shipment route and the number of ampules found to be broken upon arrival.

Number of Transfers	Number of Broken Ampules
1	16
0	9
2	17
0	12
3	22
1	13
0	8
1	15
2	19
0	11

- a. Identify the response variable and the predictor variable.
- b. Fit the regression line, $Y_i = \beta_0 + \beta_1 X_i + \varepsilon_i$
- c. Interpret the estimated values of β_0 and β_1 .
- d. Predict the number of broken ampules if shipment route needs five transfers.
- e. Find the point estimator of σ^2 .
- f. Test $\beta_1 = 10$ against $\beta_1 > 10$ at 5% level of significance.
- g. Test $\beta_2 = 5$ against $\beta_2 < 5$ at 1% level of significance.

QUESTION THREE.

[10 + 7 + 4 + 4 marks]

- 3.1 State the single-factor ANOVA model and its important features.
- 3.2 Find the least squares estimator of μ_j of the model in part 3.1.
- 3.3 An experiment was conducted to determine the effect of three methods of soil preparation on the first-year growth of slash pine seedling. Four locations were selected and each location was divided into three plots (assuming similar fertility level between the plots). Each soil preparation was applied randomly to a plot within each location. On each plot the same number of seedling was planted, and the observation recorded was the average first-year growth (in centimetres) of the seedling on each plot. Answer the following questions using only the ANOVA Table given below:

ANOVA

Source of Variation		Sum of Squares	df	Mean Square	F	Sig
SoilPrep	Location	38.000	2	19.000	10.059	.012
Error		61.667	3	20.556	10.882	.008
Total		111.333	6	1.889		
		111.000	11			

- a. Do the data provide sufficient evidence to indicate a difference in the mean growth for the three soil preparations? Explain.
- b. Is there evidence to indicate a difference in mean growth for the four locations? Explain.

QUESTION FOUR.

[8 + 3 + 14 marks]

- 4.1 Compare Cell Means Model and Factor Effects Model for single factor studies in terms of similarities and differences in the F-test of analysis of variance.
- 4.2 A consumer organization studied the effect of age of automobile owner on size of cash offer for a used car by utilizing 4 persons in each of three age groups who acted as the owner of a used car. A six-year-old car was selected for the experiment, the "owners" solicited cash offers for this car from 12 dealers selected at random from the dealers in the region. Randomization was used in assigning the dealers to the "owners." Assume that the Cell Means Model is applicable. The offers (in thousand Emalangeni) follow:

Age Group	Dealer			
	1	2	3	4
1	23	25	21	22
2	28	27	27	29
3	23	20	25	21

- a. Identify the dependent variable, factor studied and factor levels.
- b. Complete the computation of the ANOVA table and conduct the F test. Clearly state all the steps in the test including the conclusion.

QUESTION FIVE.

[2 + 2 + 5 + 5 + 5 + 3 + 3 marks]

The following output was obtained from running the model, $Y_i = \beta_0 + \beta_1 X_i + \varepsilon_i$ using SPSS:

ANOVA^b

Model	Sum of Squares	df	Mean Square	F	Sig.
1 Regression	4.847	1	4.847	26.482	.000 ^a
Residual	2.563	14	.183		
Total	7.410	15			

- a. Predictors: (Constant), X
 b. Dependent Variable: Y

Coefficients^a

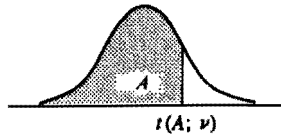
Model	Unstandardized Coefficients		Standardized Coefficients	t	Sig.
	B	Std. Error	Beta		
1 (Constant)	-1.571	.822		-1.911	.077
X	.819	.159	.809	5.146	.000

- a. Dependent Variable: Y

- State the fitted regression line.
- State the null and alternative hypotheses for the F-test of the above ANOVA table.
- Perform the F-test and clearly state the conclusion.
- Test $\beta_0 = -2$ against $\beta_0 \neq -2$ at $\alpha = 0.05$.
- Test $\beta_1 = 1$ against $\beta_1 < 1$ at $\alpha = 0.01$.
- Test $\beta_0 = 0$ against $\beta_0 \neq 0$ at $\alpha = 0.05$.
- Test $\beta_1 = 0$ against $\beta_1 \neq 0$ at $\alpha = 0.01$.

TABLE A.2 Percentiles of the *t* Distribution

Entry is $t(A; \nu)$ where $P\{t(\nu) \leq t(A; \nu)\} = A$



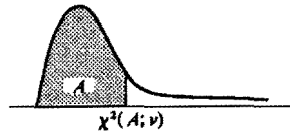
ν	A						
	.60	.70	.80	.85	.90	.95	.975
1	0.325	0.727	1.376	1.963	3.078	6.314	12.706
2	0.289	0.617	1.061	1.386	1.886	2.920	4.303
3	0.277	0.584	0.978	1.250	1.638	2.353	3.182
4	0.271	0.569	0.941	1.190	1.533	2.132	2.776
5	0.267	0.559	0.920	1.156	1.476	2.015	2.571
6	0.265	0.553	0.906	1.134	1.440	1.943	2.447
7	0.263	0.549	0.896	1.119	1.415	1.895	2.365
8	0.262	0.546	0.889	1.108	1.397	1.860	2.306
9	0.261	0.543	0.883	1.100	1.383	1.833	2.262
10	0.260	0.542	0.879	1.093	1.372	1.812	2.228
11	0.260	0.540	0.876	1.088	1.363	1.796	2.201
12	0.259	0.539	0.873	1.083	1.356	1.782	2.179
13	0.259	0.537	0.870	1.079	1.350	1.771	2.160
14	0.258	0.537	0.868	1.076	1.345	1.761	2.145
15	0.258	0.536	0.866	1.074	1.341	1.753	2.131
16	0.258	0.535	0.865	1.071	1.337	1.746	2.120
17	0.257	0.534	0.863	1.069	1.333	1.740	2.110
18	0.257	0.534	0.862	1.067	1.330	1.734	2.101
19	0.257	0.533	0.861	1.066	1.328	1.729	2.093
20	0.257	0.533	0.860	1.064	1.325	1.725	2.086
21	0.257	0.532	0.859	1.063	1.323	1.721	2.080
22	0.256	0.532	0.858	1.061	1.321	1.717	2.074
23	0.256	0.532	0.858	1.060	1.319	1.714	2.069
24	0.256	0.531	0.857	1.059	1.318	1.711	2.064
25	0.256	0.531	0.856	1.058	1.316	1.708	2.060
26	0.256	0.531	0.856	1.058	1.315	1.706	2.056
27	0.256	0.531	0.855	1.057	1.314	1.703	2.052
28	0.256	0.530	0.855	1.056	1.313	1.701	2.048
29	0.256	0.530	0.854	1.055	1.311	1.699	2.045
30	0.256	0.530	0.854	1.055	1.310	1.697	2.042
40	0.255	0.529	0.851	1.050	1.303	1.684	2.021
60	0.254	0.527	0.848	1.045	1.296	1.671	2.000
120	0.254	0.526	0.845	1.041	1.289	1.658	1.980
∞	0.253	0.524	0.842	1.036	1.282	1.645	1.960

TABLE A.2 (concluded) Percentiles of the *t* Distribution

ν	A						
	.98	.985	.99	.9925	.995	.9975	.9995
1	15.895	21.205	31.821	42.434	63.657	127.322	636.590
2	4.849	5.643	6.965	8.073	9.925	14.089	31.598
3	3.482	3.896	4.541	5.047	5.841	7.453	12.924
4	2.999	3.298	3.747	4.088	4.604	5.598	8.610
5	2.757	3.003	3.365	3.634	4.032	4.773	6.869
6	2.612	2.829	3.143	3.372	3.707	4.317	5.959
7	2.517	2.715	2.998	3.203	3.499	4.029	5.408
8	2.449	2.634	2.896	3.085	3.355	3.833	5.041
9	2.398	2.574	2.821	2.998	3.250	3.690	4.781
10	2.359	2.527	2.764	2.932	3.169	3.581	4.587
11	2.328	2.491	2.718	2.879	3.106	3.497	4.437
12	2.303	2.461	2.681	2.836	3.055	3.428	4.318
13	2.282	2.436	2.650	2.801	3.012	3.372	4.221
14	2.264	2.415	2.624	2.771	2.977	3.326	4.140
15	2.249	2.397	2.602	2.746	2.947	3.286	4.073
16	2.235	2.382	2.583	2.724	2.921	3.252	4.015
17	2.224	2.368	2.567	2.706	2.898	3.222	3.965
18	2.214	2.356	2.552	2.689	2.878	3.197	3.922
19	2.205	2.346	2.539	2.674	2.861	3.174	3.883
20	2.197	2.336	2.528	2.661	2.845	3.153	3.849
21	2.189	2.328	2.518	2.649	2.831	3.135	3.819
22	2.183	2.320	2.508	2.639	2.819	3.119	3.792
23	2.177	2.313	2.500	2.629	2.807	3.104	3.768
24	2.172	2.307	2.492	2.620	2.797	3.091	3.745
25	2.167	2.301	2.485	2.612	2.787	3.078	3.725
26	2.162	2.296	2.479	2.605	2.779	3.067	3.707
27	2.158	2.291	2.473	2.598	2.771	3.057	3.690
28	2.154	2.286	2.467	2.592	2.763	3.047	3.674
29	2.150	2.282	2.462	2.586	2.756	3.038	3.659
30	2.147	2.278	2.457	2.581	2.750	3.030	3.646
40	2.123	2.250	2.423	2.542	2.704	2.971	3.551
60	2.099	2.223	2.390	2.504	2.660	2.915	3.460
120	2.076	2.196	2.358	2.468	2.617	2.860	3.373
∞	2.054	2.170	2.326	2.432	2.576	2.807	3.291

TABLE A.3 Percentiles of the χ^2 Distribution

Entry is $\chi^2(A; \nu)$ where $P\{\chi^2(\nu) \leq \chi^2(A; \nu)\} = A$

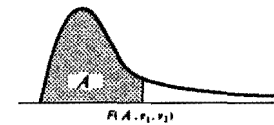


ν	A									
	.005	.010	.025	.050	.100	.900	.950	.975	.990	.995
1	0.00393	0.0157	0.03982	0.07393	0.1533	2.71	3.84	5.02	6.63	7.88
2	0.0100	0.0201	0.0506	0.103	0.211	4.61	5.99	7.38	9.21	10.60
3	0.072	0.115	0.216	0.352	0.584	6.25	7.81	9.35	11.34	12.84
4	0.207	0.297	0.484	0.711	1.064	7.78	9.49	11.14	13.28	14.86
5	0.412	0.554	0.831	1.145	1.61	9.24	11.07	12.83	15.09	16.75
6	0.676	0.872	1.24	1.64	2.20	10.64	12.59	14.45	16.81	18.55
7	0.989	1.24	1.69	2.17	2.83	12.02	14.07	16.01	18.48	20.28
8	1.34	1.65	2.18	2.73	3.49	13.36	15.51	17.53	20.09	21.96
9	1.73	2.09	2.70	3.33	4.17	14.68	16.92	19.02	21.67	23.59
10	2.16	2.56	3.25	3.94	4.87	15.99	18.31	20.48	23.21	25.19
11	2.60	3.05	3.82	4.57	5.58	17.28	19.68	21.92	24.73	26.76
12	3.07	3.57	4.40	5.23	6.30	18.55	21.03	23.34	26.22	28.30
13	3.57	4.11	5.01	5.89	7.04	19.81	22.36	24.74	27.69	29.82
14	4.07	4.66	5.63	6.57	7.79	21.06	23.68	26.12	29.14	31.32
15	4.60	5.23	6.26	7.26	8.55	22.31	25.00	27.49	30.58	32.80
16	5.14	5.81	6.91	7.96	9.31	23.54	26.30	28.85	32.00	34.27
17	5.70	6.41	7.56	8.67	10.09	24.77	27.59	30.19	33.41	35.72
18	6.26	7.01	8.23	9.39	10.86	25.99	28.87	31.53	34.81	37.16
19	6.84	7.63	8.91	10.12	11.65	27.20	30.14	32.85	36.19	38.58
20	7.43	8.26	9.59	10.85	12.44	28.41	31.41	34.17	37.57	40.00
21	8.03	8.90	10.28	11.59	13.24	29.62	32.67	35.48	38.93	41.40
22	8.64	9.54	10.98	12.34	14.04	30.81	33.92	36.78	40.29	42.80
23	9.24	10.20	11.69	13.09	14.85	32.01	35.17	38.08	41.64	44.18
24	9.89	10.86	12.40	13.85	15.66	33.20	36.42	39.36	42.98	45.56
25	10.52	11.52	13.12	14.61	16.47	34.38	37.65	40.65	44.31	46.93
26	11.16	12.20	13.84	15.38	17.29	35.56	38.89	41.92	45.64	48.29
27	11.81	12.88	14.57	16.15	18.11	36.74	40.11	43.19	46.96	49.64
28	12.46	13.56	15.31	16.93	18.94	37.92	41.34	44.46	48.28	50.99
29	13.12	14.26	16.05	17.71	19.77	39.09	42.56	45.72	49.59	52.34
30	13.79	14.95	16.79	18.49	20.60	40.26	43.77	46.98	50.89	53.67
40	20.71	22.16	24.43	26.51	29.05	51.81	55.76	59.34	63.69	66.77
50	27.99	29.71	32.36	34.76	37.69	63.17	67.50	71.42	76.15	79.49
60	35.53	37.48	40.48	43.19	46.46	74.40	79.08	83.30	88.38	91.95
70	43.28	45.44	48.76	51.74	55.33	85.53	90.53	95.02	100.4	104.2
80	51.17	53.54	57.15	60.39	64.28	96.58	101.9	106.6	112.3	116.3
90	59.20	61.75	65.65	69.13	73.29	107.6	113.1	118.1	124.1	128.3
100	67.33	70.06	74.22	77.93	82.36	118.5	124.3	129.6	135.8	140.2

Source: Reprinted, with permission, from C. M. Thompson, "Table of Percentage Points of the Chi-Square Distribution," *Biometrika* 32 (1941), pp. 188-89.

TABLE A.4 Percentiles of the F Distribution

Entry is $F(A; \nu_1, \nu_2)$ where $P\{F(\nu_1, \nu_2) \leq F(A; \nu_1, \nu_2)\} = A$



$$F(A; \nu_1, \nu_2) = \frac{1}{F(1-A; \nu_2, \nu_1)}$$

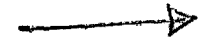


TABLE A.4 (continued) Percentiles of the F Distribution

Den. df	α	Numerator df									
		1	2	3	4	5	6	7	8	9	
30	.50	0.466	0.709	0.807	0.858	0.890	0.912	0.927	0.939	0.948	
	.90	2.88	2.49	2.28	2.14	2.05	1.98	1.93	1.88	1.85	
	.95	4.17	3.32	2.92	2.69	2.53	2.42	2.33	2.27	2.21	
	.975	5.57	4.18	3.59	3.25	3.03	2.87	2.75	2.65	2.57	
	.99	7.56	5.39	4.51	4.02	3.70	3.47	3.30	3.17	3.07	
	.995	9.18	6.35	5.24	4.62	4.23	3.95	3.74	3.58	3.45	
	.999	13.3	8.77	7.05	6.12	5.53	5.12	4.82	4.58	4.39	
	60	.50	0.461	0.701	0.798	0.849	0.880	0.901	0.917	0.928	0.937
		.90	2.79	2.39	2.18	2.04	1.95	1.87	1.82	1.77	1.74
		.95	4.00	3.15	2.76	2.53	2.37	2.25	2.17	2.10	2.04
.975		5.29	3.93	3.34	3.01	2.79	2.63	2.51	2.41	2.33	
.99		7.08	4.98	4.13	3.65	3.34	3.12	2.95	2.82	2.72	
.995		8.49	5.80	4.73	4.14	3.76	3.49	3.29	3.13	3.01	
.999		12.0	7.77	6.17	5.31	4.76	4.37	4.09	3.86	3.69	
120		.50	0.458	0.697	0.793	0.844	0.875	0.896	0.912	0.923	0.932
		.90	2.75	2.35	2.13	1.99	1.90	1.82	1.77	1.72	1.68
		.95	3.92	3.07	2.68	2.45	2.29	2.18	2.09	2.02	1.96
	.975	5.15	3.80	3.23	2.89	2.67	2.52	2.39	2.30	2.22	
	.99	6.85	4.79	3.95	3.48	3.17	2.96	2.79	2.66	2.56	
	.995	8.18	5.54	4.50	3.92	3.55	3.28	3.09	2.93	2.81	
	.999	11.4	7.32	5.78	4.95	4.42	4.04	3.77	3.55	3.38	
	∞	.50	0.455	0.693	0.789	0.839	0.870	0.891	0.907	0.918	0.927
		.90	2.71	2.30	2.08	1.94	1.85	1.77	1.72	1.67	1.63
		.95	3.84	3.00	2.60	2.37	2.21	2.10	2.01	1.94	1.88
.975		5.02	3.69	3.12	2.79	2.57	2.41	2.29	2.19	2.11	
.99		6.63	4.61	3.78	3.32	3.02	2.80	2.64	2.51	2.41	
.995		7.88	5.30	4.28	3.72	3.35	3.09	2.90	2.74	2.62	
.999		10.8	6.91	5.42	4.62	4.10	3.74	3.47	3.27	3.10	

TABLE A.4 (concluded) Percentiles of the F Distribution

Den. df	α	Numerator df									
		10	12	15	20	24	30	60	120	∞	
30	.50	0.955	0.966	0.978	0.989	0.994	1.00	1.01	1.02	1.02	
	.90	1.82	1.77	1.72	1.67	1.64	1.61	1.54	1.50	1.46	
	.95	2.16	2.09	2.01	1.93	1.89	1.84	1.74	1.68	1.62	
	.975	2.51	2.41	2.31	2.20	2.14	2.07	1.94	1.87	1.79	
	.99	2.98	2.84	2.70	2.55	2.47	2.39	2.21	2.11	2.01	
	.995	3.34	3.18	3.01	2.82	2.73	2.63	2.42	2.30	2.18	
	.999	4.24	4.00	3.75	3.49	3.36	3.22	2.92	2.76	2.59	
	60	.50	0.945	0.956	0.967	0.978	0.983	0.989	1.00	1.01	1.01
		.90	1.71	1.66	1.60	1.54	1.51	1.48	1.40	1.35	1.29
		.95	1.99	1.92	1.84	1.75	1.70	1.65	1.53	1.47	1.39
.975		2.27	2.17	2.06	1.94	1.88	1.82	1.67	1.58	1.48	
.99		2.63	2.50	2.35	2.20	2.12	2.03	1.84	1.73	1.60	
.995		2.90	2.74	2.57	2.39	2.29	2.19	1.96	1.83	1.69	
.999		3.54	3.32	3.08	2.83	2.69	2.55	2.25	2.08	1.89	
120		.50	0.939	0.950	0.961	0.972	0.978	0.983	0.994	1.00	1.01
		.90	1.65	1.60	1.55	1.48	1.45	1.41	1.32	1.26	1.19
		.95	1.91	1.83	1.75	1.66	1.61	1.55	1.43	1.35	1.25
	.975	2.16	2.05	1.95	1.82	1.76	1.69	1.53	1.43	1.31	
	.99	2.47	2.34	2.19	2.03	1.95	1.86	1.66	1.53	1.38	
	.995	2.71	2.54	2.37	2.19	2.09	1.98	1.75	1.61	1.43	
	.999	3.24	3.02	2.78	2.53	2.40	2.26	1.95	1.77	1.54	
	∞	.50	0.934	0.945	0.956	0.967	0.972	0.978	0.989	0.994	1.00
		.90	1.60	1.55	1.49	1.42	1.38	1.34	1.24	1.17	1.00
		.95	1.83	1.75	1.67	1.57	1.52	1.46	1.32	1.22	1.00
.975		2.05	1.94	1.83	1.71	1.64	1.57	1.39	1.27	1.00	
.99		2.32	2.18	2.04	1.88	1.79	1.70	1.47	1.32	1.00	
.995		2.52	2.36	2.19	2.00	1.90	1.79	1.53	1.36	1.00	
.999		2.96	2.74	2.51	2.27	2.13	1.99	1.66	1.45	1.00	

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