UNIVERSITY OF SWAZILAND

FINAL EXAMINATION PAPER 2013

TITLE OF PAPER	:	SAMPLE SURVEY THEORY
COURSE CODE	:	ST306
TIME ALLOWED	:	TWO (2) HOURS
REQUIREMENTS	:	CALCULATOR AND STATISTICAL TABLES
INSTRUCTIONS	:	ANSWER ANY THREE QUESTIONS

Question 1 [20 marks, 3+3+3+3+4+4]

(a) For each of the following surveys, describe the target population, sampling frame, sampling unit, and observation unit. Discuss any possible sources of selection bias or inaccuracy of responses.

(i) A student wants to estimate the percentage of mutual funds whose shares went up in price last week. She selects every tenth fund listing in the mutual fund pages of the newspaper and calculates the percentage of those in which the share price increased.

(ii) A sample of 8 architects was chosen in a city with 14 architects and architectural firms. To select a survey sample, each architect was contacted by telephone in order of appearance in the telephone directory. The first 8 agreeing to be interviewed formed the sample.

(iii) To estimate how many hooks in the library need rebinding, a librarian uses a random number table to randomly select 100 locations on library shelves. He then walks to each location, looks at the book that resides at that spot, and records whether the book needs rebinding or not.

(iv) A survey is conducted to find the average weight of cows in a region. A list of all farms is available for the region, and 50 farms are selected at random. Then the weight of each cow at the 50 selected farms is recorded.

(b) Senturia et al. (1994) describe a survey taken to study how many children have access to guns in their households. Questionnaires were distributed to all parents who attended selected clinics in the Chicago area during a 1-week period for well or sick-child visits.

(i) Suppose the quantity of interest is percentage of the households with guns. Describe why this is a cluster sample. What is the psu? The ssu? Is it a one-stage or two-stage cluster sample? How would you estimate the percentage of households with guns and the standard error of your estimate?

(ii) What is the sampling population for this study? Do you think this sampling procedure results in a representative sample of households with children? Why, or why not?

Question 2 [20 marks, 10+5+5]

(a) Foresters want to estimate the average age of trees in a stand. Determining age is cumbersome because one needs to count the tree rings on a core taken from the tree. In general, though, the older the tree, the larger the diameter, and diameter is easy to measure. The foresters measure the diameter of all 1132 trees and find that the population mean equals 10.3. They then randomly select 20 trees for age measurement.

Tree No.	Diameter, x	Age, y	Tree No.	Diameter, x	Age, y
1	12.0	125	11	5.7	61
2	11.4	119	12	8.0	80
3	7.9	83	13	10.3	114
4	9.0	85	14	12.0	147
5	10.5	99	15	9.2	122
6	7.9	117	16	8.5	106
7	7.3	69	17	7.0	82
8	10.2	133	18	10.7	88
9	11.7	154	19	9.3	97
10	11.3	168	20	8.2	99

Estimate the population mean age of trees in the stand and give an approximate standard error for your estimate.

(b) An accounting firm is interested in estimating the error rate in a compliance audit it is conducting. The population contains 828 claims, and the firm audits an SRS of 85 of those claims. In each of the 85 sampled claims, 215 fields are checked for errors. One claim has errors in 4 of the 215 fields, 1 claim has three errors, 4 claims have two errors, 22 claims have one error, and the remaining 57 claims have no errors. (Data courtesy of Fritz Scheuren.)

(i) Treating the claims as psu's and the observations for each field as ssu's, estimate the error rate for all 828 claims. Give a standard error for your estimate.

(ii) Estimate (with SE) the total number of errors in the 828 claims.

Question 3 [20 marks, 4+4+12]

(a) Mayr et al. (1994) took an SRS of 240 children aged 2 to 6 years who visited their pediatric outpatient clinic. They found the following frequency distribution for free (unassisted) walking among the children:

Age (months)	9	10	11	12	13	14	15	16	17	18	19	20
Number of children	13	35	44	69	36	24	7	3	2	5	1	1

(i) Find the mean, standard error, and a 95% CI for the average age for onset of free walking.

(ii) Suppose the researchers want to do another study in a different region and want a 95% confidence interval for the mean age of onset of walking to have margin of error 0.5. Using the estimated standard deviation for these data, what sample size would they need to take?

(b) The following data are from a stratified sample of faculty, using the areas biological sciences, physical sciences, social sciences, and humanities as the strata. Proportional allocation was used in this sample.

Stratum	**Number of Faculty Members in Stratum**	**Number of Faculty Members in Sample**
Biological sciences	102	7
Physical sciences	310	19
Social sciences	217	13
Humanities	178	11
Total	807	50

The frequency table for number of publications in the strata is given below.

Number of Refereed Publications	Number of Faculty Members			
	Biological	Physical	Social	Humanities
0	1	10	9	8
1	2	2	0	2
2	0	0	1	0
3	1	1	0	1
4	0	2	2	0
5	2	1	0	0
6	0	1	1	0
7	1	0	0	0
8	0	2	0	0

(i) Estimate the total number of refereed publications by faculty members in the college and give the standard error.

(ii) Estimate the proportion of faculty with no refereed publications and give the standard error.

Question 4 [20 marks, 4+4+4+2+2+2+2]

(a) A letter in the December 1995 issue of Dell Champion Variety Puzzles stated: "I've noticed over the last several issues there have been no winners from the South in your contests. You always say that winners are picked at random, so does this mean you're getting fewer entries from the South?" In response, the editors took a random sample of 1000 entries from the last few contests and found that 175 of those came from the South.

(i) Find a 95% CI for the percentage of entries that come from the South.

(ii) According to Statistical Abstract of the United States, 30.9% of the U. S. population live in states that the editors considered to be in the South. Is there evidence from your confidence interval that the percentage of entries from the South differs from the percentage of persons living in the South?

(b) A city council of a small city wants to know the proportion of eligible voters who oppose having an incinerator built for burning Phoenix garbage, just outside city limits. They randomly select 100 residential numbers from the city's telephone book that contains 3000 such numbers. Each selected residence is then called and asked for (a) the total number of eligible voters and (b) the number of voters opposed to the incinerator. A total of 157 voters are surveyed; of these, 23 refuse to answer the question. Of the remaining 134 voters, 112 oppose the incinerator, so the council estimates the proportion by

$$\hat{p} = \frac{112}{134} = 0.83582$$

with

$$\hat{V}(\hat{p}) = \frac{0.83582(1-0.83582)}{134} = 0.00102.$$

Are these estimates valid? Why, or why not? For each of the following situations, indicate how you might use ratio or regression estimation.

(i) Estimate the proportion of time devoted to sports in television news broadcasts in your city.

(ii) Estimate the average number of fish caught per hour by anglers visiting a lake in August.

(iii) Estimate the average amount that undergraduate students spent on textbooks at your university in the fall semester.

(iv) Estimate the total weight of usable meat (discarding bones, fat, and skin) in a shipment of chickens.

Question 5 [20 marks, 4+4+6+6]

Suppose a city has 90,000 dwelling units, of which 35,000 are houses, 45,000 are apartments, and 10,000 are condominiums. You believe that the mean electricity usage is about twice as much for houses as for apartments or condominiums and that the standard deviation is proportional to the mean.

(a) How would you allocate a sample of 900 observations if you want to estimate the mean electricity consumption for all households in the city?

(b) Now suppose that you want to estimate the overall proportion of households in which energy conservation is practiced. You have strong reason to believe that about 45% of house dwellers use some sort of energy conservation and that the corresponding percentages are 25% for apartment dwellers and 3% for condominium residents. What gain would proportional allocation offer over simple random sampling?

(c) Someone else has taken a small survey, using an SRS, of energy usage in houses. On the basis of the survey, each house is categorized as having electric heating or some other kind of heating. The January electricity consumption in kilowatt-hours for each house is recorded (y_i) and the results are given below:

Type of Heating	Number of Houses	Sample Mean	Sample Variance
Electric	24	972	202,396
Nonelectric	36	463	96,721
Total	60		

From other records, it is known that 16,450 of the 35,000 houses have electric heating, and 18,550 have nonelectric heating.

(i) Using the sample, give an estimate and its standard error of the proportion of houses with electric heating. Does your 95% CI include the true proportion?

(ii) Give an estimate and its standard error of the average number of kilowatt-hours used by houses in the city. What type of estimator did you use, and why did you choose that estimator?

Useful formulas

$$s^2 = \frac{\sum_{i=1}^{n}(y_i - \bar{y})^2}{n-1} \qquad \sum_{i=1}^{n}(y_i - \bar{y})^2 = \sum_{i=1}^{n} y_i^2 - \frac{\sum_{i=1}^{n} y_i}{n}$$

$$\hat{\mu}_{srs} = \bar{y} \qquad \hat{\mathsf{V}}(\hat{\mu}_{srs}) = \left(\frac{N-n}{N}\right)\frac{s^2}{n}$$

$$\hat{\tau}_{srs} = N\hat{\mu}_{srs} \qquad \hat{\mathsf{V}}(\hat{\tau}_{srs}) = N^2\hat{\mathsf{V}}(\hat{\mu}_{srs})$$

$$\hat{p}_{srs} = \sum_{i=1}^{n}\frac{y_i}{n} \qquad \left(\frac{N-n}{N}\right)\frac{\hat{p}(1-\hat{p})}{n-1}\left(\frac{N-n}{N}\right)$$

$$\hat{\tau}_{hh} = \frac{1}{n}\sum_{i=1}^{n}\frac{y_i}{p_i} \qquad \hat{\mathsf{V}}(\hat{\mu}_{hh}) = \frac{1}{n(n-1)}\sum_{i=1}^{n}\left(\frac{y_i}{p_i} - \hat{\tau}_{hh}\right)^2$$

$$\hat{\mu}_{hh} = \frac{\hat{\tau}_{hh}}{N} \qquad \hat{\mathsf{V}}(\hat{\mu}_{hh}) = \frac{1}{N^2}\hat{\mathsf{V}}(\hat{\tau}_{hh})$$

$$\hat{\tau}_{ht} = \sum_{i=1}^{\nu}\frac{y_i}{\pi_i} \qquad \hat{\mathsf{V}}(\hat{\tau}_{ht}) = \sum_{i=1}^{\nu}\left(\frac{1}{\pi_i^2} - \frac{1}{\pi_i}\right)y_i^2 +$$

$$2\sum_{i=1}^{\nu}\sum_{j>i}^{\nu}\left(\frac{1}{\pi_i\pi_j} - \frac{1}{\pi_{ij}}\right)y_iy_j$$

$$\hat{\mu}_{ht} = \frac{\hat{\tau}_{ht}}{N} \qquad \hat{\mathsf{V}}(\hat{\mu}_{ht}) = \frac{1}{N^2}\hat{\mathsf{V}}(\hat{\tau}_{ht})$$

$$\hat{r} = \frac{\sum_{i=1}^{n} y_i}{\sum_{i=1}^{n} x_i} \qquad \hat{\mathsf{V}}(\hat{r}) = \left(\frac{N-n}{Nn\mu_x^2}\right)\frac{\sum_{i=1}^{n}(y_i - rx_i)^2}{n-1}$$

$$\hat{\mu}_r = r\mu_x \qquad \hat{\mathsf{V}}(\hat{\mu}_r) = \left(\frac{N-n}{Nn}\right)\frac{\sum_{i=1}^{n}(y_i - rx_i)^2}{n-1}$$

$$\hat{\tau}_r = Nr\mu_x = r\tau_x \qquad \hat{\mathsf{V}}(\hat{\tau}_r) = \frac{N(N-n)}{n}\frac{\sum_{i=1}^{n}(y_i - rx_i)^2}{n-1}$$

$$\hat{\mu}_L = a + b\mu_x \qquad \hat{\mathsf{V}}(\hat{\mu}_L) = \frac{N-n}{Nn(n-2)}\sum_{i=1}^{n}(y_i - a - bx_i)^2$$

$$\hat{\tau}_L = N\mu_L \qquad \hat{\mathsf{V}}(\hat{\tau}_L) = \frac{N(N-n)}{n(n-2)}\sum_{i=1}^{n}(y_i - a - bx_i)^2$$

$$\hat{\mu}_{str} = \sum_{h=1}^{L}\frac{N_h}{N}\bar{y}_h \qquad \hat{\mathsf{V}}(\hat{\mu}_{str}) = \frac{1}{N^2}\sum_{h=1}^{L}N_h^2\left(\frac{N_h - n_h}{N_h}\right)\frac{s_h^2}{n_h}$$

$$\hat{\tau}_{str} = N\hat{\mu}_{str} \qquad \hat{\mathsf{V}}(\hat{\tau}_{str}) = N^2\hat{\mathsf{V}}(\hat{\mu}_{str})$$

$$\hat{p}_{str} = \sum_{h=1}^{L}\frac{N_h}{N}\hat{p}_h \qquad \hat{\mathsf{V}}(\hat{p}_{str}) = \frac{1}{N^2}\sum_{h=1}^{L}N_h^2\left(\frac{N_h - n_h}{N_h}\right)\left(\frac{\hat{p}_h(1-\hat{p}_h)}{n_h - 1}\right)$$

$$\hat{\mu}_{pstr} = \sum_{h=1}^{L}w_h\bar{y}_h \qquad \hat{\mathsf{V}}(\hat{\mu}_{pstr}) = \frac{1}{n}\left(\frac{N-n}{N}\right)\sum_{h=1}^{L}w_hs_h^2 + \frac{1}{n^2}\sum_{h=1}^{L}(1-w_h)s_h^2$$

$$\hat{\tau}_{cl} = \frac{M}{nL}\sum_{i=1}^{n}\sum_{j=1}^{L} y_{ij} = \frac{N}{n}\sum_{i=1}^{n}\sum_{j=1}^{L} y_{ij} = \frac{N}{n}\sum_{i=1}^{n} y_i = N\bar{y}$$

$$\hat{\mu}_{cl} = \frac{1}{nL}\sum_{i=1}^{n}\sum_{j=1}^{L} y_{ij} = \frac{1}{nL}\sum_{i=1}^{n} y_i = \frac{\bar{y}}{L} = \frac{\hat{\tau}_{cl}}{M}$$

where $\bar{y} = \frac{1}{n}\sum_{i=1}^{n} y_i = \frac{\hat{\tau}_{cl}}{N}$

$$\hat{\mathsf{V}}(\hat{\tau}_{cl}) = N(N-n)\frac{s_u^2}{n} \qquad \hat{\mathsf{V}}(\hat{\mu}_{cl}) = \frac{N(N-n)}{M^2}\frac{s_u^2}{n}$$

where $s_u^2 = \frac{\sum_{i=1}^{n}(y_i-\bar{y})^2}{n-1}$.

$$\hat{\mu}_1 = \bar{y} = \frac{\hat{\tau}_{cl}}{N} \qquad \hat{\mathsf{V}}(\hat{\mu}_1 = \frac{N-n}{N}\frac{s_u^2}{n}$$

The formulas for systematic sampling are the same as those used for one-stage cluster sampling. Change the subscript *cl* to *sys* to denote the fact that data were collected under systematic sampling.

$$\hat{\mu}_{c(a)} = \frac{\sum_{i=1}^{n} y_i}{\sum_{i=1}^{n} M_i} = \frac{\sum_{i=1}^{n} y_i}{m} \qquad \hat{\mathsf{V}}(\hat{\mu}_{c(a)}) = \frac{(N-n)N}{n(n-1)M^2}\sum_{i=1}^{n} M_i^2(\bar{y}-\hat{\mu}_{c(a)})^2$$

$$\hat{\mu}_{c(b)} = \frac{N}{M}\frac{\sum_{i=1}^{n} y_i}{n} = \frac{N}{nM}\sum_{i=1}^{n} y_i \qquad \hat{\mathsf{V}}(\hat{\mu}_{c(b)}) = \frac{(N-n)N}{n(n-1)M^2}\sum_{i=1}^{n}(y_i-\bar{y})^2 = \frac{(N-n)N}{nM^2}s_u^2$$

$$\hat{p}_c = \frac{\sum_{i=1}^{n} p_i}{n} \qquad \hat{\mathsf{V}}(\hat{p}_c) = \left(\frac{N-Nn}{nN}\right)\sum_{i=1}^{n}\frac{(p_i-\hat{p}_c)^2}{n-1} = \left(\frac{1-f}{n}\right)\sum_{i=1}^{n}\frac{(p_i-\hat{p}_c)^2}{n-1}$$

$$\hat{p}_c = \frac{\sum_{i=1}^{n} y_i}{\sum_{i=1}^{n} M_i} \qquad \hat{\mathsf{V}}(\hat{p}_c) = \left(\frac{1-f}{n\bar{m}^2}\right)\frac{\sum_{i=1}^{n}(y_i-\hat{p}_c M_i)^2}{n-1}$$

To estimate τ, multiply $\hat{\mu}_{c(\cdot)}$ by M. To get the estimated variances, multiply $\hat{\mathsf{V}}(\hat{\mu}_{c(\cdot)})$ by M^2. If M is not known, substitute M with Nm/n. $\bar{m} = \sum_{i=1}^{n} M_i/n$.

n for μ SRS $\quad n = \frac{N\sigma^2}{(N-1)(d^2/z^2)+\sigma^2}$

n for τ SRS $\quad n = \frac{N\sigma^2}{(N-1)(d^2/z^2N^2)+\sigma^2}$

n for p SRS $\quad n = \frac{Np(1-p)}{(N-1)(d^2/z^2)+p(1-p)}$

n for μ SYS $\quad n = \frac{N\sigma^2}{(N-1)(d^2/z^2)+\sigma^2}$

n for τ SYS $\quad n = \frac{N\sigma^2}{(N-1)(d^2/z^2N^2)+\sigma^2}$

n for μ STR $\quad n = \frac{\sum_{h=1}^{L} N_h^2(\sigma_h^2/w_h)}{N^2(d^2/z^2)+\sum_{h=1}^{L} N_h\sigma_h^2}$

n for τ STR $\quad n = \frac{\sum_{h=1}^{L} N_h^2(\sigma_h^2/w_h)}{N^2(d^2/z^2N^2)+\sum_{h=1}^{L} N_h\sigma_h^2}$

where $w_h = \frac{n_h}{n}$.

Allocations for STR μ:

$$n_h = (c - c_0)\left(\frac{N_h\sigma_h/\sqrt{c_h}}{\sum_{k=1}^{L} N_k\sigma_k\sqrt{c_k}}\right) \qquad (c - c_0) = \frac{\left(\sum_{k=1}^{L} N_k\sigma_k\sqrt{c_k}\right)\left(\sum_{k=1}^{L} N_k\sigma_k\sqrt{c_k}\right)}{N^2(d^2/z^2) + \sum_{k=1}^{L} N_k\sigma_k^2}$$

$$n_h = n\left(\frac{N_h}{N}\right) \qquad n = \frac{\sum_{k=1}^{L} N_k\sigma_k}{N^2(d^2/z^2) + \frac{1}{N}\sum_{k=1}^{L} N_k\sigma_k^2}$$

$$n_h = n\left(\frac{N_h\sigma_h}{\sum_{k=1}^{L} N_k\sigma_k}\right) \qquad n = \frac{\left(\sum_{k=1}^{L} N_k\sigma_k\right)^2}{N^2(d^2/z^2) + \sum_{k=1}^{L} N_k\sigma_k^2}$$

Allocations for STR τ:

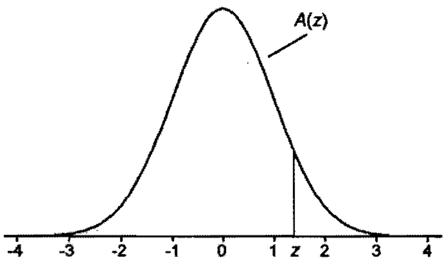
change $N^2(d^2/z^2)$ to $N^2(d^2/z^2N^2)$

Allocations for STR p:

$$n_h = n\left(\frac{N_i\sqrt{p_h(1-p_h)/c_h}}{\sum_{k=1}^{L} N_k\sqrt{p_k(1-p_k)/c_k}}\right) \qquad n = \frac{\sum_{k=1}^{L} N_k p_k(1-p_k)/w_k}{N^2(d^2/z^2) + \sum_{k=1}^{L} N_k p_k(1-p_k)}$$

TABLE A.1

Cumulative Standardized Normal Distribution



$A(z)$ is the integral of the standardized normal distribution from $-\infty$ to z (in other words, the area under the curve to the left of z). It gives the probability of a normal random variable not being more than z standard deviations above its mean. Values of z of particular importance:

z	$A(z)$	
1.645	0.9500	Lower limit of right 5% tail
1.960	0.9750	Lower limit of right 2.5% tail
2.326	0.9900	Lower limit of right 1% tail
2.576	0.9950	Lower limit of right 0.5% tail
3.090	0.9990	Lower limit of right 0.1% tail
3.291	0.9995	Lower limit of right 0.05% tail

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998
3.5	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998
3.6	0.9998	0.9998	0.9999							

TABLE A.2

t Distribution: Critical Values of t

Degrees of freedom	Two-tailed test: / One-tailed test:			*Significance level*			
	Two-tailed test:	10%	5%	2%	1%	0.2%	0.1%
	One-tailed test:	5%	2.5%	1%	0.5%	0.1%	0.05%
1		6.314	12.706	31.821	63.657	318.309	636.619
2		2.920	4.303	6.965	9.925	22.327	31.599
3		2.353	3.182	4.541	5.841	10.215	12.924
4		2.132	2.776	3.747	4.604	7.173	8.610
5		2.015	2.571	3.365	4.032	5.893	6.869
6		1.943	2.447	3.143	3.707	5.208	5.959
7		1.894	2.365	2.998	3.499	4.785	5.408
8		1.860	2.306	2.896	3.355	4.501	5.041
9		1.833	2.262	2.821	3.250	4.297	4.781
10		1.812	2.228	2.764	3.169	4.144	4.587
11		1.796	2.201	2.718	3.106	4.025	4.437
12		1.782	2.179	2.681	3.055	3.930	4.318
13		1.771	2.160	2.650	3.012	3.852	4.221
14		1.761	2.145	2.624	2.977	3.787	4.140
15		1.753	2.131	2.602	2.947	3.733	4.073
16		1.746	2.120	2.583	2.921	3.686	4.015
17		1.740	2.110	2.567	2.898	3.646	3.965
18		1.734	2.101	2.552	2.878	3.610	3.922
19		1.729	2.093	2.539	2.861	3.579	3.883
20		1.725	2.086	2.528	2.845	3.552	3.850
21		1.721	2.080	2.518	2.831	3.527	3.819
22		1.717	2.074	2.508	2.819	3.505	3.792
23		1.714	2.069	2.500	2.807	3.485	3.768
24		1.711	2.064	2.492	2.797	3.467	3.745
25		1.708	2.060	2.485	2.787	3.450	3.725
26		1.706	2.056	2.479	2.779	3.435	3.707
27		1.703	2.052	2.473	2.771	3.421	3.690
28		1.701	2.048	2.467	2.763	3.408	3.674
29		1.699	2.045	2.462	2.756	3.396	3.659
30		1.697	2.042	2.457	2.750	3.385	3.646
32		1.694	2.037	2.449	2.738	3.365	3.622
34		1.691	2.032	2.441	2.728	3.348	3.601
36		1.688	2.028	2.434	2.719	3.333	3.582
38		1.686	2.024	2.429	2.712	3.319	3.566
40		1.684	2.021	2.423	2.704	3.307	3.551
42		1.682	2.018	2.418	2.698	3.296	3.538
44		1.680	2.015	2.414	2.692	3.286	3.526
46		1.679	2.013	2.410	2.687	3.277	3.515
48		1.677	2.011	2.407	2.682	3.269	3.505
50		1.676	2.009	2.403	2.678	3.261	3.496
60		1.671	2.000	2.390	2.660	3.232	3.460
70		1.667	1.994	2.381	2.648	3.211	3.435
80		1.664	1.990	2.374	2.639	3.195	3.416
90		1.662	1.987	2.368	2.632	3.183	3.402
100		1.660	1.984	2.364	2.626	3.174	3.390
120		1.658	1.980	2.358	2.617	3.160	3.373
150		1.655	1.976	2.351	2.609	3.145	3.357
200		1.653	1.972	2.345	2.601	3.131	3.340
300		1.650	1.968	2.339	2.592	3.118	3.323
400		1.649	1.966	2.336	2.588	3.111	3.315
500		1.648	1.965	2.334	2.586	3.107	3.310
600		1.647	1.964	2.333	2.584	3.104	3.307
∞		1.645	1.960	2.326	2.576	3.090	3.291