UNIVERSITY OF SWAZILAND

SUPPLEMENTARY EXAMINATION PAPER 2013

TITLE OF PAPER	:	SAMPLE SURVEY THEORY
COURSE CODE	:	ST306
TIME ALLOWED	:	TWO (2) HOURS
REQUIREMENTS	:	CALCULATOR AND STATISTICAL TABLES
INSTRUCTIONS	:	ANSWER ANY THREE QUESTIONS

Question 1 [20 marks, 6+8+6]

A simple random sample of 1 in 20 households in a small town provided the following data about the availability of cars and the number of adults in households.

Number of cars (y_i) in the household	Adults in household (x_i)					Total
	1	2	3	4	5	
0	58	127	9	6	0	200
1	68	140	27	4	1	240
2	4	30	5	8	3	50
3	0	3	4	2	1	10
Total	130	300	45	20	5	500

Obtain point estimates, and approximate 95% confidence intervals for the following (NB: Summing over all 500 households, $\sum x_i y_i = 795$):

(a) the total number of cars in the town's households,

(b) the ratio of cars per adult in the town's households,

(c) the proportion of households with 1 or more cars per adult.

Question 2 [20 marks, 10+10]

A campus population of size $N = 9000$ is to be surveyed by a stratified sample for the prevalence of a certain disease, based upon three strata of respective sizes $N_h = 1000, 3000, 5000$ for $h = 1, 2, 3$. The costs of sampling individuals from these strata are estimated to be respectively 40, 20, and 10 Swazi Emalangeni (SZL) per person. The campus health authorities believe that roughly 1% of stratum 1, 5% of stratum 2, and 12% of stratum 3 will test positive for the disease.

(a) What is the optimal number of individuals to sample in each stratum if the total budget for data-collection in the survey is $SZL20000$.

(b) Suppose that the same population were to be sampled by SRS. About how much would the SRS cost if you want to achieve the same MSE as in (a) in estimating the proportion of the population who have the disease ?

Question 3 [20 marks, 9+3+8]

(a) Consider a population of farms on a 25 × 25 grid of varying sizes and shapes. If we randomly select a single square on this grid, then letting x_i = the area of farm i and $A = 625$ total units, the probability that farm i is selected is: $p_i = \frac{x_i}{A} = \frac{x_i}{625}$.

y_i =Workers	$p_i = \frac{x_i}{A} = \frac{\text{Size of Farm}}{\text{Total Area}}$
2	5/625
8	28/625
4	12/625
3	13/625

The table above shows a replacement sample of 4 farms selected with probability-proportional-to-size (PPS). Compute:

(i) The estimated number of workers (and associated standard errors).

(ii) The estimated number of farms.

using the Horvitz-Thompson estimator.

(b) For a health survey of a large population, estimates are wanted for two proportions, each measuring the yearly incidence of a disease? For designing the sample, we guess that one occurs with a frequency of 50 percent and the other with a frequency of only 1 percent. To obtain the same standard error of $\frac{1}{2}$ percent, how large an srs is needed for each disease. The large difference in the needed n causes a re-evaluation of the requirements. Now the same coefficient of variation of 0.05 is declared desirable for each disease; how large a sample is needed for each disease?

Question 4 **[20 marks, 10+2+4+4]**

(a) A sampling experiment is conducted by sampling 100 individuals at random out of a target population of 2000 and tabulating their monthly incomes (y_k) and educational levels (x_k = highest grade attained, including college as 13 to 16). The results of the study can be summarized as follows:

$$\bar{x} = 12.8 \qquad \bar{y} = 3072 \qquad s_y^2 = 1994^2 \qquad s_x^2 = 3.5^2$$

$$s_{xy} = \frac{1}{99}\sum_{i=1}^{n}(x_i - \bar{x})(y_i - \bar{y}) = 2805$$

Assume that it is known that the populationwide average educational level $\mu_X = 13.4$.

Give an approximately unbiased 95% two-sided confidence interval for average income based instead upon the parameters determined in the whole population from the linear regression model:

$$E(y_k) = \beta_0 + \beta_1 x_k \quad ; \quad \text{var}(y_k) = \sigma$$

(b) The Columbus Police Department (CPD) has been installing traffic light camera systems to identify and ticket auto drivers who fail to stop at red traffic lights. When the system senses that an automobile has entered the intersection while the traffic light was red, the cameras take a photo of the front and back of the car, as well as record a short video of the potential violation. At a later time, a police officer looks at the photos and watches the video. If the officer believes there was a traffic violation, the owner of the car (as identified by the license plate) is fined \$50.

The CPD annually evaluates the officer assessment of the photos. For this evaluation, a supervisor reviews a simple random sample of all the photos taken that year and either confirms the original officer's opinion, or finds an error. The goal of this study is to estimate the proportion of errors made that year.

In 2009, the supervisor examined 200 of the 35385 camera-recorded incidents. Of these, she disagreed with 8 of the original officers' decisions. For this problem, assume there is no non-sampling error.

(i) Estimate the true proportion of mistakes made by the original officers.

(ii) Estimate the standard error for your estimate in part (a).

(iii) Are you confident that the officers made mistakes on more than 1% of the recorded incidents? Justify your answer.

Question 5 [20 marks, 2+6+6+6]

A simple random sample of 10 hospitals was selected from a population of 33 hospitals that had received state funding to upgrade their emergency medical services. Within each of the selected hospitals, the records of all patients hospitalised in the past 12 months for traumatic injuries (i.e. accidents, poisonings, violence, burns, etc.) were examined. The numbers of patients hospitalised for trauma conditions and the numbers who died for the selected hospitals are given below.

Hospital	Number of patients hospitalised for trauma conditions	Number with trauma conditions who died
1	560	4
2	190	4
3	260	2
4	370	4
5	190	4
6	130	0
7	170	9
8	170	2
9	60	0
10	110	1

(a) Explain why this design may be considered as a cluster sample. What are the first-stage and second-stage units?

(b) Obtain a point estimate and an approximate 95% confidence interval for the total number of persons hospitalised for trauma conditions for the 33 hospitals. State the properties of your estimator.

(c) Obtain a point estimate of the proportion of persons who died among those hospitalised for trauma conditions for the 33 hospitals, using the cluster totals. Hence calculate an approximate 95% confidence interval for this proportion, and comment on the validity of the assumptions necessary for this calculation.

(d) Give reasons why, for this survey, cluster sampling might be preferred to stratified random sampling. What might be the drawbacks of cluster sampling? Discuss, with reasons, any improvements you might make if another survey was being planned on the same topic.

Useful formulas

$$s^2 = \frac{\sum_{i=1}^n (y_i - \bar{y})^2}{n-1}$$

$$\sum_{i=1}^n (y_i - \bar{y})^2 = \sum_{i=1}^n y_i^2 - \frac{\sum_{i=1}^n y_i}{n}$$

$$\hat{\mu}_{srs} = \bar{y}$$

$$\hat{\mathsf{V}}(\hat{\mu}_{srs}) = \left(\frac{N-n}{N}\right)\frac{s^2}{n}$$

$$\hat{\tau}_{srs} = N\hat{\mu}_{srs}$$

$$\hat{\mathsf{V}}(\hat{\tau}_{srs}) = N^2\hat{\mathsf{V}}(\hat{\mu}_{srs})$$

$$\hat{p}_{srs} = \sum_{i=1}^n \frac{y_i}{n}$$

$$\left(\frac{N-n}{N}\right)\frac{\hat{p}(1-\hat{p})}{n-1}\left(\frac{N-n}{N}\right)$$

$$\hat{\tau}_{hh} = \frac{1}{n}\sum_{i=1}^n \frac{y_i}{p_i}$$

$$\hat{\mathsf{V}}(\hat{\mu}_{hh}) = \frac{1}{n(n-1)}\sum_{i=1}^n \left(\frac{y_i}{p_i} - \hat{\tau}_{hh}\right)$$

$$\hat{\mu}_{hh} = \frac{\hat{\tau}_{hh}}{N}$$

$$\hat{\mathsf{V}}(\hat{\mu}_{hh}) = \frac{1}{N^2}\hat{\mathsf{V}}(\hat{\tau}_{hh})$$

$$\hat{\tau}_{ht} = \sum_{i=1}^{\nu} \frac{y_i}{\pi_i}$$

$$\hat{\mathsf{V}}(\hat{\tau}_{ht}) = \sum_{i=1}^{\nu}\left(\frac{1}{\pi_i^2} - \frac{1}{\pi_i}\right)y_i^2 + 2\sum_{i=1}^{\nu}\sum_{j>i}^{\nu}\left(\frac{1}{\pi_i\pi_j} - \frac{1}{\pi_{ij}}\right)y_i y_j$$

$$\hat{\mu}_{ht} = \frac{\hat{\tau}_{ht}}{N}$$

$$\hat{\mathsf{V}}(\hat{\mu}_{ht}) = \frac{1}{N^2}\hat{\mathsf{V}}(\hat{\tau}_{ht})$$

$$\hat{r} = \frac{\sum_{i=1}^n y_i}{\sum_{i=1}^n x_i}$$

$$\hat{\mathsf{V}}(\hat{r}) = \left(\frac{N-n}{Nn\mu_x^2}\right)\frac{\sum_{i=1}^n (y_i - r x_i)^2}{n-1}$$

$$\hat{\mu}_r = r\mu_x$$

$$\hat{\mathsf{V}}(\hat{\mu}_r) = \left(\frac{N-n}{Nn}\right)\frac{\sum_{i=1}^n (y_i - r x_i)^2}{n-1}$$

$$\hat{\tau}_r = Nr\mu_x = r\tau_x$$

$$\hat{\mathsf{V}}(\hat{\tau}_r) = \frac{N(N-n)}{n}\frac{\sum_{i=1}^n (y_i - r x_i)^2}{n-1}$$

$$\hat{\mu}_L = a + b\mu_x$$

$$\hat{\mathsf{V}}(\hat{\mu}_L) = \frac{N-n}{Nn(n-1)}\sum_{i=1}^n (y_i - a - bx_i)^2$$

$$\hat{\tau}_L = N\mu_L$$

$$\hat{\mathsf{V}}(\hat{\tau}_L) = \frac{N(N-n)}{n(n-1)}\sum_{i=1}^n (y_i - a - bx_i)^2$$

$$\hat{\mu}_{str} = \sum_{h=1}^L \frac{N_h}{N}\bar{y}_h$$

$$\hat{\mathsf{V}}(\hat{\mu}_{str}) = \frac{1}{N^2}\sum_{h=1}^L N_h^2\left(\frac{N_h - n_h}{N_h}\right)\frac{s_h^2}{n_h}$$

$$\hat{\tau}_{str} = N\hat{\mu}_{str}$$

$$\hat{\mathsf{V}}(\hat{\tau}_{str}) = N^2\hat{\mathsf{V}}(\hat{\mu}_{str})$$

$$\hat{p}_{str} = \sum_{h=1}^L \frac{N_h}{N}\hat{p}_h$$

$$\hat{\mathsf{V}}(\hat{p}_{str}) = \frac{1}{N^2}\sum_{h=1}^L N_h^2\left(\frac{N_h - n_h}{N_h}\right)\left(\frac{\hat{p}_h(1-\hat{p}_h)}{n_h - 1}\right)$$

$$\hat{\mu}_{pstr} = \sum_{h=1}^L w_h\bar{y}_h$$

$$\hat{\mathsf{V}}(\hat{\mu}_{pstr}) = \frac{1}{n}\left(\frac{N-n}{N}\right)\sum_{h=1}^L w_h s_h^2 + \frac{1}{n^2}\sum_{h=1}^L (1 - w_h)s_h^2$$

$$\hat{\tau}_{cl} = \frac{M}{nL}\sum_{i=1}^{n}\sum_{j=1}^{L} y_{ij} = \frac{N}{n}\sum_{i=1}^{n}\sum_{j=1}^{L} y_{ij} = \frac{N}{n}\sum_{i=1}^{n} y_i = N\bar{y}$$

$$\hat{\mu}_{cl} = \frac{1}{nL}\sum_{i=1}^{n}\sum_{j=1}^{L} y_{ij} = \frac{1}{nL}\sum_{i=1}^{n} y_i = \frac{\bar{y}}{L} = \frac{\hat{\tau}_{cl}}{M}$$

where $\bar{y} = \frac{1}{n}\sum_{i=1}^{n} y_i = \frac{\hat{\tau}_{cl}}{N}$

$$\hat{V}(\hat{\tau}_{cl}) = N(N-n)\frac{s_u^2}{n} \qquad \hat{V}(\hat{\mu}_{cl}) = \frac{N(N-n)}{M^2}\frac{s_u^2}{n}$$

where $s_u^2 = \frac{\sum_{i=1}^{n}(y_i - \bar{y})^2}{n-1}$.

$$\hat{\mu}_1 = \bar{y} = \frac{\hat{\tau}_{cl}}{N} \qquad \hat{V}(\hat{\mu}_1 = \frac{N-n}{N}\frac{s_u^2}{n}$$

The formulas for systematic sampling are the same as those used for one-stage cluster sampling. Change the subscript *cl* to *sys* to denote the fact that data were collected under systematic sampling.

$$\hat{\mu}_{c(a)} = \frac{\sum_{i=1}^{n} y_i}{\sum_{i=1}^{n} M_i} = \frac{\sum_{i=1}^{n} y_i}{m} \qquad \hat{V}(\hat{\mu}_{c(a)}) = \frac{(N-n)N}{n(n-1)M^2}\sum_{i=1}^{n} M_i^2(\bar{y} - \hat{\mu}_{c(a)})^2$$

$$\hat{\mu}_{c(b)} = \frac{N}{M}\frac{\sum_{i=1}^{n} y_i}{n} = \frac{N}{nM}\sum_{i=1}^{n} y_i \qquad \hat{V}(\hat{\mu}_{c(b)}) = \frac{(N-n)N}{n(n-1)M^2}\sum_{i=1}^{n}(y_i - \bar{y})^2 = \frac{(N-n)N}{nM^2}s_u^2$$

$$\hat{p}_c = \frac{\sum_{i=1}^{n} p_i}{n} \qquad \hat{V}(\hat{p}_c) = \left(\frac{N-Nn}{nN}\right)\sum_{i=1}^{n}\frac{(p_i - \hat{p}_c)^2}{n-1} = \left(\frac{1-f}{n}\right)\sum_{i=1}^{n}\frac{(p_i - \hat{p}_c)^2}{n-1}$$

$$\hat{p}_c = \frac{\sum_{i=1}^{n} y_i}{\sum_{i=1}^{n} M_i} \qquad \hat{V}(\hat{p}_c) = \left(\frac{1-f}{n\bar{m}^2}\right)\frac{\sum_{i=1}^{n}(y_i - \hat{p}_c M_i)^2}{n-1}$$

To estimate τ, multiply $\hat{\mu}_{c(\cdot)}$ by M. To get the estimated variances, multiply $\hat{V}(\hat{\mu}_{c(\cdot)})$ by M^2. If M is not known, substitute M with Nm/n. $\bar{m} = \sum_{i=1}^{n} M_i/n$.

n for μ SRS	$n = \frac{N\sigma^2}{(N-1)(d^2/z^2) + \sigma^2}$
n for τ SRS	$n = \frac{N\sigma^2}{(N-1)(d^2/z^2N^2) + \sigma^2}$
n for p SRS	$n = \frac{Np(1-p)}{(N-1)(d^2/z^2) + p(1-p)}$
n for μ SYS	$n = \frac{N\sigma^2}{(N-1)(d^2/z^2) + \sigma^2}$
n for τ SYS	$n = \frac{N\sigma^2}{(N-1)(d^2/z^2N^2) + \sigma^2}$
n for μ STR	$n = \frac{\sum_{h=1}^{L} N_h^2(\sigma_h^2/w_h)}{N^2(d^2/z^2) + \sum_{h=1}^{L} N_h\sigma_h^2}$
n for τ STR	$n = \frac{\sum_{h=1}^{L} N_h^2(\sigma_h^2/w_h)}{N^2(d^2/z^2N^2) + \sum_{h=1}^{L} N_h\sigma_h^2}$

where $w_h = \frac{n_h}{n}$.

Allocations for STR μ:

$$n_h = (c - c_0)\left(\frac{N_h\sigma_h/\sqrt{c_h}}{\sum_{k=1}^{L} N_k\sigma_k\sqrt{c_k}}\right) \qquad (c - c_0) = \frac{\left(\sum_{k=1}^{L} N_k\sigma_k/\sqrt{c_k}\right)\left(\sum_{k=1}^{L} N_k\sigma_k\sqrt{c_k}\right)}{N^2(d^2/z^2) + \sum_{k=1}^{L} N_k\sigma_k^2}$$

$$n_h = n\left(\frac{N_h}{N}\right) \qquad n = \frac{\sum_{k=1}^{L} N_k\sigma_k}{N^2(d^2/z^2) + \frac{1}{N}\sum_{k=1}^{L} N_k\sigma_k^2}$$

$$n_h = n\left(\frac{N_h\sigma_h}{\sum_{k=1}^{L} N_k\sigma_k}\right) \qquad n = \frac{\left(\sum_{k=1}^{L} N_k\sigma_k\right)^2}{N^2(d^2/z^2) + \sum_{k=1}^{L} N_k\sigma_k^2}$$

Allocations for STR τ:

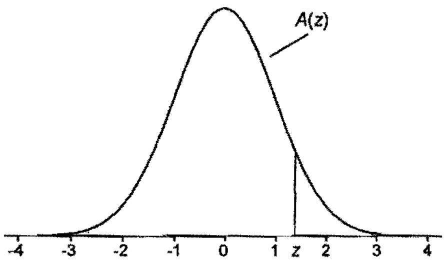
change $N^2(d^2/z^2)$ to $N^2(d^2/z^2N^2)$

Allocations for STR p:

$$n_h = n\left(\frac{N_i\sqrt{p_h(1-p_h)/c_h}}{\sum_{k=1}^{L} N_k\sqrt{p_k(1-p_k)/c_k}}\right) \qquad n = \frac{\sum_{k=1}^{L} N_k p_k(1-p_k)/w_k}{N^2(d^2/z^2) + \sum_{k=1}^{L} N_k p_k(1-p_k)}$$

TABLE A.1

Cumulative Standardized Normal Distribution



$A(z)$ is the integral of the standardized normal distribution from $-\infty$ to z (in other words, the area under the curve to the left of z). It gives the probability of a normal random variable not being more than z standard deviations above its mean. Values of z of particular importance:

z	$A(z)$	
1.645	0.9500	Lower limit of right 5% tail
1.960	0.9750	Lower limit of right 2.5% tail
2.326	0.9900	Lower limit of right 1% tail
2.576	0.9950	Lower limit of right 0.5% tail
3.090	0.9990	Lower limit of right 0.1% tail
3.291	0.9995	Lower limit of right 0.05% tail

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998
3.5	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998
3.6	0.9998	0.9998	0.9999							

TABLE A.2

t Distribution: Critical Values of t

		Significance level					
Degrees of freedom	*Two-tailed test:*	10%	5%	2%	1%	0.2%	0.1%
	One-tailed test:	5%	2.5%	1%	0.5%	0.1%	0.05%
1		6.314	12.706	31.821	63.657	318.309	636.619
2		2.920	4.303	6.965	9.925	22.327	31.599
3		2.353	3.182	4.541	5.841	10.215	12.924
4		2.132	2.776	3.747	4.604	7.173	8.610
5		2.015	2.571	3.365	4.032	5.893	6.869
6		1.943	2.447	3.143	3.707	5.208	5.959
7		1.894	2.365	2.998	3.499	4.785	5.408
8		1.860	2.306	2.896	3.355	4.501	5.041
9		1.833	2.262	2.821	3.250	4.297	4.781
10		1.812	2.228	2.764	3.169	4.144	4.587
11		1.796	2.201	2.718	3.106	4.025	4.437
12		1.782	2.179	2.681	3.055	3.930	4.318
13		1.771	2.160	2.650	3.012	3.852	4.221
14		1.761	2.145	2.624	2.977	3.787	4.140
15		1.753	2.131	2.602	2.947	3.733	4.073
16		1.746	2.120	2.583	2.921	3.686	4.015
17		1.740	2.110	2.567	2.898	3.646	3.965
18		1.734	2.101	2.552	2.878	3.610	3.922
19		1.729	2.093	2.539	2.861	3.579	3.883
20		1.725	2.086	2.528	2.845	3.552	3.850
21		1.721	2.080	2.518	2.831	3.527	3.819
22		1.717	2.074	2.508	2.819	3.505	3.792
23		1.714	2.069	2.500	2.807	3.485	3.768
24		1.711	2.064	2.492	2.797	3.467	3.745
25		1.708	2.060	2.485	2.787	3.450	3.725
26		1.706	2.056	2.479	2.779	3.435	3.707
27		1.703	2.052	2.473	2.771	3.421	3.690
28		1.701	2.048	2.467	2.763	3.408	3.674
29		1.699	2.045	2.462	2.756	3.396	3.659
30		1.697	2.042	2.457	2.750	3.385	3.646
32		1.694	2.037	2.449	2.738	3.365	3.622
34		1.691	2.032	2.441	2.728	3.348	3.601
36		1.688	2.028	2.434	2.719	3.333	3.582
38		1.686	2.024	2.429	2.712	3.319	3.566
40		1.684	2.021	2.423	2.704	3.307	3.551
42		1.682	2.018	2.418	2.698	3.296	3.538
44		1.680	2.015	2.414	2.692	3.286	3.526
46		1.679	2.013	2.410	2.687	3.277	3.515
48		1.677	2.011	2.407	2.682	3.269	3.505
50		1.676	2.009	2.403	2.678	3.261	3.496
60		1.671	2.000	2.390	2.660	3.232	3.460
70		1.667	1.994	2.381	2.648	3.211	3.435
80		1.664	1.990	2.374	2.639	3.195	3.416
90		1.662	1.987	2.368	2.632	3.183	3.402
100		1.660	1.984	2.364	2.626	3.174	3.390
120		1.658	1.980	2.358	2.617	3.160	3.373
150		1.655	1.976	2.351	2.609	3.145	3.357
200		1.653	1.972	2.345	2.601	3.131	3.340
300		1.650	1.968	2.339	2.592	3.118	3.323
400		1.649	1.966	2.336	2.588	3.111	3.315
500		1.648	1.965	2.334	2.586	3.107	3.310
600		1.647	1.964	2.333	2.584	3.104	3.307
∞		1.645	1.960	2.326	2.576	3.090	3.291