

UNIVERSITY OF SWAZILAND

SUPPLEMENTARY EXAMINATION PAPER 2013

TITLE OF PAPER : NON-PARAMETRIC ANALYSIS

COURSE CODE : ST409

TIME ALLOWED : 2 (TWO) HOURS

**REQUIRMENTS : STATISTICAL TABLES
AND CALCULATOR**

**INSTRUCTIONS : ANSWER ANY THREE (3) QUESTIONS.
ALL QUESTIONS CARRY EQUAL MARKS.**

**THIS PAPER IS NOT TO BE OPENED UNTIL PERMISSION HAS BEEN
GRANTED BY THE INVIGILATOR**

For all questions, clearly state the null & alternate hypotheses, the test statistics, the decision rule, the level of significance, the decision, the conclusions.

QUESTION ONE.

[20 marks]

A simple experiment was designed to see if flint in area A tended to have the same degree of hardness as flint in area B. Four sample pieces of flint were collected in area A and five sample pieces of flint were collected in area B. To determine which of two pieces of flint was harder, the two pieces were rubbed against each other. The piece sustaining less damage was judged the harder of the two. In this manner all nine pieces of flint were ordered according to hardness. The rank 1 was assigned to the softest piece, rank 2 to the next softest, and so on.

Origin of Piece	Rank
A	1
A	2
A	3
B	4
A	5
B	6
B	7
B	8
B	9

Use the Mann-Whitney test to determine whether the hardness of the flint in both areas are equal. Use 5% level of significance.

QUESTION TWO.

[20 marks]

A random sample of size 9, X_1, \dots, X_9 is obtained from one population, and a random sample of size 15, Y_1, \dots, Y_{15} is obtained from a second population as follows:

X_i : 7.6, 9.3, 8.7, 8.6, 10.6, 10.1, 11.2, 9.9, 8.4

Y_i : 8.2, 6.8, 6.5, 5.9, 5.2, 9.1, 5.7, 12.5, 11.5, 12.3, 10.8, 14.6, 11.3, 13.4, 9.8

Use the Smirnov test with $\alpha = 0.10$ to test whether the two populations have same distribution function..

QUESTION THREE.

[10 + 10 marks]

- a. Of 16 cars inspected during a safety campaign, 6 were found to be unsafe. Test the hypothesis that no more than 10% of the cars in the population are unsafe. Use 5% level of significance.
- b. The following table shows the information on cotton crop insurance for the years 1976 to 2000.

Year	# of Crops insured	Year	# of Crops insured
1976	19,479	1989	15,375
1977	26,667	1990	21,312
1978	63,969	1991	26,526
1979	57,715	1992	24,865
1980	38,086	1993	21,152
1981	38,434	1994	23,458
1982	24,196	1995	25,774
1983	19,319	1996	32,646
1984	29,975	1997	31,786
1985	25,451	1998	24,821
1986	20,410	1999	19,593
1987	19,940	2000	14,960
1988	15,628		

Do these data indicate a downward trend in the number of crops insured? Use the Cox-Stuart test for trend with $\alpha = 0.05$.

QUESTION FOUR.

[20 marks]

Two judges rated the participants in a dance contest as follows:

Judge	Contestant							
	A	B	C	D	E	F	G	H
1	5	2	6	3	4	1	8	7
2	3	1	7	4	5	2	6	8

Are the two rankings independent? Use either Spearman's ρ test or Kendall's τ test with $\alpha = 0.05$.

TABLE A1 Normal Distribution*

p	Selected values									
	0.000	0.001	0.002	0.003	0.004	0.005	0.006	0.007	0.008	0.009
0.00		-3.0902	-2.8782	-2.7478	-2.6521	-2.5758	-2.5121	-2.4573	-2.4089	-2.3656
0.01	-2.3263	-2.2904	-2.2571	-2.2262	-2.1973	-2.1701	-2.1444	-2.1201	-2.0969	-2.0749
0.02	-2.0537	-2.0335	-2.0141	-1.9954	-1.9774	-1.9600	-1.9431	-1.9268	-1.9110	-1.8957
0.03	-1.8808	-1.8663	-1.8522	-1.8384	-1.8250	-1.8119	-1.7991	-1.7866	-1.7744	-1.7624
0.04	-1.7507	-1.7392	-1.7279	-1.7169	-1.7060	-1.6954	-1.6849	-1.6747	-1.6646	-1.6546
0.05	-1.6449	-1.6352	-1.6258	-1.6164	-1.6072	-1.5982	-1.5893	-1.5805	-1.5718	-1.5632
0.06	-1.5548	-1.5464	-1.5382	-1.5301	-1.5220	-1.5141	-1.5063	-1.4985	-1.4909	-1.4833
0.07	-1.4758	-1.4684	-1.4611	-1.4538	-1.4466	-1.4395	-1.4325	-1.4255	-1.4187	-1.4118
0.08	-1.4051	-1.3984	-1.3917	-1.3852	-1.3787	-1.3722	-1.3658	-1.3595	-1.3532	-1.3469
0.09	-1.3408	-1.3346	-1.3285	-1.3225	-1.3165	-1.3106	-1.3047	-1.2988	-1.2930	-1.2873
0.10	-1.2816	-1.2759	-1.2702	-1.2646	-1.2591	-1.2536	-1.2481	-1.2426	-1.2372	-1.2319
0.11	-1.2265	-1.2212	-1.2160	-1.2107	-1.2055	-1.2004	-1.1952	-1.1901	-1.1850	-1.1800
0.12	-1.1750	-1.1700	-1.1650	-1.1601	-1.1552	-1.1503	-1.1455	-1.1407	-1.1359	-1.1311
0.13	-1.1264	-1.1217	-1.1170	-1.1123	-1.1077	-1.1031	-1.0985	-1.0939	-1.0893	-1.0848
0.14	-1.0803	-1.0758	-1.0714	-1.0669	-1.0625	-1.0581	-1.0537	-1.0494	-1.0450	-1.0407
0.15	-1.0364	-1.0322	-1.0279	-1.0237	-1.0194	-1.0152	-1.0110	-1.0069	-1.0027	-0.9986
0.16	-0.9945	-0.9904	-0.9863	-0.9822	-0.9782	-0.9741	-0.9701	-0.9661	-0.9621	-0.9581
0.17	-0.9542	-0.9502	-0.9463	-0.9424	-0.9385	-0.9346	-0.9307	-0.9269	-0.9230	-0.9192
0.18	-0.9154	-0.9116	-0.9078	-0.9040	-0.9002	-0.8965	-0.8927	-0.8890	-0.8853	-0.8816
0.19	-0.8779	-0.8742	-0.8705	-0.8669	-0.8633	-0.8596	-0.8560	-0.8524	-0.8488	-0.8452
0.20	-0.8416	-0.8381	-0.8345	-0.8310	-0.8274	-0.8239	-0.8204	-0.8169	-0.8134	-0.8099
0.21	-0.8064	-0.8030	-0.7995	-0.7961	-0.7926	-0.7892	-0.7858	-0.7824	-0.7790	-0.7756
0.22	-0.7722	-0.7688	-0.7655	-0.7621	-0.7588	-0.7554	-0.7521	-0.7488	-0.7454	-0.7421
0.23	-0.7388	-0.7356	-0.7323	-0.7290	-0.7257	-0.7225	-0.7192	-0.7160	-0.7128	-0.7095
0.24	-0.7063	-0.7031	-0.6999	-0.6967	-0.6935	-0.6903	-0.6871	-0.6840	-0.6808	-0.6776

TABLE A1 (Continued)

p	0.000	0.001	0.002	0.003	0.004	0.005	0.006	0.007	0.008	0.009
0.25	-0.6745	-0.6713	-0.6682	-0.6651	-0.6620	-0.6588	-0.6557	-0.6526	-0.6495	-0.6464
0.26	-0.6433	-0.6403	-0.6372	-0.6341	-0.6311	-0.6280	-0.6250	-0.6219	-0.6189	-0.6158
0.27	-0.6128	-0.6098	-0.6068	-0.6038	-0.6008	-0.5978	-0.5948	-0.5918	-0.5888	-0.5858
0.28	-0.5828	-0.5799	-0.5769	-0.5740	-0.5710	-0.5681	-0.5651	-0.5622	-0.5592	-0.5563
0.29	-0.5534	-0.5505	-0.5476	-0.5446	-0.5417	-0.5388	-0.5359	-0.5330	-0.5302	-0.5273
0.30	-0.5244	-0.5215	-0.5187	-0.5158	-0.5129	-0.5101	-0.5072	-0.5044	-0.5015	-0.4987
0.31	-0.4959	-0.4930	-0.4902	-0.4874	-0.4845	-0.4817	-0.4789	-0.4761	-0.4733	-0.4705
0.32	-0.4677	-0.4649	-0.4621	-0.4593	-0.4565	-0.4538	-0.4510	-0.4482	-0.4454	-0.4427
0.33	-0.4399	-0.4372	-0.4344	-0.4316	-0.4289	-0.4261	-0.4234	-0.4207	-0.4179	-0.4152
0.34	-0.4125	-0.4097	-0.4070	-0.4043	-0.4016	-0.3989	-0.3961	-0.3934	-0.3907	-0.3880
0.35	-0.3853	-0.3826	-0.3799	-0.3772	-0.3745	-0.3719	-0.3692	-0.3665	-0.3638	-0.3611
0.36	-0.3585	-0.3558	-0.3531	-0.3505	-0.3478	-0.3451	-0.3425	-0.3398	-0.3372	-0.3345
0.37	-0.3319	-0.3292	-0.3266	-0.3239	-0.3213	-0.3186	-0.3160	-0.3134	-0.3107	-0.3081
0.38	-0.3055	-0.3029	-0.3002	-0.2976	-0.2950	-0.2924	-0.2898	-0.2871	-0.2845	-0.2819
0.39	-0.2793	-0.2767	-0.2741	-0.2715	-0.2689	-0.2663	-0.2637	-0.2611	-0.2585	-0.2559
0.40	-0.2533	-0.2508	-0.2482	-0.2456	-0.2430	-0.2404	-0.2378	-0.2352	-0.2327	-0.2301
0.41	-0.2275	-0.2250	-0.2224	-0.2198	-0.2172	-0.2147	-0.2121	-0.2096	-0.2070	-0.2045
0.42	-0.2019	-0.1993	-0.1968	-0.1942	-0.1917	-0.1891	-0.1866	-0.1840	-0.1815	-0.1789
0.43	-0.1764	-0.1738	-0.1713	-0.1687	-0.1662	-0.1637	-0.1611	-0.1586	-0.1560	-0.1535
0.44	-0.1510	-0.1484	-0.1459	-0.1434	-0.1408	-0.1383	-0.1358	-0.1332	-0.1307	-0.1282
0.45	-0.1257	-0.1231	-0.1206	-0.1181	-0.1156	-0.1130	-0.1105	-0.1080	-0.1055	-0.1030
0.46	-0.1004	-0.0979	-0.0954	-0.0929	-0.0904	-0.0878	-0.0853	-0.0828	-0.0803	-0.0778
0.47	-0.0753	-0.0728	-0.0702	-0.0677	-0.0652	-0.0627	-0.0602	-0.0577	-0.0552	-0.0527
0.48	-0.0502	-0.0476	-0.0451	-0.0426	-0.0401	-0.0376	-0.0351	-0.0326	-0.0301	-0.0276
0.49	-0.0251	-0.0226	-0.0201	-0.0175	-0.0150	-0.0125	-0.0100	-0.0075	-0.0050	-0.0025
0.50	0.0000	0.0025	0.0050	0.0075	0.0100	0.0125	0.0150	0.0175	0.0201	0.0226
0.51	0.0251	0.0276	0.0301	0.0326	0.0351	0.0376	0.0401	0.0426	0.0451	0.0476
0.52	0.0502	0.0527	0.0552	0.0577	0.0602	0.0627	0.0652	0.0677	0.0702	0.0728
0.53	0.0753	0.0778	0.0803	0.0828	0.0853	0.0878	0.0904	0.0929	0.0954	0.0979
0.54	0.1004	0.1030	0.1055	0.1080	0.1105	0.1130	0.1156	0.1181	0.1206	0.1231

Table A1 (Continued)

p	0.000	0.001	0.002	0.003	0.004	0.005	0.006	0.007	0.008	0.009
0.55	0.1257	0.1282	0.1307	0.1332	0.1358	0.1383	0.1408	0.1434	0.1459	0.1484
0.56	0.1510	0.1535	0.1560	0.1586	0.1611	0.1637	0.1662	0.1687	0.1713	0.1738
0.57	0.1764	0.1789	0.1815	0.1840	0.1866	0.1891	0.1917	0.1942	0.1968	0.1993
0.58	0.2019	0.2045	0.2070	0.2096	0.2121	0.2147	0.2173	0.2198	0.2224	0.2250
0.59	0.2275	0.2301	0.2327	0.2353	0.2378	0.2404	0.2430	0.2456	0.2482	0.2508
0.60	0.2533	0.2559	0.2585	0.2611	0.2637	0.2663	0.2689	0.2715	0.2741	0.2767
0.61	0.2793	0.2819	0.2845	0.2871	0.2898	0.2924	0.2950	0.2976	0.3002	0.3029
0.62	0.3055	0.3081	0.3107	0.3134	0.3160	0.3186	0.3213	0.3239	0.3266	0.3292
0.63	0.3319	0.3345	0.3372	0.3398	0.3425	0.3451	0.3478	0.3505	0.3531	0.3558
0.64	0.3585	0.3611	0.3638	0.3665	0.3692	0.3719	0.3745	0.3772	0.3799	0.3826
0.65	0.3853	0.3880	0.3907	0.3934	0.3961	0.3989	0.4016	0.4043	0.4070	0.4097
0.66	0.4125	0.4152	0.4179	0.4207	0.4234	0.4261	0.4289	0.4316	0.4344	0.4372
0.67	0.4399	0.4427	0.4454	0.4482	0.4510	0.4538	0.4565	0.4593	0.4621	0.4649
0.68	0.4677	0.4705	0.4733	0.4761	0.4789	0.4817	0.4845	0.4874	0.4902	0.4930
0.69	0.4959	0.4987	0.5015	0.5044	0.5072	0.5101	0.5129	0.5158	0.5187	0.5215
0.70	0.5244	0.5273	0.5302	0.5330	0.5359	0.5388	0.5417	0.5446	0.5476	0.5505
0.71	0.5534	0.5563	0.5592	0.5622	0.5651	0.5681	0.5710	0.5740	0.5769	0.5799
0.72	0.5828	0.5858	0.5888	0.5918	0.5948	0.5978	0.6008	0.6038	0.6068	0.6098
0.73	0.6128	0.6158	0.6189	0.6219	0.6250	0.6280	0.6311	0.6341	0.6372	0.6403
0.74	0.6433	0.6464	0.6495	0.6526	0.6557	0.6588	0.6620	0.6651	0.6682	0.6713
0.75	0.6745	0.6776	0.6808	0.6840	0.6871	0.6903	0.6935	0.6967	0.6999	0.7031
0.76	0.7063	0.7095	0.7128	0.7160	0.7192	0.7225	0.7257	0.7290	0.7323	0.7356
0.77	0.7388	0.7421	0.7454	0.7488	0.7521	0.7554	0.7588	0.7621	0.7655	0.7688
0.78	0.7722	0.7756	0.7790	0.7824	0.7858	0.7892	0.7926	0.7961	0.7995	0.8030
0.79	0.8064	0.8099	0.8134	0.8169	0.8204	0.8239	0.8274	0.8310	0.8345	0.8381
0.80	0.8416	0.8452	0.8488	0.8524	0.8560	0.8596	0.8633	0.8669	0.8705	0.8742
0.81	0.8779	0.8816	0.8853	0.8890	0.8927	0.8965	0.9002	0.9040	0.9078	0.9116
0.82	0.9154	0.9192	0.9230	0.9269	0.9307	0.9346	0.9385	0.9424	0.9463	0.9502

Table A1 (Continued)

p	0.000	0.001	0.002	0.003	0.004	0.005	0.006	0.007	0.008	0.009
0.83	0.9542	0.9581	0.9621	0.9661	0.9701	0.9741	0.9782	0.9822	0.9863	0.9904
0.84	0.9945	0.9986	1.0027	1.0069	1.0110	1.0152	1.0194	1.0237	1.0279	1.0322
0.85	1.0364	1.0407	1.0450	1.0494	1.0537	1.0581	1.0625	1.0669	1.0714	1.0758
0.86	1.0803	1.0848	1.0893	1.0939	1.0985	1.1031	1.1077	1.1123	1.1170	1.1217
0.87	1.1264	1.1311	1.1359	1.1407	1.1455	1.1503	1.1552	1.1601	1.1650	1.1700
0.88	1.1750	1.1800	1.1850	1.1901	1.1952	1.2004	1.2055	1.2107	1.2160	1.2212
0.89	1.2265	1.2319	1.2372	1.2426	1.2481	1.2536	1.2591	1.2646	1.2702	1.2759
0.90	1.2816	1.2873	1.2930	1.2988	1.3047	1.3106	1.3165	1.3225	1.3285	1.3346
0.91	1.3406	1.3469	1.3532	1.3595	1.3658	1.3722	1.3787	1.3852	1.3917	1.3984
0.92	1.4051	1.4116	1.4181	1.4255	1.4325	1.4395	1.4466	1.4538	1.4611	1.4684
0.93	1.4756	1.4823	1.4895	1.4965	1.5035	1.5111	1.5220	1.5301	1.5382	1.5464
0.94	1.5546	1.5622	1.5710	1.5805	1.5893	1.5982	1.6072	1.6164	1.6258	1.6352
0.95	1.6449	1.6546	1.6646	1.6747	1.6849	1.6954	1.7060	1.7169	1.7279	1.7392
0.96	1.7507	1.7624	1.7744	1.7866	1.7991	1.8119	1.8250	1.8384	1.8522	1.8663
0.97	1.8808	1.8957	1.9110	1.9268	1.9431	1.9600	1.9774	1.9954	2.0141	2.0335
0.98	2.0537	2.0749	2.0969	2.1201	2.1444	2.1701	2.1973	2.2262	2.2571	2.2904
0.99	2.3263	2.3656	2.4089	2.4573	2.5121	2.5758	2.6521	2.7478	2.8782	3.0902

SOURCE: Generated by R. L. Iman. Used with permission.

* The entries in this table are quantiles z_p of the standard normal random variable Z selected so $P(Z \leq z_p) = p$ and $P(Z > z_p) = 1 - p$. Note that the value of p to two decimal places determines which row to use; the third decimal place of p determines which column to use to find z_p .

n	y	p = 0.50	0.55	0.60	0.65	0.70	0.75	0.80	0.85	0.90	0.95
19	0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	1	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	2	0.0004	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	3	0.0022	0.0005	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	4	0.0096	0.0028	0.0006	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	5	0.0318	0.0109	0.0031	0.0007	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000
	6	0.0835	0.0342	0.0116	0.0031	0.0006	0.0001	0.0000	0.0000	0.0000	0.0000
	7	0.1796	0.0871	0.0352	0.0114	0.0028	0.0005	0.0000	0.0000	0.0000	0.0000
	8	0.3328	0.1841	0.0885	0.0347	0.0105	0.0023	0.0003	0.0000	0.0000	0.0000
	9	0.5000	0.3290	0.1861	0.0875	0.0326	0.0089	0.0016	0.0001	0.0000	0.0000
	10	0.6762	0.5060	0.3325	0.1855	0.0839	0.0287	0.0067	0.0008	0.0000	0.0000
	11	0.8204	0.6831	0.5122	0.3344	0.1820	0.0775	0.0233	0.0041	0.0003	0.0000
	12	0.9165	0.8273	0.6919	0.5188	0.3345	0.1749	0.0676	0.0163	0.0017	0.0000
	13	0.9682	0.9223	0.8371	0.7032	0.5261	0.3322	0.1631	0.0537	0.0086	0.0002
	14	0.9904	0.9720	0.9304	0.8500	0.7178	0.5346	0.3267	0.0537	0.0086	0.0002
	15	0.9978	0.9923	0.9770	0.9409	0.8668	0.7369	0.5449	0.3159	0.1150	0.0132
	16	0.9996	0.9985	0.9945	0.9830	0.9538	0.8887	0.7631	0.5587	0.2946	0.0665
	17	1.0000	0.9998	0.9992	0.9969	0.9896	0.9690	0.9171	0.8015	0.5797	0.2453
	18	1.0000	1.0000	0.9999	0.9997	0.9989	0.9958	0.9856	0.9544	0.8649	0.6726
	19	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
20	0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	1	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	2	0.0002	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	3	0.0013	0.0003	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	4	0.0059	0.0015	0.0003	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	5	0.0207	0.0064	0.0016	0.0003	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	6	0.0577	0.0214	0.0065	0.0015	0.0003	0.0000	0.0000	0.0000	0.0000	0.0000
	7	0.1316	0.0580	0.0210	0.0060	0.0013	0.0002	0.0000	0.0000	0.0000	0.0000
	8	0.2517	0.1308	0.0565	0.0196	0.0051	0.0009	0.0001	0.0000	0.0000	0.0000
	9	0.4119	0.2493	0.1275	0.0532	0.0171	0.0039	0.0006	0.0000	0.0000	0.0000
	10	0.5881	0.4086	0.2447	0.1218	0.0480	0.0139	0.0026	0.0002	0.0000	0.0000
	11	0.7483	0.5857	0.4044	0.2376	0.1133	0.0409	0.0100	0.0013	0.0001	0.0000
	12	0.8684	0.7480	0.5841	0.3990	0.2277	0.1018	0.0321	0.0059	0.0004	0.0000
	13	0.9423	0.8701	0.7500	0.5834	0.3920	0.2142	0.0867	0.0219	0.0024	0.0000
	14	0.9793	0.9447	0.8744	0.7546	0.5836	0.3828	0.1958	0.0673	0.0113	0.0003
	15	0.9941	0.9811	0.9490	0.8818	0.7625	0.5852	0.3704	0.1702	0.0432	0.0026
	16	0.9987	0.9951	0.9840	0.9556	0.8929	0.7748	0.5886	0.3523	0.1330	0.0159
	17	0.9998	0.9991	0.9964	0.9879	0.9645	0.9087	0.7939	0.5951	0.3231	0.0755
	18	1.0000	0.9999	0.9995	0.9979	0.9924	0.9757	0.9308	0.8244	0.6083	0.2642
	19	1.0000	1.0000	1.0000	0.9998	0.9992	0.9968	0.9885	0.9612	0.8784	0.6415
	20	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

* Y has the binomial distribution with parameters n and p. The entries are the values of $P(Y \leq y) = \sum_{i=0}^y \binom{n}{i} p^i (1-p)^{n-i}$, for p ranging from 0.05 to 0.95. For n larger than 20, the rth quantile y_r of a binomial random variable may be approximated using $y_r \approx np + z_r \sqrt{np(1-p)}$, where z_r is the rth quantile of a standard normal random variable, obtained from Table A1.

TABLE A7 Quantiles of the Mann-Whitney Test Statistic*

n	p	m = 2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20		
2	0.001	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	
	0.005	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	
	0.01	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	
	0.025	3	3	3	3	3	3	3	4	4	4	5	5	5	5	5	5	6	6	6	6	
	0.05	3	3	3	4	4	4	4	5	5	5	5	6	6	7	7	7	7	7	7	7	7
	0.10	3	4	4	5	5	5	5	6	6	7	7	8	8	8	9	9	10	10	10	10	11
3	0.001	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	
	0.005	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	
	0.01	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	
	0.025	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	
	0.05	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	
	0.10	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6
4	0.001	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	
	0.005	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	
	0.01	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	
	0.025	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	
	0.05	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	
	0.10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10
5	0.001	15	15	15	15	15	15	15	15	15	15	15	15	15	15	15	15	15	15	15	15	
	0.005	15	15	15	15	15	15	15	15	15	15	15	15	15	15	15	15	15	15	15	15	
	0.01	15	15	15	15	15	15	15	15	15	15	15	15	15	15	15	15	15	15	15	15	
	0.025	15	15	15	15	15	15	15	15	15	15	15	15	15	15	15	15	15	15	15	15	
	0.05	15	15	15	15	15	15	15	15	15	15	15	15	15	15	15	15	15	15	15	15	
	0.10	15	15	15	15	15	15	15	15	15	15	15	15	15	15	15	15	15	15	15	15	15
6	0.001	21	21	21	21	21	21	21	21	21	21	21	21	21	21	21	21	21	21	21	21	
	0.005	21	21	21	21	21	21	21	21	21	21	21	21	21	21	21	21	21	21	21	21	
	0.01	21	21	21	21	21	21	21	21	21	21	21	21	21	21	21	21	21	21	21	21	
	0.025	21	21	21	21	21	21	21	21	21	21	21	21	21	21	21	21	21	21	21	21	
	0.05	21	21	21	21	21	21	21	21	21	21	21	21	21	21	21	21	21	21	21	21	
	0.10	21	21	21	21	21	21	21	21	21	21	21	21	21	21	21	21	21	21	21	21	21
7	0.001	28	28	28	28	28	28	28	28	28	28	28	28	28	28	28	28	28	28	28	28	
	0.005	28	28	28	28	28	28	28	28	28	28	28	28	28	28	28	28	28	28	28	28	
	0.01	28	28	28	28	28	28	28	28	28	28	28	28	28	28	28	28	28	28	28	28	
	0.025	28	28	28	28	28	28	28	28	28	28	28	28	28	28	28	28	28	28	28	28	
	0.05	28	28	28	28	28	28	28	28	28	28	28	28	28	28	28	28	28	28	28	28	28
	0.10	28	28	28	28	28	28	28	28	28	28	28	28	28	28	28	28	28	28	28	28	28
8	0.001	36	36	36	36	36	36	36	36	36	36	36	36	36	36	36	36	36	36	36	36	
	0.005	36	36	36	36	36	36	36	36	36	36	36	36	36	36	36	36	36	36	36	36	
	0.01	36	36	36	36	36	36	36	36	36	36	36	36	36	36	36	36	36	36	36	36	
	0.025	36	36	36	36	36	36	36	36	36	36	36	36	36	36	36	36	36	36	36	36	
	0.05	36	36	36	36	36	36	36	36	36	36	36	36	36	36	36	36	36	36	36	36	
	0.10	36	36	36	36	36	36	36	36	36	36	36	36	36	36	36	36	36	36	36	36	36

TABLE A7 (Continued)

n	p	m = 2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	
9	0.001	45	45	45	47	48	49	51	53	54	56	58	60	61	63	65	67	69	71	71
	0.005	45	46	47	49	51	53	55	57	59	62	64	66	68	70	73	75	77	79	79
	0.01	45	47	49	51	53	55	57	60	62	64	67	69	72	74	77	79	82	84	84
	0.025	46	48	50	53	56	58	61	63	66	69	72	74	77	80	83	85	88	91	91
	0.05	47	50	52	55	58	61	64	67	70	73	76	79	82	85	88	91	94	97	97
	0.10	48	51	55	58	61	64	68	71	74	77	81	84	87	91	94	98	101	104	104
10	0.001	55	55	56	57	59	61	62	64	66	68	70	73	75	77	79	81	83	85	85
	0.005	55	56	58	60	62	65	67	69	72	74	77	80	82	85	87	90	93	95	95
	0.01	55	57	59	62	64	67	69	72	75	78	80	83	86	89	92	94	97	100	100
	0.025	56	59	61	64	67	70	73	76	79	82	85	89	92	95	98	101	104	108	108
	0.05	57	60	63	67	70	73	76	80	83	87	90	93	97	100	104	107	111	114	114
	0.10	59	62	66	69	73	77	80	84	88	92	95	99	103	107	110	114	118	122	122
11	0.001	66	66	67	69	71	73	75	77	79	82	84	87	89	91	94	96	99	101	101
	0.005	66	67	69	72	74	77	80	83	85	88	91	94	97	100	103	106	109	112	112
	0.01	66	68	71	74	76	79	82	85	89	92	95	98	101	104	108	111	114	117	117
	0.025	67	70	73	76	80	83	86	90	93	97	100	104	107	111	114	118	122	125	125
	0.05	68	72	75	79	83	86	90	94	98	101	105	109	113	117	121	124	128	132	132
	0.10	70	74	78	82	86	90	94	98	103	107	111	115	119	124	128	132	136	140	140
12	0.001	78	78	79	81	83	86	88	91	93	96	98	102	104	106	110	113	116	118	118
	0.005	78	80	82	85	88	91	94	97	100	103	106	110	113	116	120	123	126	130	130
	0.01	78	81	84	87	90	93	96	100	103	107	110	114	117	121	125	128	132	135	135
	0.025	80	83	86	90	93	97	101	105	108	112	116	120	124	128	132	136	140	144	144
	0.05	81	84	88	92	96	100	105	109	111	117	121	126	130	134	139	143	147	151	151
	0.10	83	87	91	96	100	105	109	114	118	123	128	132	137	142	146	151	155	160	160
13	0.001	91	91	93	95	97	100	103	106	109	112	115	118	121	124	127	130	134	137	137
	0.005	91	93	95	99	102	105	109	112	116	119	123	126	130	134	137	141	145	149	149
	0.01	92	94	97	101	104	108	112	115	119	123	127	131	135	139	143	147	151	155	155
	0.025	93	96	100	104	108	112	116	120	125	129	133	137	142	146	151	155	159	164	164
	0.05	94	98	102	107	111	116	120	125	129	134	139	143	148	153	157	162	167	172	172
	0.10	96	101	105	110	115	120	125	130	135	140	145	150	155	160	166	171	176	181	181
14	0.001	105	105	107	109	112	115	118	121	125	128	131	135	138	142	145	149	152	156	156
	0.005	105	107	110	113	117	121	124	128	132	136	140	144	148	152	156	160	164	169	169
	0.01	106	108	112	116	119	123	128	132	136	140	144	149	153	157	162	166	171	175	175
	0.025	107	111	115	119	123	128	132	137	142	146	151	156	161	165	170	175	180	184	184
	0.05	109	113	117	122	127	132	137	142	147	152	1								

TABLE A7 (Continued)

n	p	m = 2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
17	0.001	153	154	156	159	163	167	171	175	179	183	188	192	197	201	206	211	215	220
	0.005	153	156	160	164	169	173	178	183	188	193	198	203	208	214	219	224	229	235
	0.01	154	158	162	167	172	177	182	187	192	198	203	209	214	220	225	231	236	242
	0.025	156	160	165	171	176	182	188	193	199	205	211	217	223	229	235	241	247	253
	0.05	157	163	169	174	180	187	193	199	205	211	218	224	231	237	243	250	256	263
18	0.001	171	172	175	178	182	186	190	195	199	204	209	214	218	223	228	233	238	243
	0.005	171	174	178	183	188	193	198	203	209	214	219	225	230	236	242	247	253	259
	0.01	172	176	181	186	191	196	202	208	213	219	225	231	237	242	248	254	260	266
	0.025	174	179	184	190	196	202	208	214	220	227	233	239	246	252	258	265	271	278
	0.05	176	181	188	194	200	207	213	220	227	233	240	247	254	260	267	274	281	288
19	0.001	190	191	194	198	202	206	211	216	220	225	231	236	241	246	251	257	262	268
	0.005	191	194	198	203	208	213	219	224	230	236	242	248	254	260	265	272	278	284
	0.01	192	195	200	206	211	217	223	229	235	241	247	254	260	266	273	279	285	292
	0.025	193	198	204	210	216	223	229	236	243	249	256	263	269	276	283	290	297	304
	0.05	195	201	208	214	221	228	235	242	249	256	263	271	278	285	292	300	307	314
20	0.001	210	211	214	218	223	227	232	237	243	248	253	259	265	270	276	281	287	293
	0.005	211	214	219	224	229	235	241	247	253	259	265	271	278	284	290	297	303	310
	0.01	212	216	221	227	233	239	245	251	258	264	271	278	284	291	298	304	311	318
	0.025	213	219	225	231	238	245	251	259	266	273	280	287	294	301	309	316	323	330
	0.05	215	222	229	236	243	250	258	265	273	280	288	295	303	311	318	326	334	341
0.10	218	226	233	241	249	257	265	273	281	289	297	305	313	321	330	338	346	354	

For n or m greater than 20, the pth quantile w_p of the Mann-Whitney test statistic may be approximated by

$$w_p = n(N + 1)/2 + z_p \sqrt{nm(N + 1)/12}$$

where z_p is the pth quantile of a standard normal random variable, obtained from Table A1, and where $N = m + n$.

*The entries in this table are quantiles w_p of the Mann-Whitney test statistic T, given by Equation 5.1.1, for selected values of p. Note that $P(T < w_p) \leq p$. Upper quantiles may be found from the equation

$$w_p = n(n + m + 1) - w_{1-p}$$

Critical regions correspond to values of T less than (or greater than) but not equal to the appropriate quantile.

TABLE A8 Quantiles of the Kruskal-Wallis Test Statistic for Small Sample Sizes*

Sample Sizes	$W_{0.95}$	$W_{0.99}$	$W_{0.995}$
2, 2, 2	3.7143	4.5714	4.5714
3, 2, 1	3.8571	4.2857	4.2857
3, 2, 2	4.4643	4.5000	5.3571
3, 3, 1	4.0000	4.5714	5.1429
3, 3, 2	4.2500	5.1389	6.2500
3, 3, 3	4.6000	5.0867	6.4889
4, 2, 1	4.0179	4.8214	4.8214
4, 2, 2	4.1667	5.1250	6.0000
4, 3, 1	3.8889	5.0000	5.8333
4, 3, 2	4.4444	5.4000	6.3000
4, 3, 3	4.7000	5.7273	6.7091
4, 4, 1	4.0667	4.8667	6.1667
4, 4, 2	4.4455	5.2364	6.8727
4, 4, 3	4.7730	5.5758	7.1364
4, 4, 4	4.5000	5.6538	7.5385
5, 2, 1	4.0500	4.4500	5.2500
5, 2, 2	4.2933	5.0400	6.1333
5, 3, 1	3.8400	4.8711	6.4000
5, 3, 2	4.4946	5.1055	6.8218
5, 3, 3	4.4121	5.5152	6.9818
5, 4, 1	3.9600	4.8600	6.8400
5, 4, 2	4.5182	5.2682	7.1182
5, 4, 3	4.5231	5.6308	7.3949
5, 4, 4	4.6187	5.6176	7.7440
5, 5, 1	4.0364	4.9091	6.8364
5, 5, 2	4.5077	5.2462	7.2692
5, 5, 3	4.5363	5.6264	7.5429
5, 5, 4	4.5200	5.6429	7.7914
5, 5, 5	4.5000	5.6600	7.9800

Source: Adapted from Iman, Quade, and Alexander (1975), with permission from the American Mathematical Society.
 *The null hypothesis may be rejected at the level α if the Kruskal-Wallis test statistic, given by Equation 5.2.5, exceeds the $1 - \alpha$ quantile given in the table.

TABLE A10 Quantiles of Spearman's ρ^a

n	$\rho = 0.900$	0.950	0.975	0.990	0.995	0.999
4	0.8000	0.8000				
5	0.7000	0.8000	0.9000	0.9000		
6	0.6000	0.7714	0.8286	0.8857	0.9429	
7	0.5357	0.6786	0.7500	0.8571	0.8929	0.9643
8	0.5000	0.6190	0.7143	0.8095	0.8571	0.9286
9	0.4667	0.5833	0.6833	0.7667	0.8167	0.9000
10	0.4424	0.5515	0.6364	0.7333	0.7818	0.8667
11	0.4182	0.5273	0.6091	0.7000	0.7455	0.8364
12	0.3986	0.4965	0.5804	0.6713	0.7203	0.8112
13	0.3791	0.4780	0.5549	0.6429	0.6978	0.7857
14	0.3626	0.4593	0.5341	0.6220	0.6747	0.7670
15	0.3500	0.4429	0.5179	0.6000	0.6500	0.7464
16	0.3382	0.4265	0.5000	0.5794	0.6324	0.7265
17	0.3260	0.4118	0.4853	0.5637	0.6152	0.7083
18	0.3148	0.3994	0.4696	0.5480	0.5975	0.6904
19	0.3070	0.3895	0.4579	0.5333	0.5825	0.6737
20	0.2977	0.3789	0.4451	0.5203	0.5684	0.6586
21	0.2909	0.3688	0.4351	0.5078	0.5545	0.6455
22	0.2829	0.3597	0.4241	0.4963	0.5426	0.6318
23	0.2767	0.3518	0.4150	0.4852	0.5306	0.6186
24	0.2704	0.3435	0.4061	0.4748	0.5200	0.6070
25	0.2646	0.3362	0.3977	0.4654	0.5100	0.5962
26	0.2588	0.3299	0.3894	0.4564	0.5002	0.5856
27	0.2540	0.3236	0.3822	0.4481	0.4915	0.5757
28	0.2490	0.3175	0.3749	0.4401	0.4828	0.5660
29	0.2443	0.3113	0.3685	0.4320	0.4744	0.5567
30	0.2400	0.3059	0.3620	0.4251	0.4665	0.5479

For n greater than 30 the approximate quantiles of ρ may be obtained from

$$w_p \approx \frac{z_p}{\sqrt{n-1}}$$

where z_p is the p th quantile of a standard normal random variable obtained from Table A1.

SOURCE: Adapted from Glasser and Winter (1961), with corrections, with permission from the *Biometrika* Trustees.

* The entries in this table are selected quantiles w_p of the Spearman rank correlation coefficient ρ when used as a test statistic. The lower quantiles may be obtained from the equation

$$w_p = -w_{1-p}$$

The critical region corresponds to values of ρ smaller than (or greater than) but not including the appropriate quantile. Note that the median of ρ is 0.

TABLE A11 Quantiles of the Kendall test statistic $T = N_c - N_d$. Quantiles of Kendall's τ are given in parentheses. Lower quantiles are the negative of the upper quantiles, $w_p = -w_{1-p}$.

n	$p = 0.900$	0.950	0.975	0.990	0.995
4	4 (0.6667)	4 (0.6667)	6 (1.0000)	6 (1.0000)	6 (1.0000)
5	6 (0.6000)	6 (0.6000)	8 (0.8000)	8 (0.8000)	10 (1.0000)
6	7 (0.4667)	9 (0.6000)	11 (0.7333)	11 (0.7333)	13 (0.8667)
7	9 (0.4286)	11 (0.5238)	13 (0.6190)	15 (0.7143)	17 (0.8095)
8	10 (0.3571)	14 (0.5000)	16 (0.5714)	18 (0.6429)	20 (0.7143)
9	12 (0.3333)	16 (0.4444)	18 (0.5000)	22 (0.6111)	24 (0.6667)
10	15 (0.3333)	19 (0.4222)	21 (0.4667)	25 (0.5556)	27 (0.6000)
11	17 (0.3091)	21 (0.3818)	25 (0.4545)	29 (0.5273)	31 (0.5636)
12	18 (0.2727)	24 (0.3636)	28 (0.4242)	34 (0.5152)	36 (0.5455)
13	22 (0.2821)	26 (0.3333)	32 (0.4103)	38 (0.4872)	42 (0.5285)
14	23 (0.2527)	31 (0.3407)	35 (0.3846)	41 (0.4505)	45 (0.4945)
15	27 (0.2571)	33 (0.3143)	39 (0.3714)	47 (0.4476)	51 (0.4857)
16	28 (0.2333)	36 (0.3000)	44 (0.3667)	50 (0.4167)	56 (0.4667)
17	32 (0.2353)	40 (0.2941)	48 (0.3529)	56 (0.4118)	62 (0.4559)
18	35 (0.2288)	43 (0.2810)	51 (0.3333)	61 (0.3987)	67 (0.4379)
19	37 (0.2164)	47 (0.2749)	55 (0.3216)	65 (0.3801)	73 (0.4269)
20	40 (0.2105)	50 (0.2632)	60 (0.3158)	70 (0.3684)	78 (0.4105)
21	42 (0.2000)	54 (0.2571)	64 (0.3048)	76 (0.3619)	84 (0.4000)
22	45 (0.1948)	59 (0.2554)	69 (0.2987)	81 (0.3506)	89 (0.3853)
23	49 (0.1937)	63 (0.2490)	73 (0.2885)	87 (0.3439)	97 (0.3834)
24	52 (0.1884)	66 (0.2391)	78 (0.2826)	92 (0.3333)	102 (0.3696)
25	56 (0.1867)	70 (0.2333)	84 (0.2800)	98 (0.3267)	108 (0.3600)
26	59 (0.1815)	75 (0.2308)	89 (0.2738)	105 (0.3231)	115 (0.3538)
27	61 (0.1738)	79 (0.2251)	93 (0.2650)	111 (0.3162)	123 (0.3504)
28	66 (0.1746)	84 (0.2222)	98 (0.2593)	116 (0.3069)	128 (0.3386)
29	68 (0.1675)	88 (0.2167)	104 (0.2562)	124 (0.3054)	136 (0.3350)
30	73 (0.1678)	93 (0.2138)	109 (0.2506)	129 (0.2966)	143 (0.3287)
31	75 (0.1613)	97 (0.2086)	115 (0.2473)	135 (0.2903)	149 (0.3204)
32	80 (0.1613)	102 (0.2056)	120 (0.2419)	142 (0.2863)	158 (0.3185)
33	84 (0.1591)	106 (0.2008)	126 (0.2386)	150 (0.2841)	164 (0.3106)
34	87 (0.1551)	111 (0.1979)	131 (0.2335)	155 (0.2763)	173 (0.3084)
35	91 (0.1529)	115 (0.1933)	137 (0.2303)	163 (0.2739)	179 (0.3008)
36	94 (0.1492)	120 (0.1905)	144 (0.2286)	170 (0.2698)	188 (0.2984)
37	98 (0.1471)	126 (0.1892)	150 (0.2252)	176 (0.2643)	198 (0.2943)

TABLE A11 (Continued)

n	p = 0.900	0.950	0.975	0.990	0.995
18	103 (0.1465)	131 (0.1863)	155 (0.2205)	183 (0.2603)	203 (0.2888)
19	107 (0.1444)	137 (0.1849)	161 (0.2173)	191 (0.2578)	211 (0.2848)
10	110 (0.1372)	142 (0.1821)	168 (0.2154)	198 (0.2538)	220 (0.2821)
11	114 (0.1390)	146 (0.1780)	174 (0.2122)	206 (0.2512)	228 (0.2780)
12	119 (0.1382)	151 (0.1754)	181 (0.2102)	213 (0.2474)	235 (0.2729)
13	123 (0.1362)	157 (0.1739)	187 (0.2071)	221 (0.2447)	245 (0.2713)
14	128 (0.1353)	162 (0.1712)	194 (0.2051)	228 (0.2410)	252 (0.2664)
15	132 (0.1333)	168 (0.1697)	200 (0.2020)	236 (0.2383)	262 (0.2646)
16	135 (0.1304)	173 (0.1671)	207 (0.2000)	245 (0.2367)	271 (0.2618)
17	141 (0.1304)	179 (0.1656)	213 (0.1970)	253 (0.2340)	279 (0.2581)
18	144 (0.1277)	186 (0.1649)	220 (0.1950)	260 (0.2305)	288 (0.2553)
19	150 (0.1276)	190 (0.1616)	228 (0.1939)	268 (0.2279)	296 (0.2517)
20	153 (0.1249)	197 (0.1608)	233 (0.1902)	277 (0.2261)	305 (0.2490)
21	159 (0.1247)	203 (0.1592)	241 (0.1890)	285 (0.2235)	315 (0.2471)
22	162 (0.1222)	208 (0.1569)	248 (0.1870)	294 (0.2217)	324 (0.2443)
23	168 (0.1219)	214 (0.1553)	256 (0.1858)	302 (0.2192)	334 (0.2424)
24	173 (0.1209)	221 (0.1544)	263 (0.1838)	311 (0.2173)	343 (0.2397)
25	177 (0.1192)	227 (0.1529)	269 (0.1811)	319 (0.2148)	353 (0.2377)
26	182 (0.1182)	232 (0.1506)	276 (0.1792)	328 (0.2130)	362 (0.2351)
27	186 (0.1165)	240 (0.1504)	284 (0.1779)	336 (0.2105)	372 (0.2331)
28	191 (0.1155)	245 (0.1482)	291 (0.1760)	345 (0.2087)	381 (0.2305)
29	197 (0.1151)	251 (0.1467)	299 (0.1748)	355 (0.2075)	391 (0.2285)
30	202 (0.1141)	258 (0.1458)	306 (0.1729)	364 (0.2056)	402 (0.2271)

For n greater than 60, approximate quantiles of T may be obtained from

$$w_p \approx z_p \sqrt{\frac{n(n-1)(2n+5)}{18}}$$

where z_p is from the standard normal distribution given by Table A1. Approximate quantiles of τ may be obtained from

$$w_p \approx z_p \frac{\sqrt{2(2n+5)}}{3\sqrt{n(n-1)}}$$

Critical regions correspond to values of T greater than (or less than) but not including the appropriate quantile. Note that the median of T is 0. Quantiles for τ are obtained by dividing the quantiles of T by $(n-1)/2$.

SOURCE: Adapted from Table 1, Best (1974), with permission from the author.

TABLE A12 Quantiles of the Wilcoxon Signed Rank Test Statistic

n	Quantiles of the Wilcoxon Signed Rank Test Statistic									$\frac{n(n+1)}{2}$
	$W_{0.05}$	$W_{0.01}$	$W_{0.025}$	$W_{0.05}$	$W_{0.10}$	$W_{0.20}$	$W_{0.30}$	$W_{0.40}$	$W_{0.50}$	
4	0	0	0	0	1	3	3	4	5	10
5	0	0	0	1	3	4	5	6	7.5	15
6	0	0	1	3	4	6	8	9	10.5	21
7	0	1	3	4	6	9	11	12	14	28
8	1	2	4	6	9	12	14	16	18	36
9	2	4	6	9	11	15	18	20	22.5	45
10	4	6	9	11	15	19	22	25	27.5	55
11	6	8	11	14	18	23	27	30	33	66
12	8	10	14	18	22	28	32	36	39	78
13	10	13	18	22	27	33	38	42	45.5	91
14	13	16	22	26	32	39	44	48	52.5	105
15	16	20	26	31	37	45	51	55	60	120
16	20	24	30	36	43	51	58	63	68	136
17	24	28	35	42	49	58	65	71	76.5	153
18	28	33	41	48	56	65	73	80	85.5	171
19	33	38	47	54	63	74	82	89	95	190
20	38	44	53	61	70	82	91	98	105	210
21	44	50	59	68	78	91	100	108	115.5	231
22	49	56	67	76	87	100	110	119	126.5	253
23	55	63	74	84	95	110	120	130	138	276
24	62	70	82	92	105	120	131	141	150	300
25	69	77	90	101	114	131	143	153	162.5	325
26	76	85	99	111	125	145	155	165	175.5	351
27	84	94	108	120	135	154	167	178	189	378
28	92	102	117	131	146	166	180	192	203	406
29	101	111	127	141	158	179	193	206	217.5	435
30	110	121	138	152	170	191	207	220	232.5	465
31	119	131	148	164	182	205	221	235	248	496
32	129	141	160	176	195	219	236	250	264	528
33	139	152	171	188	208	233	251	266	280.5	561
34	149	163	183	201	222	248	266	282	297.5	595
35	160	175	196	214	236	263	283	299	315	630
36	172	187	209	228	251	279	299	317	333	666
37	184	199	222	242	266	295	316	335	351.5	703
38	196	212	236	257	282	312	334	353	370.5	741
39	208	225	250	272	298	329	352	372	390	780
40	221	239	265	287	314	347	371	391	410	820
41	235	253	280	303	331	365	390	411	430.5	861
42	248	267	295	320	349	384	409	431	451.5	903

TABLE A12 (Continued)

	$w_{0.005}$	$w_{0.01}$	$w_{0.025}$	$w_{0.05}$	$w_{0.10}$	$w_{0.20}$	$w_{0.30}$	$w_{0.40}$	$w_{0.50}$	$n(n+1)$
										2
43	263	282	311	337	366	403	429	452	473	946
44	277	297	328	354	385	422	450	473	495	990
45	292	313	344	372	403	442	471	495	517.5	1035
46	308	329	362	390	423	463	492	517	540.5	1081
47	324	346	379	408	442	484	514	540	564	1128
48	340	363	397	428	463	505	536	563	588	1176
49	357	381	416	447	483	527	559	587	612.5	1225
50	374	398	435	467	504	550	583	611	637.5	1275

For n larger than 50, the p th quantile w_p of the Wilcoxon signed ranks test statistic may be approximated by $w_p = [n(n+1)/4] + z_p \sqrt{n(n+1)(2n+1)/24}$, where z_p is the p th quantile of a standard normal random variable, obtained from Table A1.

SOURCE: Adapted from Harter and Owen (1970), with permission from the American Mathematical Society.

The entries in this table are quantiles w_p of the Wilcoxon signed ranks test statistic T^ , given by Equation 5.7.3, for selected values of $p \leq 0.50$. Quantiles w_p for $p > 0.50$ may be computed from the equation

$$w_p = n(n+1)/2 - w_{1-p}$$

where $n(n+1)/2$ is given in the right hand column in the table. Note that $P(T^* < w_p) \leq p$ and $P(T^* > w_p) \leq 1 - p$ if H_0 is true. Critical regions correspond to values of T^* less than (or greater than) but not including the appropriate quantile.

TABLE A13 Quantiles of the Kolmogorov Test Statistic^a

One-Sided Test		0.95	0.975	0.99	0.995	Two-Sided Test					
$p = 0.90$		0.90	0.95	0.98	0.99	$p = 0.80$		0.90	0.95	0.98	0.99
$n = 1$	0.900	0.950	0.975	0.990	0.995	$n = 21$	0.226	0.259	0.287	0.321	0.344
2	0.684	0.776	0.842	0.900	0.929	22	0.221	0.253	0.281	0.314	0.337
3	0.565	0.636	0.708	0.785	0.829	23	0.216	0.247	0.275	0.307	0.330
4	0.493	0.565	0.624	0.689	0.734	24	0.212	0.242	0.269	0.301	0.323
5	0.447	0.509	0.563	0.627	0.669	25	0.208	0.238	0.264	0.295	0.317
6	0.410	0.468	0.519	0.577	0.617	26	0.204	0.233	0.259	0.290	0.311
7	0.381	0.436	0.483	0.538	0.576	27	0.200	0.229	0.254	0.284	0.305
8	0.358	0.410	0.454	0.507	0.542	28	0.197	0.225	0.250	0.279	0.300
9	0.339	0.387	0.430	0.480	0.513	29	0.193	0.221	0.246	0.275	0.295
10	0.323	0.369	0.409	0.457	0.489	30	0.190	0.218	0.242	0.270	0.290
11	0.308	0.352	0.391	0.437	0.466	31	0.187	0.214	0.238	0.266	0.285
12	0.296	0.338	0.375	0.419	0.449	32	0.184	0.211	0.234	0.262	0.281
13	0.285	0.325	0.361	0.404	0.432	33	0.182	0.208	0.231	0.258	0.277
14	0.275	0.314	0.349	0.390	0.418	34	0.179	0.205	0.227	0.254	0.273
15	0.266	0.304	0.338	0.377	0.404	35	0.177	0.202	0.224	0.251	0.269
16	0.258	0.295	0.327	0.366	0.392	36	0.174	0.199	0.221	0.247	0.265
17	0.250	0.286	0.318	0.355	0.381	37	0.172	0.196	0.218	0.244	0.262
18	0.244	0.279	0.309	0.346	0.371	38	0.170	0.194	0.215	0.241	0.258
19	0.237	0.271	0.301	0.337	0.361	39	0.168	0.191	0.213	0.238	0.255
20	0.232	0.265	0.294	0.329	0.352	40	0.165	0.189	0.210	0.235	0.252
							Approximation for $n > 40$				
							$\frac{1.07}{\sqrt{n}}$	$\frac{1.22}{\sqrt{n}}$	$\frac{1.36}{\sqrt{n}}$	$\frac{1.52}{\sqrt{n}}$	$\frac{1.63}{\sqrt{n}}$

SOURCE: Adapted from Table 1 of Miller (1956). Used with permission of the American Statistical Association.

^aThe entries in this table are selected quantiles w_p of the Kolmogorov test statistics T , T^+ , and T^- as defined by Equation 6.1.1 for two-sided tests and by Equations 6.1.2 and 6.1.3 for one-sided tests. Reject H_0 at the level α if T exceeds the $1 - \alpha$ quantile given in this table. These quantiles are exact for $n \leq 40$ in the two-tailed test. The other quantiles are approximations that are equal to the exact quantiles in most cases. A better approximation for $n > 40$ results if $(n + \sqrt{n}/10)^{1/2}$ is used instead of \sqrt{n} in the denominator.

TABLE A19 Quantiles of the Smirnov Test Statistic for Two Samples of Equal Size n^a

One-Sided Test: $p = 0.90$						One-Sided Test: $p = 0.90$					
Two-Sided Test: $p = 0.80$						Two-Sided Test: $p = 0.80$					
n	0.95	0.975	0.99	0.995	0.995	n	0.90	0.95	0.975	0.99	0.995
3	2/3	2/3				22	7/22	8/22	8/22	10/22	10/22
4	3/4	3/4	3/4			23	7/23	8/23	9/23	10/23	10/23
5	3/5	3/5	4/5	4/5	4/5	24	7/24	8/24	9/24	10/24	11/24
6	3/6	4/6	4/6	5/6	5/6	25	7/25	8/25	9/25	10/25	11/25
7	4/7	4/7	5/7	5/7	5/7	26	7/26	8/26	9/26	10/26	11/26
8	4/8	4/8	5/8	5/8	6/8	27	7/27	8/27	9/27	11/27	11/27
9	4/9	5/9	5/9	6/9	6/9	28	8/28	9/28	10/28	11/28	12/28
10	4/10	5/10	6/10	6/10	7/10	29	8/29	9/29	10/29	11/29	12/29
11	5/11	5/11	6/11	7/11	7/11	30	8/30	9/30	10/30	11/30	12/30
12	5/12	5/12	6/12	7/12	7/12	31	8/31	9/31	10/31	11/31	12/31
13	5/13	6/13	6/13	7/13	8/13	32	8/32	9/32	10/32	12/32	12/32
14	5/14	6/14	7/14	7/14	8/14	33	8/33	9/33	11/33	12/33	13/33
15	5/15	6/15	7/15	8/15	8/15	34	8/34	10/34	11/34	12/34	13/34
16	6/16	6/16	7/16	8/16	9/16	35	8/35	10/35	11/35	12/35	13/35
17	6/17	7/17	7/17	8/17	9/17	36	9/36	10/36	11/36	12/36	13/36
18	6/18	7/18	8/18	9/18	9/18	37	9/37	10/37	11/37	13/37	13/37
19	6/19	7/19	8/19	9/19	9/19	38	9/38	10/38	11/38	13/38	14/38
20	6/20	7/20	8/20	9/20	10/20	39	9/39	10/39	11/39	13/39	14/39
21	6/21	7/21	8/21	9/21	10/21	40	9/40	10/40	12/40	13/40	14/40
Approximation for $n > 40$:							$\frac{1.52}{\sqrt{n}}$	$\frac{1.73}{\sqrt{n}}$	$\frac{1.92}{\sqrt{n}}$	$\frac{2.15}{\sqrt{n}}$	$\frac{2.30}{\sqrt{n}}$

SOURCE. Adapted from Birnbaum and Hall (1960), with permission from the Institute of Mathematical Statistics.

^aThe entries in this table are selected quantiles w_α of the Smirnov two-sample test statistic T defined by Equations 6.3.2 and 6.3.3 for the one-tailed test and defined by Equation 6.3.1 for the two-tailed test. Reject H_0 at the level α if T exceeds the $1 - \alpha$ quantile of T as given in this table. The test statistic is a discrete random variable, so the exact level of significance may be less than the apparent α used in this table.

TABLE A20 Quantiles of the Smirnov Test Statistic for Two Samples of Different Size n and m^a

One-Sided Test:		$p = 0.90$	0.95	0.975	0.99	0.995
Two-Sided Test:		$p = 0.80$	0.90	0.95	0.99	0.99
$N_1 = 1$	$N_2 = 9$	17/18				
	10	9/10				
$N_1 = 2$	$N_2 = 3$	5/6				
	4	3/4				
	5	4/5	4/5			
	6	5/6	5/6			
	7	5/7	6/7			
	8	3/4	7/8	7/8		
	9	7/9	8/9	8/9		
	10	7/10	4/5	4/5	9/10	
$N_1 = 3$	$N_2 = 4$	3/4	3/4			
	5	2/3	4/5	4/5		
	6	2/3	2/3	5/6		
	7	2/3	5/7	6/7	6/7	
	8	5/8	3/4	3/4	7/8	
	9	2/3	2/3	7/9	8/9	8/9
	10	3/5	7/10	4/5	9/10	9/10
	12	7/12	2/3	3/4	5/6	11/12
$N_1 = 4$	$N_2 = 5$	3/5	3/4	4/5	4/5	
	6	7/12	2/3	3/4	5/6	5/6
	7	17/28	5/7	3/4	6/7	6/7
	8	5/8	5/8	3/4	7/8	7/8
	9	5/9	2/3	3/4	7/9	8/9
	10	11/20	13/20	7/10	4/5	4/5
	12	7/12	2/3	2/3	3/4	5/6
	16	9/16	5/8	11/16	3/4	13/16
$N_1 = 5$	$N_2 = 6$	3/5	2/3	2/3	5/6	5/6
	7	4/7	23/35	5/7	29/35	6/7
	8	11/20	5/8	27/40	4/5	4/5
	9	5/9	3/5	31/45	7/9	4/5
	10	1/2	3/5	7/10	7/10	4/5
	15	8/15	3/5	2/3	11/15	11/15
	20	1/2	11/20	3/5	7/10	3/4
$N_1 = 6$	$N_2 = 7$	23/42	4/7	29/42	5/7	5/6
	8	1/2	7/12	2/3	3/4	3/4
	9	1/2	5/9	2/3	13/18	7/9
	10	1/2	17/30	19/30	7/10	11/15
	12	1/2	7/12	7/12	2/3	3/4
	18	4/9	5/9	11/18	2/3	13/18
	24	11/24	1/2	7/12	5/8	2/3

TABLE A20 (Continued)

One-Sided Test:		$p = 0.90$	0.95	0.975	0.99	0.995
Two-Sided Test:		$p = 0.80$	0.90	0.95	0.99	0.99
7	$N_2 = 8$	27/56	33/56	5/8	41/56	3/4
	9	31/63	5/9	40/63	5/7	47/63
	10	33/70	39/70	43/70	7/10	5/7
	14	3/7	1/2	4/7	9/14	5/7
	28	3/7	13/28	15/28	17/28	9/14
8	$N_2 = 9$	4/9	13/24	5/8	2/3	3/4
	10	19/40	21/40	23/40	27/40	7/10
	12	11/24	1/2	7/12	5/8	2/3
	16	7/16	1/2	9/16	5/8	5/8
	32	13/32	7/16	1/2	9/16	19/32
9	$N_2 = 10$	7/15	1/2	26/45	2/3	31/45
	12	4/9	1/2	5/9	11/18	2/3
	15	19/45	22/45	8/15	3/5	29/45
	18	7/18	4/9	1/2	5/9	11/18
	36	13/36	5/12	17/36	19/36	5/9
10	$N_2 = 15$	2/5	7/15	1/2	17/30	19/30
	20	2/5	9/20	1/2	11/20	3/5
	40	7/20	2/5	9/20	1/2	—
12	$N_2 = 15$	23/60	9/20	1/2	11/20	7/12
	16	3/8	7/16	23/48	13/24	7/12
	18	13/36	5/12	17/36	19/36	5/9
	20	11/30	5/12	7/15	31/60	17/30
15	$N_2 = 20$	7/20	2/5	13/30	29/60	31/60
16	$N_2 = 20$	27/80	31/80	17/40	19/40	41/80

Large sample approximation	$1.07\sqrt{\frac{m+n}{mn}}$	$1.22\sqrt{\frac{m+n}{mn}}$	$1.36\sqrt{\frac{m+n}{mn}}$	$1.52\sqrt{\frac{m+n}{mn}}$	$1.63\sqrt{\frac{m+n}{mn}}$
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NOTE. Adapted from Massey (1952), with permission from the Institute of Mathematical Statistics. Entries in this table are selected quantiles w_p of the Smirnov test statistic T for two samples, defined in equations 6.3.1, 6.3.2, and 6.3.3. To enter the table let N_1 be the smaller sample size and let N_2 be the larger sample size. Reject H_0 at the level α if T exceeds $w_{1-\alpha}$ as given in this table. If n and m are not covered by this table, use the large sample approximation given at the end of the table, or consult exact tables of n and Jennrich, which appear in Harter and Owen (1970) for $n, m \leq 100$.

TABLE A21 The t Distribution*

Degrees of Freedom	$p = 0.6$	0.75	0.9	0.95	0.975	0.99	0.995	0.9975	0.999	0.9995
1	0.325	1.000	3.078	6.314	12.706	31.821	63.657	127.32	318.31	636.62
2	0.289	0.816	1.886	2.920	4.303	6.965	9.925	14.089	22.327	31.598
3	0.277	0.765	1.638	2.353	3.182	4.541	5.841	7.453	10.214	12.924
4	0.271	0.741	1.533	2.132	2.776	3.747	4.604	5.598	7.173	8.610
5	0.267	0.727	1.476	2.015	2.571	3.365	4.032	4.773	5.893	6.869
6	0.265	0.718	1.440	1.943	2.447	3.143	3.707	4.317	5.208	5.959
7	0.263	0.711	1.415	1.895	2.365	2.998	3.499	4.029	4.785	5.408
8	0.262	0.706	1.397	1.860	2.306	2.896	3.355	3.833	4.501	5.041
9	0.261	0.703	1.383	1.833	2.262	2.821	3.250	3.690	4.297	4.781
10	0.260	0.700	1.372	1.812	2.228	2.764	3.169	3.581	4.144	4.587
11	0.260	0.697	1.363	1.796	2.201	2.718	3.106	3.497	4.025	4.437
12	0.259	0.695	1.356	1.782	2.179	2.681	3.055	3.428	3.930	4.318
13	0.259	0.694	1.350	1.771	2.160	2.650	3.012	3.372	3.852	4.221
14	0.258	0.692	1.345	1.761	2.145	2.624	2.977	3.326	3.787	4.140
15	0.258	0.691	1.341	1.753	2.131	2.602	2.947	3.286	3.733	4.073
16	0.258	0.690	1.337	1.746	2.120	2.583	2.921	3.252	3.686	4.015
17	0.257	0.689	1.333	1.740	2.110	2.567	2.898	3.222	3.646	3.965
18	0.257	0.688	1.330	1.734	2.101	2.552	2.878	3.197	3.610	3.922
19	0.257	0.688	1.328	1.729	2.093	2.539	2.861	3.174	3.579	3.883
20	0.257	0.687	1.325	1.725	2.086	2.528	2.845	3.153	3.552	3.850
21	0.257	0.686	1.323	1.721	2.080	2.518	2.831	3.135	3.527	3.819
22	0.256	0.686	1.321	1.717	2.074	2.508	2.819	3.119	3.505	3.792
23	0.256	0.685	1.319	1.714	2.069	2.500	2.807	3.104	3.485	3.767
24	0.256	0.685	1.318	1.711	2.064	2.492	2.797	3.091	3.467	3.745
25	0.256	0.684	1.316	1.708	2.060	2.485	2.787	3.078	3.450	3.725
26	0.256	0.684	1.315	1.706	2.056	2.479	2.779	3.067	3.435	3.707
27	0.256	0.684	1.314	1.703	2.052	2.473	2.771	3.057	3.421	3.690
28	0.256	0.683	1.313	1.701	2.048	2.467	2.763	3.047	3.408	3.674
29	0.256	0.683	1.311	1.699	2.045	2.462	2.756	3.038	3.396	3.659
30	0.256	0.683	1.310	1.697	2.042	2.457	2.750	3.030	3.385	3.646
40	0.255	0.681	1.303	1.684	2.021	2.423	2.704	2.971	3.307	3.551
60	0.254	0.679	1.296	1.671	2.000	2.390	2.660	2.915	3.232	3.460
120	0.254	0.677	1.289	1.658	1.980	2.358	2.617	2.860	3.160	3.373
∞	0.253	0.674	1.282	1.645	1.960	2.326	2.576	2.807	3.090	3.291

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* The entries in this table are quantiles w_p of the t distribution for various degrees of freedom. Quantiles w_p for $p < 0.5$ may be computed from the equation

$$w_p = -w_{1-p}$$

Note that $w_{0.50} = 0$ for all degrees of freedom.