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# UNIVERSITY OF SWAZILAND

FINAL EXAMINATIONS 2014

**TITLE OF PAPER** : QUANTITATIVE METHODS IN DEMOGRAPHY

**COURSE CODE** : DEM 206

**TIME ALLOWED** : 2 HOURS

**REQUIREMENTS** : STATISTICAL TABLES, CALCULATOR

**INSTRUCTIONS** : 1. THIS PAPER CONSISTS OF FIVE QUESTIONS

2. ANSWER ANY 4 (FOUR) QUESTIONS

*THIS EXAMINATION PAPER SHOULD NOT BE OPENED UNTILL PERMISSION HAS  
BEEN GRANTED BY THE INVIGILATOR*

### Question 1

(a) A random sample of size 64 is taken from a normal population with  $\mu = 51.4$  and  $\sigma = 6.8$ .

What is the probability that the mean of the sample will:

i) exceed 52.9 [2]

ii) fall between 50.5 and 52.3. [4]

(b) Differentiate between point estimation and interval estimation. Give any three properties of a good estimator. [5]

(c) The large sample  $(1-\alpha)100\%$  confidence interval for the proportion  $p$  is given by  $\hat{p} \pm z_{\alpha/2} \sqrt{\hat{p}\hat{q}/n}$ , show that the sample size formula when there is no knowledge about  $p$  is given by  $n = \frac{1}{4} \left[ \frac{z_{\alpha/2}}{E} \right]^2$ , where  $E$  is the error margin or maximum error. [5]

(d) A public health survey is to be designed to estimate the proportion  $p$  of a population having defective vision. How many persons should be examined if the public health commissioner wishes to 95% certain that the error of estimation is below 0.05 when;

(i) There is no knowledge about the value of  $p$ ? [2]

(ii)  $p$  is known to be 0.3? [2]

(e) A medical research worker intends to use the mean of a random sample of size  $n = 120$  to estimate the mean blood pressure of women in their fifties. If, based on experience, he knows  $\sigma = 10.5$  mm of mercury, what can he assert with a 99% confidence about the maximum error? [3]

(f) With reference to (d) above, suppose that the research worker takes his sample and gets  $\bar{x} = 141.8$  mm of mercury. Construct a 99% confidence interval for the mean blood pressure of women in their fifties. [2]

**Question 2**

- (a) Define type I and type II error. What is meant by the power of a test? [5]
- (b) Let  $X_1, X_2, \dots, X_5$  be a random sample of SAT mathematics scores, assumed to be  $N(\mu_X, \sigma^2)$ , and let  $Y_1, Y_2, \dots, Y_8$  be an independent random sample of SAT verbal scores, assumed to be  $N(\mu_Y, \sigma^2)$ . If the following data are observed, find a 90% confidence interval for  $\mu_X - \mu_Y$ ; [10]

x:	644	493	532	462	565				
<hr/>									
y:	623	472	492	661	540	502	549	518	

- (c) A study is undertaken to compare the rates of prevalence of CF antibody to parainfluenza I virus among boys and girls in the age group 5 to 9 years. Among 113 boys tested, 34 are found to have the antibody; among 139 girls tested, 54 have the antibody. Do the data provide strong evidence that the rate of prevalence of the antibody is significantly higher in girls than boys? Use  $\alpha = 0.05$ . [10]

**Question 3**

- (a) In what ways are the  $\chi^2$  and the  $F$  distributions similar? [5]
- (b) A researcher wishes to determine whether the salaries of professional nurses employed by private hospitals are higher than those of nurses employed by government owned hospitals. She selects a sample of nurses from each type of hospital and calculates the means and standard deviations of their salaries. Assume that the populations are approximately normally distributed;

Private	Government
$\bar{x}_1 = \$26800$	$\bar{x}_2 = \$25400$
$s_1 = \$600$	$s_2 = \$450$
$n_1 = 10$	$n_2 = 8$

- (i) Test whether the variances are equal. Use  $\alpha = 0.10$ . [10]
- (ii) Can she conclude that the private hospitals pay more than the government hospitals? Use  $\alpha = 0.01$ . [10]

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**Question 4**

(a) A company wishing to improve its customer service collected hold times from 75 randomly selected incoming calls to its hot line that were put on hold. These calls had sample mean hold time  $\bar{x} = 3.4$  minutes and  $s = 2.4$  minutes. Is the claim that  $\mu > 3.0$  minutes substantiated by these data? Using the P-value method, test with  $\alpha = 0.05$ . [10]

(b) A researcher claims that adult pigs fed a special diet will have an average weight of 200 pounds. A sample of 10 pigs has an average of 192.2 pounds and a standard deviation of 3.3 pounds. At  $\alpha = 0.05$ , can the claim be rejected? Use the confidence interval method. [10]

(c) A researcher conducts a one-independent sample  $Z$  test. The  $z$  statistic for the upper-tail critical test at a 5% level of significance was  $z = 1.84$ . What is the decision for this test? [5]

**Question 5**

(a) A researcher selected a sample of 150 students from each of three area high schools and asked each student, 'Do you drive to school in a car owned by either you or your parents?' The data are shown in the table. At  $\alpha = 0.05$ , test the claim that the proportion of students who drives their own or their parents cars is the same at all three schools. [12]

	School 1	School 2	School 3	Total
Yes	18	22	16	56
No	32	28	34	94
	50	50	50	150

(b) Let  $X$  be the number of defects in printed circuit boards. A random sample of  $n = 60$  printed circuit boards is taken and the number of defects recorded. The results were as follows:

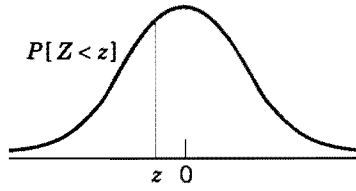
No. of defects	0	1	2	3
Observed Freq.	32	15	9	4

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Does the assumption of a Poisson distribution seem appropriate as a model for these data? Use  $\alpha = 0.05$ . [13]

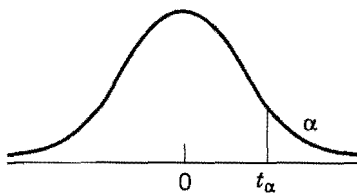
**END OF EXAMINATION**

TABLE 3 Standard Normal Probabilities



z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
-3.5	.0002	.0002	.0002	.0002	.0002	.0002	.0002	.0002	.0002	.0002
-3.4	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0002
-3.3	.0005	.0005	.0005	.0004	.0004	.0004	.0004	.0004	.0004	.0003
-3.2	.0007	.0007	.0006	.0006	.0006	.0006	.0006	.0005	.0005	.0005
-3.1	.0010	.0009	.0009	.0009	.0008	.0008	.0008	.0008	.0007	.0007
-3.0	.0013	.0013	.0013	.0012	.0012	.0011	.0011	.0011	.0010	.0010
-2.9	.0019	.0018	.0018	.0017	.0016	.0016	.0015	.0015	.0014	.0014
-2.8	.0026	.0025	.0024	.0023	.0023	.0022	.0021	.0021	.0020	.0019
-2.7	.0035	.0034	.0033	.0032	.0031	.0030	.0029	.0028	.0027	.0026
-2.6	.0047	.0045	.0044	.0043	.0041	.0040	.0039	.0038	.0037	.0036
-2.5	.0062	.0060	.0059	.0057	.0055	.0054	.0052	.0051	.0049	.0048
-2.4	.0082	.0080	.0078	.0075	.0073	.0071	.0069	.0068	.0066	.0064
-2.3	.0107	.0104	.0102	.0099	.0096	.0094	.0091	.0089	.0087	.0084
-2.2	.0139	.0136	.0132	.0129	.0125	.0122	.0119	.0116	.0113	.0110
-2.1	.0179	.0174	.0170	.0166	.0162	.0158	.0154	.0150	.0146	.0143
-2.0	.0228	.0222	.0217	.0212	.0207	.0202	.0197	.0192	.0188	.0183
-1.9	.0287	.0281	.0274	.0268	.0262	.0256	.0250	.0244	.0239	.0233
-1.8	.0359	.0351	.0344	.0336	.0329	.0322	.0314	.0307	.0301	.0294
-1.7	.0446	.0436	.0427	.0418	.0409	.0401	.0392	.0384	.0375	.0367
-1.6	.0548	.0537	.0526	.0516	.0505	.0495	.0485	.0475	.0465	.0455
-1.5	.0668	.0655	.0643	.0630	.0618	.0606	.0594	.0582	.0571	.0559
-1.4	.0808	.0793	.0778	.0764	.0749	.0735	.0721	.0708	.0694	.0681
-1.3	.0968	.0951	.0934	.0918	.0901	.0885	.0869	.0853	.0838	.0823
-1.2	.1151	.1131	.1112	.1093	.1075	.1056	.1038	.1020	.1003	.0985
-1.1	.1357	.1335	.1314	.1292	.1271	.1251	.1230	.1210	.1190	.1170
-1.0	.1587	.1562	.1539	.1515	.1492	.1469	.1446	.1423	.1401	.1379
-0.9	.1841	.1814	.1788	.1762	.1736	.1711	.1685	.1660	.1635	.1611
-0.8	.2119	.2090	.2061	.2033	.2005	.1977	.1949	.1922	.1894	.1867
-0.7	.2420	.2389	.2358	.2327	.2297	.2266	.2236	.2206	.2177	.2148
-0.6	.2743	.2709	.2676	.2643	.2611	.2578	.2546	.2514	.2483	.2451
-0.5	.3085	.3050	.3015	.2981	.2946	.2912	.2877	.2843	.2810	.2776
-0.4	.3446	.3409	.3372	.3336	.3300	.3264	.3228	.3192	.3156	.3121
-0.3	.3821	.3783	.3745	.3707	.3669	.3632	.3594	.3557	.3520	.3483
-0.2	.4207	.4168	.4129	.4090	.4052	.4013	.3974	.3936	.3897	.3859
-0.1	.4602	.4562	.4522	.4483	.4443	.4404	.4364	.4325	.4286	.4247
-0.0	.5000	.4960	.4920	.4880	.4840	.4801	.4761	.4721	.4681	.4641

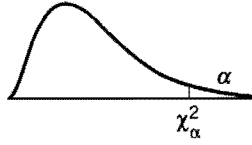


TABLE 4 Percentage Points of  $t$  Distributions

d.f. \ $\alpha$	.25	.10	.05	.025	.01	.00833	.00625	.005
1	1.000	3.078	6.314	12.706	31.821	38.204	50.923	63.657
2	.816	1.886	2.920	4.303	6.965	7.649	8.860	9.925
3	.765	1.638	2.353	3.182	4.541	4.857	5.392	5.841
4	.741	1.533	2.132	2.776	3.747	3.961	4.315	4.604
5	.727	1.476	2.015	2.571	3.365	3.534	3.810	4.032
6	.718	1.440	1.943	2.447	3.143	3.287	3.521	3.707
7	.711	1.415	1.895	2.365	2.998	3.128	3.335	3.499
8	.706	1.397	1.860	2.306	2.896	3.016	3.206	3.355
9	.703	1.383	1.833	2.262	2.821	2.933	3.111	3.250
10	.700	1.372	1.812	2.228	2.764	2.870	3.038	3.169
11	.697	1.363	1.796	2.201	2.718	2.820	2.981	3.106
12	.695	1.356	1.782	2.179	2.681	2.779	2.934	3.055
13	.694	1.350	1.771	2.160	2.650	2.746	2.896	3.012
14	.692	1.345	1.761	2.145	2.624	2.718	2.864	2.977
15	.691	1.341	1.753	2.131	2.602	2.694	2.837	2.947
16	.690	1.337	1.746	2.120	2.583	2.673	2.813	2.921
17	.689	1.333	1.740	2.110	2.567	2.655	2.793	2.898
18	.688	1.330	1.734	2.101	2.552	2.639	2.775	2.878
19	.688	1.328	1.729	2.093	2.539	2.625	2.759	2.861
20	.687	1.325	1.725	2.086	2.528	2.613	2.744	2.845
21	.686	1.323	1.721	2.080	2.518	2.601	2.732	2.831
22	.686	1.321	1.717	2.074	2.508	2.591	2.720	2.819
23	.685	1.319	1.714	2.069	2.500	2.582	2.710	2.807
24	.685	1.318	1.711	2.064	2.492	2.574	2.700	2.797
25	.684	1.316	1.708	2.060	2.485	2.566	2.692	2.787
26	.684	1.315	1.706	2.056	2.479	2.559	2.684	2.779
27	.684	1.314	1.703	2.052	2.473	2.552	2.676	2.771
28	.683	1.313	1.701	2.048	2.467	2.546	2.669	2.763
29	.683	1.311	1.699	2.045	2.462	2.541	2.663	2.756
30	.683	1.310	1.697	2.042	2.457	2.536	2.657	2.750
40	.681	1.303	1.684	2.021	2.423	2.499	2.616	2.704
60	.679	1.296	1.671	2.000	2.390	2.463	2.575	2.660
120	.677	1.289	1.658	1.980	2.358	2.428	2.536	2.617
$\infty$	.674	1.282	1.645	1.960	2.326	2.394	2.498	2.576



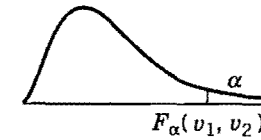
TABLE 5 Percentage Points of  $\chi^2$  Distributions



d.f. \ $\alpha$	.99	.975	.95	.90	.50	.10	.05	.025	.01
1	.0002	.001	.004	.02	.45	2.71	3.84	5.02	6.63
2	.02	.05	.10	.21	1.39	4.61	5.99	7.38	9.21
3	.11	.22	.35	.58	2.37	6.25	7.81	9.35	11.34
4	.30	.48	.71	1.06	3.36	7.78	9.49	11.14	13.28
5	.55	.83	1.15	1.61	4.35	9.24	11.07	12.83	15.09
6	.87	1.24	1.64	2.20	5.35	10.64	12.59	14.45	16.81
7	1.24	1.69	2.17	2.83	6.35	12.02	14.07	16.01	18.48
8	1.65	2.18	2.73	3.49	7.34	13.36	15.51	17.53	20.09
9	2.09	2.70	3.33	4.17	8.34	14.68	16.92	19.02	21.67
10	2.56	3.24	3.94	4.87	9.34	15.99	18.31	20.48	23.21
11	3.05	3.81	4.57	5.58	10.34	17.28	19.68	21.92	24.72
12	3.57	4.40	5.23	6.30	11.34	18.55	21.03	23.34	26.22
13	4.11	5.01	5.89	7.04	12.34	19.81	22.36	24.74	27.69
14	4.66	5.62	6.57	7.79	13.34	21.06	23.68	26.12	29.14
15	5.23	6.26	7.26	8.55	14.34	22.31	25.00	27.49	30.58
16	5.81	6.90	7.96	9.31	15.34	23.54	26.30	28.85	32.00
17	6.41	7.56	8.67	10.09	16.34	24.77	27.59	30.19	33.41
18	7.01	8.23	9.39	10.86	17.34	25.99	28.87	31.53	34.81
19	7.63	8.90	10.12	11.65	18.34	27.20	30.14	32.85	36.19
20	8.26	9.59	10.85	12.44	19.34	28.41	31.41	34.17	37.57
21	8.90	10.28	11.59	13.24	20.34	29.62	32.67	35.48	38.93
22	9.54	10.98	12.34	14.04	21.34	30.81	33.92	36.78	40.29
23	10.20	11.69	13.09	14.85	22.34	32.01	35.17	38.08	41.64
24	10.86	12.40	13.85	15.66	23.34	33.20	36.42	39.36	42.98
25	11.52	13.11	14.61	16.47	24.34	34.38	37.65	40.65	44.31
26	12.20	13.84	15.38	17.29	25.34	35.56	38.89	41.92	45.64
27	12.88	14.57	16.15	18.11	26.34	36.74	40.11	43.19	46.96
28	13.56	15.30	16.93	18.94	27.34	37.92	41.34	44.46	48.28
29	14.26	16.04	17.71	19.77	28.34	39.09	42.56	45.72	49.59
30	14.95	16.78	18.49	20.60	29.34	40.26	43.77	46.98	50.89
40	22.16	24.42	26.51	29.05	39.34	51.81	55.76	59.34	63.69
50	29.71	32.35	34.76	37.69	49.33	63.17	67.50	71.42	76.15
60	37.48	40.47	43.19	46.46	59.33	74.40	79.08	83.30	88.38
70	45.44	48.75	51.74	55.33	69.33	85.53	90.53	95.02	100.43
80	53.54	57.15	60.39	64.28	79.33	96.58	101.88	106.63	112.33
90	61.75	65.64	69.13	73.29	89.33	107.57	113.15	118.14	124.12
100	70.06	74.22	77.93	82.36	99.33	118.50	124.34	129.56	135.81

TABLE 6 Percentage Points of  $F(v_1, v_2)$  Distributions

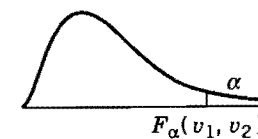
$\alpha = .10$



$v_1 \backslash v_2$	1	2	3	4	5	6	7	8	9	10	12	15	20	25	30	40	60
1	39.86	49.50	53.59	55.83	57.24	58.20	58.91	59.44	59.86	60.19	60.71	61.22	61.74	62.05	62.26	62.53	62.79
2	8.53	9.00	9.16	9.24	9.29	9.33	9.35	9.37	9.38	9.39	9.41	9.42	9.44	9.45	9.46	9.47	9.47
3	5.54	5.46	5.39	5.34	5.31	5.28	5.27	5.25	5.24	5.23	5.22	5.20	5.18	5.17	5.17	5.16	5.15
4	4.54	4.32	4.19	4.11	4.05	4.01	3.98	3.95	3.94	3.92	3.90	3.87	3.84	3.83	3.82	3.80	3.79
5	4.06	3.78	3.62	3.52	3.45	3.40	3.37	3.34	3.32	3.30	3.27	3.24	3.21	3.19	3.17	3.16	3.14
6	3.78	3.46	3.29	3.18	3.11	3.05	3.01	2.98	2.96	2.94	2.90	2.87	2.84	2.81	2.80	2.78	2.76
7	3.59	3.26	3.07	2.96	2.88	2.83	2.78	2.75	2.72	2.70	2.67	2.63	2.59	2.57	2.56	2.54	2.51
8	3.46	3.11	2.92	2.81	2.73	2.67	2.62	2.59	2.56	2.54	2.50	2.46	2.42	2.40	2.38	2.36	2.34
9	3.36	3.01	2.81	2.69	2.61	2.55	2.51	2.47	2.44	2.42	2.38	2.34	2.30	2.27	2.25	2.23	2.21
10	3.29	2.92	2.73	2.61	2.52	2.46	2.41	2.38	2.35	2.32	2.28	2.24	2.20	2.17	2.16	2.13	2.11
11	3.23	2.86	2.66	2.54	2.45	2.39	2.34	2.30	2.27	2.25	2.21	2.17	2.12	2.10	2.08	2.05	2.03
12	3.18	2.81	2.61	2.48	2.39	2.33	2.28	2.24	2.21	2.19	2.15	2.10	2.06	2.03	2.01	1.99	1.96
13	3.14	2.76	2.56	2.43	2.35	2.28	2.23	2.20	2.16	2.14	2.10	2.05	2.01	1.98	1.96	1.93	1.90
14	3.10	2.73	2.52	2.39	2.31	2.24	2.19	2.15	2.12	2.10	2.05	2.01	1.96	1.93	1.91	1.89	1.86
15	3.07	2.70	2.49	2.36	2.27	2.21	2.16	2.12	2.09	2.06	2.02	1.97	1.92	1.89	1.87	1.85	1.82
16	3.05	2.67	2.46	2.33	2.24	2.18	2.13	2.09	2.06	2.03	1.99	1.94	1.89	1.86	1.84	1.81	1.78
17	3.03	2.64	2.44	2.31	2.22	2.15	2.10	2.06	2.03	2.00	1.96	1.91	1.86	1.83	1.81	1.78	1.75
18	3.01	2.62	2.42	2.29	2.20	2.13	2.08	2.04	2.00	1.98	1.93	1.89	1.84	1.80	1.78	1.75	1.72
19	2.99	2.61	2.40	2.27	2.18	2.11	2.06	2.02	1.98	1.96	1.91	1.86	1.81	1.78	1.76	1.73	1.70
20	2.97	2.59	2.38	2.25	2.16	2.09	2.04	2.00	1.96	1.94	1.89	1.84	1.79	1.76	1.74	1.71	1.68
21	2.96	2.57	2.36	2.23	2.14	2.08	2.02	1.98	1.95	1.92	1.87	1.83	1.78	1.74	1.72	1.69	1.66
22	2.95	2.56	2.35	2.22	2.13	2.06	2.01	1.97	1.93	1.90	1.86	1.81	1.76	1.73	1.70	1.67	1.64
23	2.94	2.55	2.34	2.21	2.11	2.05	1.99	1.95	1.92	1.89	1.84	1.80	1.74	1.71	1.69	1.66	1.62
24	2.93	2.54	2.33	2.19	2.10	2.04	1.98	1.94	1.91	1.88	1.83	1.78	1.73	1.70	1.67	1.64	1.61
25	2.92	2.53	2.32	2.18	2.09	2.02	1.97	1.93	1.89	1.87	1.82	1.77	1.72	1.68	1.66	1.63	1.59
26	2.91	2.52	2.31	2.17	2.08	2.01	1.96	1.92	1.88	1.86	1.81	1.76	1.71	1.67	1.65	1.61	1.58
27	2.90	2.51	2.30	2.17	2.07	2.00	1.95	1.91	1.87	1.85	1.80	1.75	1.70	1.66	1.64	1.60	1.57
28	2.89	2.50	2.29	2.16	2.06	2.00	1.94	1.90	1.87	1.84	1.79	1.74	1.69	1.65	1.63	1.59	1.56
29	2.89	2.50	2.28	2.15	2.06	1.99	1.93	1.89	1.86	1.83	1.78	1.73	1.68	1.64	1.62	1.58	1.55
30	2.88	2.49	2.28	2.14	2.05	1.98	1.93	1.88	1.85	1.82	1.77	1.72	1.67	1.63	1.61	1.57	1.54
40	2.84	2.44	2.23	2.09	2.00	1.93	1.87	1.83	1.79	1.76	1.71	1.66	1.61	1.57	1.54	1.51	1.47
60	2.79	2.39	2.18	2.04	1.95	1.87	1.82	1.77	1.74	1.71	1.66	1.60	1.54	1.50	1.48	1.44	1.40
120	2.75	2.35	2.13	1.99	1.90	1.82	1.77	1.72	1.68	1.65	1.60	1.55	1.48	1.45	1.41	1.37	1.32
$\infty$	2.71	2.30	2.08	1.94	1.85	1.77	1.72	1.67	1.63	1.60	1.55	1.49	1.42	1.38	1.34	1.30	1.24

TABLE 6 (Continued)

$\alpha = .05$



$v_2 \backslash v_1$	1	2	3	4	5	6	7	8	9	10	12	15	20	25	30	40	60
1	161.5	199.5	215.7	224.6	230.2	234.0	236.8	238.9	240.5	241.9	243.9	246.0	248.0	249.3	250.1	251.1	252.2
2	18.51	19.00	19.16	19.25	19.30	19.33	19.35	19.37	19.38	19.40	19.41	19.43	19.45	19.46	19.46	19.47	19.48
3	10.13	9.55	9.28	9.12	9.01	8.94	8.89	8.85	8.81	8.79	8.74	8.70	8.66	8.63	8.62	8.59	8.57
4	7.71	6.94	6.59	6.39	6.26	6.16	6.09	6.04	6.00	5.96	5.91	5.86	5.80	5.77	5.75	5.72	5.69
5	6.61	5.79	5.41	5.19	5.05	4.95	4.88	4.82	4.77	4.74	4.68	4.62	4.56	4.52	4.50	4.46	4.43
6	5.99	5.14	4.76	4.53	4.39	4.28	4.21	4.15	4.10	4.06	4.00	3.94	3.87	3.83	3.81	3.77	3.74
7	5.59	4.74	4.35	4.12	3.97	3.87	3.79	3.73	3.68	3.64	3.57	3.51	3.44	3.40	3.38	3.34	3.30
8	5.32	4.46	4.07	3.84	3.69	3.58	3.50	3.44	3.39	3.35	3.28	3.22	3.15	3.11	3.08	3.04	3.01
9	5.12	4.26	3.86	3.63	3.48	3.37	3.29	3.23	3.18	3.14	3.07	3.01	2.94	2.89	2.86	2.83	2.79
10	4.96	4.10	3.71	3.48	3.33	3.22	3.14	3.07	3.02	2.98	2.91	2.85	2.77	2.73	2.70	2.66	2.62
11	4.84	3.98	3.59	3.36	3.20	3.09	3.01	2.95	2.90	2.85	2.79	2.72	2.65	2.60	2.57	2.53	2.49
12	4.75	3.89	3.49	3.26	3.11	3.00	2.91	2.85	2.80	2.75	2.69	2.62	2.54	2.50	2.47	2.43	2.38
13	4.67	3.81	3.41	3.18	3.03	2.92	2.83	2.77	2.71	2.67	2.60	2.53	2.46	2.41	2.38	2.34	2.30
14	4.60	3.74	3.34	3.11	2.96	2.85	2.76	2.70	2.65	2.60	2.53	2.46	2.39	2.34	2.31	2.27	2.22
15	4.54	3.68	3.29	3.06	2.90	2.79	2.71	2.64	2.59	2.54	2.48	2.40	2.33	2.28	2.25	2.20	2.16
16	4.49	3.63	3.24	3.01	2.85	2.74	2.66	2.59	2.54	2.49	2.42	2.35	2.28	2.23	2.19	2.15	2.11
17	4.45	3.59	3.20	2.96	2.81	2.70	2.61	2.55	2.49	2.45	2.38	2.31	2.23	2.18	2.15	2.10	2.06
18	4.41	3.55	3.16	2.93	2.77	2.66	2.58	2.51	2.46	2.41	2.34	2.27	2.19	2.14	2.11	2.06	2.02
19	4.38	3.52	3.13	2.90	2.74	2.63	2.54	2.48	2.42	2.38	2.31	2.23	2.16	2.11	2.07	2.03	1.98
20	4.35	3.49	3.10	2.87	2.71	2.60	2.51	2.45	2.39	2.35	2.28	2.20	2.12	2.07	2.04	1.99	1.95
21	4.32	3.47	3.07	2.84	2.68	2.57	2.49	2.42	2.37	2.32	2.25	2.18	2.10	2.05	2.01	1.96	1.92
22	4.30	3.44	3.05	2.82	2.66	2.55	2.46	2.40	2.34	2.30	2.23	2.15	2.07	2.02	1.98	1.94	1.89
23	4.28	3.42	3.03	2.80	2.64	2.53	2.44	2.37	2.32	2.27	2.20	2.13	2.05	2.00	1.96	1.91	1.86
24	4.26	3.40	3.01	2.78	2.62	2.51	2.42	2.36	2.30	2.25	2.18	2.11	2.03	1.97	1.94	1.89	1.84
25	4.24	3.39	2.99	2.76	2.60	2.49	2.40	2.34	2.28	2.24	2.16	2.09	2.01	1.96	1.92	1.87	1.82
26	4.23	3.37	2.98	2.74	2.59	2.47	2.39	2.32	2.27	2.22	2.15	2.07	1.99	1.94	1.90	1.85	1.80
27	4.21	3.35	2.96	2.73	2.57	2.46	2.37	2.31	2.25	2.20	2.13	2.06	1.97	1.92	1.88	1.84	1.79
28	4.20	3.34	2.95	2.71	2.56	2.45	2.36	2.29	2.24	2.19	2.12	2.04	1.96	1.91	1.87	1.82	1.77
29	4.18	3.33	2.93	2.70	2.55	2.43	2.35	2.28	2.22	2.18	2.10	2.03	1.94	1.89	1.85	1.81	1.75
30	4.17	3.32	2.92	2.69	2.53	2.42	2.33	2.27	2.21	2.16	2.09	2.01	1.93	1.88	1.84	1.79	1.74
40	4.08	3.23	2.84	2.61	2.45	2.34	2.25	2.18	2.12	2.08	2.00	1.92	1.84	1.78	1.74	1.69	1.64
60	4.00	3.15	2.76	2.53	2.37	2.25	2.17	2.10	2.04	1.99	1.92	1.84	1.75	1.69	1.65	1.59	1.53
120	3.92	3.07	2.68	2.45	2.29	2.18	2.09	2.02	1.96	1.91	1.83	1.75	1.66	1.60	1.55	1.50	1.43
$\infty$	3.84	3.00	2.61	2.37	2.21	2.10	2.01	1.94	1.88	1.83	1.75	1.67	1.57	1.51	1.46	1.39	1.32