UNIVERSITY OF SWAZILAND

DEPARTMENT OF STATISTICS AND DEMOGRAPHY

SUPPLEMENTARY EXAMINATION, 2013/14

COURSE TITLE: MATHEMATICS FOR STATISTICS

COURSE CODE: ST 202

TIME ALLOWED: THREE (3) HOURS

INSTRUCTION: ANSWER ANY THREE QUESTIONS

SPECIAL REQUIREMENTS: SCIENTIFIC CALCULATORS AND STATISTICAL TABLES

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Question 1

If x is a **Gamma-distributed** random variable (with $\alpha = 2$ and $\beta = 1$), its probability density function is given by

$$f(x) = xe^{-x}, \text{ for } x \geq 0$$

(a) Determine all critical values and critical points of $f'(x)$ and indicate the direction of the slope around each extreme point.

(10 marks)

(b) Determine all critical values and critical points of $f''(x)$ and indicate the concavity around the point(s).

(5 marks)

(c) Now, sketch the graph of the density function. **(5 marks)**

Question 2

(a) Given that $f(x,y) = xye^{xy}$, find

(i) f_y **(3 marks)**

(ii) f_{yx} **(3 marks)**

(b) Find the derivative of the functions:

(i) $y = e^{-3x} + 5$,

(ii) $y = \ln\dfrac{5x}{x+2}$,

(iii) $y = x^{\ln x}$ **(6 marks)**

(c) Find the second derivative of the function

$f(x) = x \ln\sqrt{x} + 2x$ **(3 marks)**

(d) Find the first partial derivatives of $f(x,y) = xe^{xy}$, and evaluate it at the point (1, ln2) **(2 marks)**

(e) Find the slope of the curve $x^2 + xy + y^2 = 2$ at the point (1, 2) **(3 marks)**

Question 3

(a) Let x be a discrete random variable whose probability mass function is given by

$$f(x) = \frac{1}{11}(x^2 - x = 1), \text{ for } x = 1, 2, 3.$$

(i) Find the mean and variance **(3 + 4 marks)**

(ii) Find P(X < μ) **(3 marks)**

(b) Let X and Y have the joint density given by

$$f(x, y) = \begin{cases} \frac{1}{2}(3x + y), \textit{for} & 0 < x < 1 \& 0 < y < 1 \\ 0, & \textit{elsewhere} \end{cases}$$

(i) Find μ_y and σ_y **(5 +5 marks)**

Question 4

(a) Write the following system of equations in matrix form and use determinants to solve for x, y and z.

2x + 3y + z = 10
4x - y -2z = 8
5x +2y – 3z = 6 **(7 marks)**

(b) Solve the following system of equations using the Gauss-Jordan elimination method:

x + y + 2z = 9
2x + 4y – 3z = 1
3x + 6y – 5z = 0 **(7 marks)**

(c) Suppose x = $\begin{bmatrix} 2 \\ 0 \\ -4 \end{bmatrix}$ and y = $\begin{bmatrix} 0 \\ -1 \\ -3 \end{bmatrix}$

Use vector algebra to find the least squares regression line through the set of points determined by vectors x and y. **(6 marks)**

Question 5

(a) Find eigenvalues and eigenvectors of matrix $A = \begin{bmatrix} 4 & 0 & 1 \\ -2 & 1 & 0 \\ -2 & 0 & 1 \end{bmatrix}$

(10 marks)

(b) Find the adjoint of the following matrix A and use the adjoint to find the inverse of this matrix.

$$A = \begin{bmatrix} 3 & 2 & -1 \\ 1 & 6 & 3 \\ 2 & -4 & 0 \end{bmatrix}$$

(10 marks)

END OF EXAM!!