

UNIVERSITY OF SWAZILAND



MAIN EXAMINATION PAPER 2014

**TITLE OF PAPER** : STATISTICAL INFERENCE II

**COURSE CODE** : ST 303

**TIME ALLOWED** : TWO (2) HOURS

**REQUIREMENTS** : CALCULATOR AND STATISTICAL TABLES

**INSTRUCTIONS** : ANSWER ANY THREE QUESTIONS

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INVIGILATOR**

### Question 1

Let  $X = (X_1, \dots, X_n)$  be a random sample from a Gamma distribution with shape parameters  $\alpha$  and  $\beta$  known.

The p.d.f. of  $X_i$  is

$$f(x_i|\alpha, \beta) = \frac{\beta^\alpha}{\Gamma(\alpha)} \cdot x_i^{\alpha-1} \cdot \exp(-\beta x_i), \quad x \geq 0, \alpha > 0, \beta > 0$$

- (a) Determine the log likelihood function. (5 Marks)
- (b) Show that  $\sum_{i=1}^n \ln(X_i)$ , is a sufficient statistic, and  $\bar{X}$  is not. (15 Marks)

### Question 2

The times in seconds  $X_1, X_2, \dots, X_n$  between messages arriving at a node in a telecommunications system constitute a random sample from an exponential distribution with probability density function  $f(x) = \mu^{-1} \exp\left(-\frac{x}{\mu}\right)$  ( $x > 0$ ), where  $\mu$  ( $> 0$ ) is an unknown parameter.

- a) Show that  $\hat{\mu}_n = \frac{1}{n} \sum_{i=1}^n X_i$ , is an unbiased estimator of  $\mu$  and find its variance. (5 Marks)
- b) Is  $\hat{\mu}_n$  a consistent estimator of  $\mu$ ? Justify your answer. (2 Marks)
- c) Show that  $E(\sqrt{X_i}) = \frac{1}{2} \sqrt{\pi \mu}$  and hence show that  $\tilde{\mu} = \frac{4}{\pi} \sqrt{X_1 \cdot X_2}$  is an unbiased estimator of  $\mu$ . (6 Marks)
- d) Find the relative efficiency of  $\tilde{\mu}$  compared to  $\hat{\mu}_2$ . (4 Marks)

### Question 3

Let  $X$  be  $N(\mu, 8100)$ . To test  $H_0: \mu = 530$  against the alternative hypothesis  $H_1: \mu < 530$ .

Given a random sample of size  $n = 36$ , let the critical region be defined by

$C = \{ \bar{X} : \bar{X} \leq 510.77 \}$ , where  $\bar{X}$  is the observed random sample mean.

Find the p-value for this test.

(10 marks)

If  $H_1: \mu = 509$ , find the probabilities of making the Type II error and of correctly rejecting  $H_0$ .

(15 marks)

#### Question 4

Let  $X_1, \dots, X_n \sim N(\theta, 9)$ . Thus,  $E(X_i) = \theta$ , and  $\text{Var}(X_i) = 9$ .

- a) Find the maximum likelihood estimator  $\hat{\theta}$ , (5 Marks)
- b) Find the score function and the Fisher information. (10 Marks)
- c) Find the minimum variance bound for unbiased estimator of  $\theta$ . (5 Marks)

#### Question 5

a) Suppose that  $X_1, X_2, \dots, X_n$  constitute a random sample from a distribution with probability density  $f(x|\theta)$ , where  $\theta$  is a real parameter with prior density  $\pi(\theta)$ . The loss when  $\theta$  is estimated by  $\hat{\theta}$  (a function of  $X_1, X_2, \dots, X_n$ ) is  $\ell(\hat{\theta}; \theta)$ .

- i. Define the Bayes risk of  $\hat{\theta}$  and the Bayes estimator of  $\theta$ . (4 Marks)
- ii. Show that the Bayes estimator can be found by minimising the posterior expected loss for given  $(x_1, x_2, \dots, x_n)$ . (6 Marks)

b) Let  $X_1, \dots, X_n \sim N(\theta, 1)$ . Let  $\pi$  be a  $N(0, 1)$  prior:

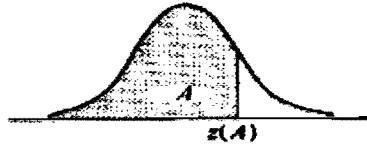
$$\pi(\theta) = \frac{1}{\sqrt{2\pi}} \cdot \exp\left(-\frac{\theta^2}{2}\right)$$

- i. Find the posterior distribution for  $\theta$ . (4 Marks)
- ii. Find the posterior mean  $\bar{\theta}$ . (6 Marks)

## Normal Distribution

Table C-1. Cumulative Probabilities of the Standard Normal Distribution.

Entry is area  $A$  under the standard normal curve from  $-\infty$  to  $z(A)$

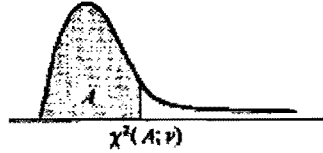


$z$	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998

## Chi-Square Distribution

Table C-2. Percentiles of the  $\chi^2$  Distribution

Entry is  $\chi^2(A; \nu)$  where  $P\{\chi^2(\nu) \leq \chi^2(A; \nu)\} = A$



$\nu$	$A$									
	.005	.010	.025	.050	.100	.900	.950	.975	.990	.995
1	0.004393	0.00157	0.00982	0.02393	0.0158	2.71	3.84	5.02	6.63	7.88
2	0.0100	0.0201	0.0506	0.103	0.211	4.61	5.99	7.38	9.21	10.60
3	0.072	0.115	0.216	0.352	0.584	6.25	7.81	9.35	11.34	12.84
4	0.207	0.297	0.484	0.711	1.064	7.78	9.49	11.14	13.28	14.86
5	0.412	0.554	0.831	1.145	1.61	9.24	11.07	12.83	15.09	16.75
6	0.676	0.872	1.24	1.64	2.20	10.64	12.59	14.45	16.81	18.55
7	0.989	1.24	1.69	2.17	2.83	12.02	14.07	16.01	18.48	20.28
8	1.34	1.65	2.18	2.73	3.49	13.36	15.51	17.53	20.09	21.96
9	1.73	2.09	2.70	3.33	4.17	14.68	16.92	19.02	21.67	23.59
10	2.16	2.56	3.25	3.94	4.87	15.99	18.31	20.48	23.21	25.19
11	2.60	3.05	3.82	4.57	5.58	17.28	19.68	21.92	24.73	26.76
12	3.07	3.57	4.40	5.23	6.30	18.55	21.03	23.34	26.22	28.30
13	3.57	4.11	5.01	5.89	7.04	19.81	22.36	24.74	27.69	29.82
14	4.07	4.66	5.63	6.57	7.79	21.06	23.68	26.12	29.14	31.32
15	4.60	5.23	6.26	7.26	8.55	22.31	25.00	27.49	30.58	32.80
16	5.14	5.81	6.91	7.96	9.31	23.54	26.30	28.85	32.00	34.27
17	5.70	6.41	7.56	8.67	10.09	24.77	27.59	30.19	33.41	35.72
18	6.26	7.01	8.23	9.39	10.86	25.99	28.87	31.53	34.81	37.16
19	6.84	7.63	8.91	10.12	11.65	27.20	30.14	32.85	36.19	38.58
20	7.43	8.26	9.59	10.85	12.44	28.41	31.41	34.17	37.57	40.00
21	8.03	8.90	10.28	11.59	13.24	29.62	32.67	35.48	38.93	41.40
22	8.64	9.54	10.98	12.34	14.04	30.81	33.92	36.78	40.29	42.80
23	9.26	10.20	11.69	13.09	14.85	32.01	35.17	38.08	41.64	44.18
24	9.89	10.86	12.40	13.85	15.66	33.20	36.42	39.36	42.98	45.56
25	10.52	11.52	13.12	14.61	16.47	34.38	37.65	40.65	44.31	46.93
26	11.16	12.20	13.84	15.38	17.29	35.56	38.89	41.92	45.64	48.29
27	11.81	12.88	14.57	16.15	18.11	36.74	40.11	43.19	46.96	49.64
28	12.46	13.56	15.31	16.93	18.94	37.92	41.34	44.46	48.28	50.99
29	13.12	14.26	16.05	17.71	19.77	39.09	42.56	45.72	49.59	52.34
30	13.79	14.95	16.79	18.49	20.60	40.26	43.77	46.98	50.89	53.67
40	20.71	22.16	24.43	26.51	29.05	51.81	55.76	59.34	63.69	66.77
50	27.99	29.71	32.36	34.76	37.69	63.17	67.50	71.42	76.15	79.49
60	35.53	37.48	40.48	43.19	46.46	74.40	79.08	83.30	88.38	91.95
70	43.28	45.44	48.76	51.74	55.33	85.53	90.53	95.02	100.4	104.2
80	51.17	53.54	57.15	60.39	64.28	96.58	101.9	106.6	112.3	116.3
90	59.20	61.75	65.65	69.13	73.29	107.6	113.1	118.1	124.1	128.3
100	67.33	70.06	74.22	77.93	82.36	118.5	124.3	129.6	135.8	140.2

## Student's Distribution (t Distribution)

Table C-4 Percentiles of the t Distribution

Entry is  $t(A; \nu)$  where  $P\{t(\nu) \leq t(A; \nu)\} = A$



$\nu$	A						
	.60	.70	.80	.85	.90	.95	.975
1	0.325	0.727	1.376	1.963	3.078	6.314	12.706
2	0.289	0.617	1.061	1.386	1.886	2.920	4.303
3	0.277	0.584	0.978	1.250	1.638	2.353	3.182
4	0.271	0.569	0.941	1.190	1.533	2.132	2.776
5	0.267	0.559	0.920	1.156	1.476	2.015	2.571
6	0.265	0.553	0.906	1.134	1.440	1.943	2.447
7	0.263	0.549	0.896	1.119	1.415	1.895	2.365
8	0.262	0.546	0.889	1.108	1.397	1.860	2.306
9	0.261	0.543	0.883	1.100	1.383	1.833	2.262
10	0.260	0.542	0.879	1.093	1.372	1.812	2.228
11	0.260	0.540	0.876	1.088	1.363	1.796	2.201
12	0.259	0.539	0.873	1.083	1.356	1.782	2.179
13	0.259	0.537	0.870	1.079	1.350	1.771	2.160
14	0.258	0.537	0.868	1.076	1.345	1.761	2.145
15	0.258	0.536	0.866	1.074	1.341	1.753	2.131
16	0.258	0.535	0.865	1.071	1.337	1.746	2.120
17	0.257	0.534	0.863	1.069	1.333	1.740	2.110
18	0.257	0.534	0.862	1.067	1.330	1.734	2.101
19	0.257	0.533	0.861	1.066	1.328	1.729	2.093
20	0.257	0.533	0.860	1.064	1.325	1.725	2.086
21	0.257	0.532	0.859	1.063	1.323	1.721	2.080
22	0.256	0.532	0.858	1.061	1.321	1.717	2.074
23	0.256	0.532	0.858	1.060	1.319	1.714	2.069
24	0.256	0.531	0.857	1.059	1.318	1.711	2.064
25	0.256	0.531	0.856	1.058	1.316	1.708	2.060
26	0.256	0.531	0.856	1.058	1.315	1.706	2.056
27	0.256	0.531	0.855	1.057	1.314	1.703	2.052
28	0.256	0.530	0.855	1.056	1.313	1.701	2.048
29	0.256	0.530	0.854	1.055	1.311	1.699	2.045
30	0.256	0.530	0.854	1.055	1.310	1.697	2.042
40	0.255	0.529	0.851	1.050	1.303	1.684	2.021
60	0.254	0.527	0.848	1.045	1.296	1.671	2.000
120	0.254	0.526	0.845	1.041	1.289	1.658	1.980
$\infty$	0.253	0.524	0.842	1.036	1.282	1.645	1.960

Table C-4 (Continued) Percentiles of the *t* Distribution

<i>v</i>	A						
	.98	.985	.99	.9925	.995	.9975	.9995
1	15.895	21.205	31.821	42.434	63.657	127.322	636.590
2	4.849	5.643	6.965	8.073	9.925	14.089	31.598
3	3.482	3.896	4.541	5.047	5.841	7.453	12.924
4	2.999	3.298	3.747	4.088	4.604	5.598	8.610
5	2.757	3.003	3.365	3.634	4.032	4.773	6.869
6	2.612	2.829	3.143	3.372	3.707	4.317	5.959
7	2.517	2.715	2.998	3.203	3.499	4.029	5.408
8	2.449	2.634	2.896	3.085	3.355	3.833	5.041
9	2.398	2.574	2.821	2.998	3.250	3.690	4.781
10	2.359	2.527	2.764	2.932	3.169	3.581	4.587
11	2.328	2.491	2.718	2.879	3.106	3.497	4.437
12	2.303	2.461	2.681	2.836	3.055	3.428	4.318
13	2.282	2.436	2.650	2.801	3.012	3.372	4.221
14	2.264	2.415	2.624	2.771	2.977	3.326	4.140
15	2.249	2.397	2.602	2.746	2.947	3.286	4.073
16	2.235	2.382	2.583	2.724	2.921	3.252	4.015
17	2.224	2.368	2.567	2.706	2.898	3.222	3.965
18	2.214	2.356	2.552	2.689	2.878	3.197	3.922
19	2.205	2.346	2.539	2.674	2.861	3.174	3.883
20	2.197	2.336	2.528	2.661	2.845	3.153	3.849
21	2.189	2.328	2.518	2.649	2.831	3.135	3.819
22	2.183	2.320	2.508	2.639	2.819	3.119	3.792
23	2.177	2.313	2.500	2.629	2.807	3.104	3.768
24	2.172	2.307	2.492	2.620	2.797	3.091	3.745
25	2.167	2.301	2.485	2.612	2.787	3.078	3.725
26	2.162	2.296	2.479	2.605	2.779	3.067	3.707
27	2.158	2.291	2.473	2.598	2.771	3.057	3.690
28	2.154	2.286	2.467	2.592	2.763	3.047	3.674
29	2.150	2.282	2.462	2.586	2.756	3.038	3.659
30	2.147	2.278	2.457	2.581	2.750	3.030	3.646
40	2.123	2.250	2.423	2.542	2.704	2.971	3.551
60	2.099	2.223	2.390	2.504	2.660	2.915	3.460
120	2.076	2.196	2.358	2.468	2.617	2.860	3.373
∞	2.054	2.170	2.326	2.432	2.576	2.807	3.291

Table C-5 (Continued) Percentiles of the F Distribution

Den. df	A	Numerator df									
		1	2	3	4	5	6	7	8	9	
1	.50	1.00	1.50	1.71	1.82	1.89	1.94	1.98	2.00	2.03	
	.90	39.9	48.5	53.6	55.8	57.2	58.2	58.9	59.4	59.9	
	.95	161	200	236	225	230	234	237	239	241	
	.975	648	800	864	900	922	937	948	957	963	
	.99	4,052	5,000	5,403	5,625	5,764	5,859	5,928	5,981	6,022	
	.995	16,211	20,000	21,615	22,500	23,056	23,437	23,715	23,925	24,091	
	.999	405,280	500,000	540,380	562,500	576,400	585,940	592,870	598,140	602,280	
	2	.50	0.667	1.00	1.13	1.21	1.25	1.28	1.30	1.32	1.33
		.90	8.53	9.00	9.16	9.24	9.29	9.33	9.35	9.37	9.38
		.95	18.5	19.0	19.2	19.2	19.3	19.3	19.4	19.4	19.4
.975		38.5	39.0	39.2	39.2	39.3	39.3	39.4	39.4	39.4	
.99		88.5	89.0	89.2	89.2	89.3	89.3	89.4	89.4	89.4	
.995		199	199	199	199	199	199	199	199	199	
.999		998.5	999.0	999.2	999.2	999.3	999.3	999.4	999.4	999.4	
3		.50	0.585	0.881	1.00	1.06	1.10	1.13	1.15	1.16	1.17
		.90	5.54	5.46	5.39	5.34	5.31	5.28	5.27	5.25	5.24
		.95	10.1	9.55	9.28	9.12	9.01	8.94	8.89	8.85	8.81
	.975	17.4	16.0	15.4	15.1	14.9	14.7	14.6	14.5	14.5	
	.99	34.1	30.8	29.5	28.7	28.2	27.9	27.7	27.5	27.3	
	.995	35.6	49.8	47.5	46.2	45.4	44.8	44.4	44.1	43.9	
	.999	167.0	148.5	141.1	137.1	134.6	132.8	131.6	130.6	129.9	
	4	.50	0.549	0.828	0.941	1.00	1.04	1.06	1.08	1.09	1.10
		.90	4.54	4.32	4.19	4.11	4.05	4.01	3.98	3.95	3.94
		.95	7.71	6.94	6.59	6.39	6.26	6.16	6.09	6.04	6.00
.975		12.2	10.6	9.98	9.60	9.36	9.20	9.07	8.98	8.90	
.99		21.2	18.0	16.7	16.0	15.5	15.2	15.0	14.8	14.7	
.995		31.3	26.3	24.3	23.2	22.5	22.0	21.6	21.4	21.1	
.999		74.1	61.2	56.2	53.4	51.7	50.5	49.7	49.0	48.5	
5		.50	0.528	0.799	0.907	0.965	1.00	1.02	1.04	1.05	1.06
		.90	4.06	3.78	3.62	3.52	3.45	3.40	3.37	3.34	3.32
		.95	6.61	5.79	5.41	5.19	5.05	4.95	4.88	4.82	4.77
	.975	10.0	8.43	7.76	7.39	7.15	6.98	6.85	6.76	6.68	
	.99	16.3	13.3	12.1	11.4	11.0	10.7	10.5	10.3	10.2	
	.995	22.8	18.3	16.5	15.6	14.9	14.5	14.2	14.0	13.8	
	.999	47.2	37.1	33.2	31.1	29.8	28.8	28.2	27.6	27.2	
	6	.50	0.515	0.780	0.886	0.942	0.977	1.00	1.02	1.03	1.04
		.90	3.78	3.46	3.29	3.18	3.11	3.05	3.01	2.98	2.96
		.95	5.99	5.14	4.76	4.53	4.39	4.28	4.21	4.15	4.10
.975		8.81	7.26	6.60	6.23	5.99	5.82	5.70	5.60	5.52	
.99		13.7	10.9	9.78	9.15	8.75	8.47	8.26	8.10	7.98	
.995		18.6	14.5	12.9	12.0	11.3	11.1	10.8	10.6	10.4	
.999		35.5	27.0	23.7	21.9	20.8	20.0	19.5	19.0	18.7	
7		.50	0.506	0.767	0.871	0.926	0.960	0.983	1.00	1.01	1.02
		.90	3.59	3.26	3.07	2.96	2.88	2.83	2.78	2.75	2.72
		.95	5.59	4.74	4.35	4.12	3.97	3.87	3.79	3.73	3.68
	.975	8.07	6.54	5.89	5.52	5.29	5.12	4.99	4.90	4.82	
	.99	12.2	9.55	8.45	7.85	7.46	7.19	6.99	6.84	6.72	
	.995	16.2	12.4	10.9	10.1	9.52	9.16	8.89	8.68	8.51	
	.999	29.2	21.7	18.8	17.2	16.2	15.5	15.0	14.6	14.3	

Table C-5 (Continued) Percentiles of the F Distribution

Den. df	A	Numerator df									
		10	12	15	20	24	30	60	120	∞	
1	.50	2.04	2.07	2.09	2.12	2.13	2.15	2.17	2.18	2.20	
	.90	60.2	60.7	61.2	61.7	62.0	62.3	62.8	63.1	63.3	
	.95	242	244	246	248	249	250	252	253	254	
	.975	969	977	985	993	997	1,001	1,010	1,014	1,018	
	.99	6,056	6,106	6,157	6,209	6,235	6,261	6,313	6,339	6,366	
	.995	24,224	24,426	24,630	24,836	24,940	25,044	25,253	25,359	25,464	
	.999	605,620	610,670	615,760	620,910	623,500	626,100	631,340	633,970	636,620	
	2	.50	1.34	1.36	1.38	1.39	1.40	1.41	1.43	1.43	1.44
		.90	9.39	9.41	9.42	9.44	9.45	9.46	9.47	9.48	9.49
		.95	19.4	19.4	19.4	19.4	19.5	19.5	19.5	19.5	19.5
.975		39.4	39.4	39.4	39.4	39.5	39.5	39.5	39.5	39.5	
.99		99.4	99.4	99.4	99.4	99.5	99.5	99.5	99.5	99.5	
.995		199	199	199	199	199	199	199	199	200	
.999		999.4	999.4	999.4	999.4	999.5	999.5	999.5	999.5	999.5	
3		.50	1.18	1.20	1.21	1.23	1.23	1.24	1.25	1.26	1.27
		.90	5.23	5.22	5.20	5.18	5.18	5.17	5.15	5.14	5.13
		.95	8.79	8.74	8.70	8.66	8.64	8.62	8.57	8.53	8.53
	.975	14.4	14.3	14.3	14.2	14.1	14.1	14.0	13.9	13.9	
	.99	27.2	27.1	26.9	26.7	26.6	26.5	26.3	26.2	26.1	
	.995	43.7	43.4	43.1	42.8	42.6	42.5	42.1	42.0	41.8	
	.999	129.2	128.3	127.4	126.4	125.9	125.4	124.3	124.0	123.5	
	4	.50	1.11	1.13	1.14	1.15	1.16	1.16	1.18	1.18	1.19
		.90	3.92	3.90	3.87	3.84	3.83	3.82	3.79	3.78	3.76
		.95	5.96	5.91	5.86	5.80	5.77	5.75	5.69	5.66	5.63
.975		8.84	8.75	8.66	8.56	8.51	8.46	8.36	8.31	8.26	
.99		14.5	14.4	14.2	14.0	13.9	13.8	13.7	13.6	13.5	
.995		21.0	20.7	20.4	20.2	20.0	19.9	19.6	19.5	19.3	
.999		48.1	47.4	46.8	46.1	45.8	45.4	44.7	44.4	44.1	
5		.50	1.07	1.09	1.10	1.11	1.12	1.12	1.14	1.14	1.15
		.90	3.30	3.27	3.24	3.21	3.19	3.17	3.14	3.12	3.11
		.95	4.74	4.68	4.62	4.56	4.53	4.50	4.43	4.40	4.37
	.975	6.62	6.52	6.43	6.33	6.28	6.23	6.12	6.07	6.02	
	.99	10.1	9.89	9.72	9.55	9.47	9.38	9.20	9.11	9.02	
	.995	13.6	13.4	13.1	12.9	12.8	12.7	12.4	12.3	12.1	
	.999	26.9	26.4	25.9	25.4	25.1	24.9	24.3	24.1	23.8	
	6	.50	1.05	1.06	1.07	1.08	1.09	1.10	1.11	1.12	1.12
		.90	2.94	2.90	2.87	2.84	2.82	2.80	2.76	2.74	2.72
		.95	4.06	4.00	3.94	3.87	3.84	3.81	3.74	3.70	3.67
.975		5.46	5.37	5.27	5.17	5.12	5.07	4.96	4.90	4.85	
.99		7.87	7.72	7.56	7.40	7.31	7.23	7.06	6.97	6.88	
.995		10.2	10.0	9.81	9.59	9.47	9.36	9.12	9.00	8.88	
.999		18.4	18.0	17.6	17.1	16.9	16.7	16.2	16.0	15.7	
7		.50	1.03	1.04	1.05	1.07	1.07	1.08	1.09	1.10	1.10
		.90	2.70	2.67	2.63	2.59	2.58	2.56	2.51	2.49	2.47
		.95	3.64	3.57	3.51	3.44	3.41	3.38	3.30	3.27	3.23
	.975	4.76	4.67	4.57	4.47	4.42	4.36	4.25	4.20	4.14	
	.99	6.62	6.47	6.31	6.16	6.07	5.99	5.82	5.74	5.65	
	.995	8.38	8.18	7.97	7.75	7.65	7.53	7.31	7.19	7.08	
	.999	14.1	13.7	13.3	12.9	12.7	12.5	12.1	11.9	11.7	



Table C-5 (Continued) Percentiles of the F Distribution

Den. df	4	Numerator df								
		1	2	3	4	5	6	7	8	9
8	.50	0.499	0.757	0.860	0.915	0.948	0.971	0.988	1.00	1.01
	.90	3.46	3.13	2.92	2.81	2.73	2.67	2.62	2.59	2.56
	.95	5.32	4.46	4.07	3.84	3.69	3.58	3.50	3.44	3.39
	.975	7.57	6.06	5.42	5.05	4.82	4.65	4.53	4.43	4.36
	.99	11.3	8.65	7.59	7.01	6.63	6.37	6.18	6.03	5.91
	.995	14.7	11.0	9.60	8.81	8.30	7.95	7.69	7.50	7.34
.999	25.4	18.5	15.8	14.4	13.3	12.9	12.4	12.0	11.8	
9	.50	0.484	0.749	0.852	0.906	0.939	0.962	0.978	0.990	1.00
	.90	3.36	3.01	2.81	2.69	2.61	2.55	2.51	2.47	2.44
	.95	5.12	4.26	3.86	3.63	3.48	3.37	3.29	3.23	3.18
	.975	7.21	5.71	5.08	4.72	4.48	4.32	4.20	4.10	4.03
	.99	10.6	8.02	6.99	6.42	6.04	5.80	5.61	5.47	5.35
	.995	13.6	10.4	8.72	7.96	7.47	7.13	6.88	6.69	6.54
.999	22.9	16.4	13.9	12.6	11.7	11.1	10.7	10.4	10.3	
10	.50	0.490	0.743	0.845	0.899	0.932	0.954	0.971	0.983	0.992
	.90	3.29	2.92	2.73	2.61	2.52	2.46	2.41	2.38	2.35
	.95	4.96	4.10	3.71	3.48	3.33	3.22	3.14	3.07	3.02
	.975	6.94	5.46	4.83	4.47	4.24	4.07	3.95	3.85	3.78
	.99	10.0	7.56	6.55	5.99	5.64	5.39	5.20	5.06	4.94
	.995	12.8	9.43	8.08	7.34	6.87	6.54	6.30	6.12	5.97
.999	21.0	14.9	12.6	11.3	10.5	9.93	9.52	9.20	8.96	
12	.50	0.484	0.735	0.835	0.888	0.921	0.943	0.959	0.972	0.981
	.90	3.18	2.81	2.61	2.48	2.39	2.33	2.28	2.24	2.21
	.95	4.75	3.89	3.49	3.26	3.11	3.00	2.91	2.85	2.80
	.975	6.55	5.10	4.47	4.12	3.89	3.73	3.61	3.51	3.44
	.99	9.33	6.93	5.93	5.41	5.06	4.82	4.64	4.50	4.39
	.995	11.8	8.51	7.23	6.52	6.07	5.76	5.52	5.35	5.20
.999	18.6	13.0	10.8	9.63	8.89	8.38	8.00	7.71	7.48	
15	.50	0.478	0.726	0.826	0.878	0.911	0.933	0.949	0.960	0.970
	.90	3.07	2.70	2.49	2.36	2.27	2.21	2.16	2.12	2.09
	.95	4.54	3.68	3.29	3.06	2.90	2.79	2.71	2.64	2.59
	.975	6.20	4.77	4.15	3.80	3.58	3.41	3.29	3.20	3.12
	.99	8.68	6.36	5.42	4.89	4.56	4.32	4.14	4.00	3.89
	.995	10.8	7.70	6.48	5.90	5.37	5.07	4.85	4.67	4.54
.999	16.6	11.3	9.34	8.25	7.57	7.09	6.74	6.47	6.26	
20	.50	0.472	0.718	0.816	0.868	0.900	0.922	0.938	0.950	0.959
	.90	2.97	2.59	2.38	2.25	2.16	2.09	2.04	2.00	1.96
	.95	4.35	3.49	3.10	2.87	2.71	2.60	2.51	2.45	2.39
	.975	5.87	4.46	3.86	3.51	3.29	3.13	3.01	2.91	2.84
	.99	8.10	5.83	4.84	4.43	4.10	3.87	3.70	3.56	3.46
	.995	9.94	6.99	5.82	5.17	4.76	4.47	4.26	4.09	3.96
.999	14.8	9.95	8.10	7.10	6.46	6.02	5.69	5.44	5.24	
24	.50	0.469	0.714	0.812	0.863	0.895	0.917	0.932	0.944	0.953
	.90	2.93	2.54	2.33	2.19	2.10	2.04	1.98	1.94	1.91
	.95	4.26	3.40	3.01	2.78	2.62	2.51	2.42	2.36	2.30
	.975	5.72	4.32	3.72	3.38	3.15	2.99	2.87	2.78	2.70
	.99	7.62	5.61	4.72	4.22	3.90	3.67	3.50	3.36	3.26
	.995	9.55	6.66	5.52	4.89	4.49	4.20	3.99	3.83	3.69
.999	14.0	9.34	7.53	6.59	5.98	5.53	5.23	4.99	4.80	

Table C-5 (Continued) Percentiles of the F Distribution

Den. df	4	Numerator df								
		10	12	15	20	24	30	40	120	$\infty$
8	.50	1.02	1.03	1.04	1.05	1.06	1.07	1.08	1.08	1.09
	.90	2.54	2.50	2.46	2.42	2.40	2.38	2.34	2.32	2.29
	.95	3.35	3.28	3.22	3.15	3.12	3.08	3.01	2.97	2.93
	.975	4.30	4.20	4.10	4.00	3.95	3.89	3.78	3.73	3.67
	.99	5.81	5.67	5.52	5.36	5.28	5.20	5.03	4.95	4.86
	.995	7.21	7.01	6.81	6.61	6.50	6.40	6.18	6.06	5.95
.999	11.5	11.2	10.8	10.5	10.3	10.1	9.73	9.53	9.33	
9	.50	1.01	1.02	1.03	1.04	1.05	1.05	1.07	1.07	1.08
	.90	2.42	2.38	2.34	2.30	2.28	2.25	2.21	2.18	2.16
	.95	3.14	3.07	3.01	2.94	2.90	2.86	2.79	2.75	2.71
	.975	3.96	3.87	3.77	3.67	3.61	3.56	3.45	3.39	3.33
	.99	5.26	5.11	4.96	4.81	4.73	4.65	4.48	4.40	4.31
	.995	6.42	6.23	6.03	5.83	5.73	5.62	5.41	5.30	5.19
.999	9.89	9.57	9.24	8.90	8.72	8.55	8.19	8.00	7.81	
10	.50	1.00	1.01	1.02	1.03	1.04	1.05	1.06	1.06	1.07
	.90	2.32	2.28	2.24	2.20	2.18	2.16	2.11	2.08	2.06
	.95	2.98	2.91	2.84	2.77	2.74	2.70	2.62	2.58	2.54
	.975	3.72	3.62	3.52	3.42	3.37	3.31	3.20	3.14	3.08
	.99	4.85	4.71	4.56	4.41	4.33	4.25	4.08	4.00	3.91
	.995	5.85	5.66	5.47	5.27	5.17	5.07	4.86	4.75	4.64
.999	8.75	8.45	8.13	7.80	7.64	7.47	7.12	6.94	6.76	
12	.50	0.989	1.00	1.01	1.02	1.03	1.03	1.05	1.05	1.06
	.90	2.19	2.15	2.10	2.06	2.04	2.01	1.96	1.93	1.90
	.95	2.75	2.69	2.62	2.54	2.51	2.47	2.38	2.34	2.30
	.975	3.37	3.28	3.18	3.07	3.02	2.96	2.83	2.79	2.72
	.99	4.30	4.16	4.01	3.86	3.78	3.70	3.54	3.45	3.36
	.995	5.09	4.91	4.72	4.53	4.43	4.33	4.12	4.01	3.90
.999	7.79	7.00	6.71	6.40	6.25	6.09	5.76	5.59	5.42	
15	.50	0.977	0.989	1.00	1.01	1.02	1.02	1.03	1.04	1.05
	.90	2.06	2.02	1.97	1.92	1.90	1.87	1.82	1.79	1.76
	.95	2.54	2.48	2.40	2.33	2.29	2.25	2.16	2.11	2.07
	.975	3.06	2.96	2.86	2.76	2.70	2.64	2.52	2.46	2.40
	.99	3.80	3.67	3.52	3.37	3.29	3.21	3.05	2.96	2.89
	.995	4.42	4.25	4.07	3.88	3.79	3.69	3.48	3.37	3.26
.999	6.08	5.81	5.54	5.25	5.10	4.95	4.64	4.48	4.31	
20	.50	0.966	0.977	0.989	1.00	1.01	1.01	1.02	1.03	1.03
	.90	1.94	1.89	1.84	1.79	1.77	1.74	1.68	1.64	1.61
	.95	2.35	2.28	2.20	2.12	2.08	2.04	1.95	1.90	1.84
	.975	2.77	2.68	2.57	2.46	2.41	2.35	2.22	2.16	2.09
	.99	3.37	3.23	3.09	2.94	2.86	2.78	2.61	2.52	2.42
	.995	3.85	3.68	3.50	3.32	3.22	3.12	2.92	2.81	2.69
.999	5.08	4.82	4.56	4.29	4.15	4.00	3.70	3.54	3.38	
24	.50	0.961	0.972	0.983	0.994	1.00	1.01	1.02	1.02	1.03
	.90	1.88	1.83	1.78	1.73	1.70	1.67	1.61	1.57	1.53
	.95	2.25	2.18	2.11	2.03	1.98	1.94	1.84	1.79	1.73
	.975	2.64	2.54	2.44	2.33	2.27	2.21	2.08	2.01	1.94
	.99	3.17	3.03	2.89	2.74	2.66	2.58	2.40	2.31	2.21
	.995	3.59	3.42	3.25	3.06	2.97	2.87	2.66	2.55	2.43
.999	4.64	4.39	4.14	3.87	3.74	3.59	3.29	3.14	2.97	

Table C-5 (Continued) Percentiles of the F Distribution

Den. df A	Numerator df								
	1	2	3	4	5	6	7	8	9
30 .50	0.466	0.709	0.807	0.858	0.890	0.912	0.927	0.939	0.948
.90	2.88	2.49	2.28	2.14	2.05	1.98	1.93	1.88	1.85
.95	4.17	3.32	2.92	2.69	2.53	2.42	2.33	2.27	2.21
.975	5.57	4.18	3.59	3.25	3.03	2.87	2.75	2.65	2.57
.99	7.36	5.39	4.51	4.02	3.70	3.47	3.30	3.17	3.07
.995	9.18	6.35	5.24	4.62	4.23	3.95	3.74	3.58	3.45
.999	13.3	8.77	7.05	6.12	5.53	5.12	4.82	4.58	4.39
60 .50	0.461	0.701	0.798	0.849	0.880	0.901	0.917	0.928	0.937
.90	2.79	2.39	2.18	2.04	1.95	1.87	1.82	1.77	1.74
.95	4.00	3.15	2.76	2.53	2.37	2.25	2.17	2.10	2.04
.975	5.29	3.93	3.34	3.01	2.79	2.63	2.51	2.41	2.33
.99	7.08	4.98	4.13	3.65	3.34	3.12	2.95	2.82	2.72
.995	8.49	5.80	4.73	4.14	3.76	3.49	3.29	3.13	3.01
.999	12.0	7.77	6.17	5.31	4.76	4.37	4.09	3.86	3.69
120 .50	0.458	0.697	0.793	0.844	0.875	0.896	0.912	0.923	0.932
.90	2.75	2.35	2.13	1.99	1.90	1.82	1.77	1.72	1.68
.95	3.92	3.07	2.68	2.45	2.29	2.18	2.09	2.02	1.96
.975	5.15	3.80	3.23	2.89	2.67	2.52	2.39	2.30	2.22
.99	6.85	4.79	3.95	3.48	3.17	2.96	2.79	2.66	2.56
.995	8.18	5.54	4.50	3.92	3.55	3.28	3.09	2.93	2.81
.999	11.4	7.32	5.78	4.95	4.42	4.04	3.77	3.55	3.38
∞ .50	0.455	0.693	0.789	0.839	0.870	0.891	0.907	0.918	0.927
.90	2.71	2.30	2.08	1.94	1.85	1.77	1.72	1.67	1.63
.95	3.84	3.00	2.60	2.37	2.21	2.10	2.01	1.94	1.88
.975	5.02	3.69	3.12	2.79	2.57	2.41	2.29	2.19	2.11
.99	6.63	4.61	3.78	3.32	3.02	2.80	2.64	2.51	2.41
.995	7.88	5.30	4.28	3.72	3.35	3.09	2.90	2.74	2.62
.999	10.8	6.91	5.42	4.62	4.10	3.74	3.47	3.27	3.10