

**UNIVERSITY OF SWAZILAND**

**FINAL EXAMINATION PAPER 2014**

**TITLE OF PAPER : INTRODUCTION TO REGRESSION ANALYSIS**

**COURSE CODE : ST304**

**TIME ALLOWED : 2 (TWO) HOURS**

**REQUIRMENTS : STATISTICAL TABLES AND CALCULATOR**

**INSTRUCTIONS : ANSWER ANY 4 (FOUR) QUESTIONS.  
ALL QUESTIONS CARRY EQUAL MARKS.**

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BEEN GRANTED BY THE INVIGILATOR**

**QUESTION ONE.**

[ 6 + 3 + 2 + 3 + 5 + 6 marks ]

- 1.1. Express the General Linear Regression Model with  $p$  predictor variables, including all assumptions, in matrix terms.
- 1.2. The following results were computed from a multiple regression model with  $n = 12$ :

$$(XX)^{-1} = \begin{bmatrix} 4.174 & -0.175 & -0.069 \\ & 0.009 & 0.002 \\ & & 0.001 \end{bmatrix}, XY = \begin{bmatrix} 184 \\ 2426 \\ 4865 \end{bmatrix}, \text{ and } MSE = 9.171$$

- Find the estimated regression function.
- Estimate the variance-covariance matrix.
- Construct a 90% confidence interval for  $\beta_0$ .
- Construct a 95% joint confidence interval for  $\beta_1$  and  $\beta_2$ .
- Test the hypothesis,  $\beta_1 = 0$  against  $\beta_1 < 0$ .

**QUESTION TWO.**

[ 3 + 3 + 4 + 6 + 9 marks ]

- What is Multicollinearity?
- Discuss the effects of perfectly uncorrelated predictor variables on a response variable in a first-order regression model.
- State the important problems arise when multicollinearity is present in the model.
- What is Variance Inflation Factor (VIF)? Explain how you can use it to detect multicollinearity?
- Discuss six remedial measures for serious multicollinearity present in the regression model.

**QUESTION THREE.**

[ 8 + 8 + 4 + 5 marks ]

- What is Heteroscedasticity? What happens to OLS estimator in the presence of heteroscedasticity?
- What is Generalized Least Squares (GLS) method? Explain how the GLS method works to obtain estimates of regression coefficients when heteroscedasticity is present.
- Explain the Goldfeld-Quandt test procedure how it helps to detect Heteroscedasticity
- Carry out the Spearman's rank correlation test to detect heteroscedasticity at the 5% level of significance if  $\sum d_i^2 = 114$ ; where  $d_i$  is the difference in the rank assigned to the predictor variable and the absolute value of the residuals in a sample of 10 units.

**QUESTION FOUR.**

[ 4 + 4 + 6 + 4 + 7 marks ]

- 4.1 Define Autocorrelation. State the causes of positively autocorrelated error terms in economic data.
- 4.2 State the Generalized Multiple Regression model when the random error terms follow a first order autoregressive process.
- 4.3 The least-squares regression equation  $\hat{Y}_t = 98.1 + 5.0X_t$  has been fitted to the following data. Use Durbin-Watson test statistic to test the existence of positive autocorrelation at  $\alpha = 0.01$  and also consider  $\sum (e_t - e_{t-1})^2 = 309.26$ .

| $X_t$ | $Y_t$ | $X_t$ | $Y_t$ | $X_t$ | $Y_t$ |
|-------|-------|-------|-------|-------|-------|
| 1     | 97.0  | 6     | 125.5 | 11    | 150.6 |
| 2     | 109.7 | 7     | 132.9 | 12    | 157.8 |
| 3     | 110.3 | 8     | 139.7 | 13    | 158.3 |
| 4     | 121.5 | 9     | 145.6 | 14    | 166.9 |
| 5     | 130.4 | 10    | 149.4 | 15    | 175.8 |

- 4.4 Given a sample of 40 observations and 5 predictor variables with 5% level of significance, what can you say about autocorrelation (positive or negative) if
  - (i)  $D = 1.05$ ?
  - (ii)  $D = 1.60$ ?
  - (iii)  $D = 2.10$ ?
  - (iv)  $D = 3.50$ ?

[You must show all your work regarding your conclusions]

- 4.5 To eliminate the problem of autocorrelated errors, we use transformed variables. There are three methods for this purpose. State and compare those methods and discuss how to select any specific method.

**QUESTION FIVE.**

[ 2 + 3 + 4 + 4 + 4 + 4 + 4 marks ]

The following tables are the partial SPSS output of a multiple regression problem running with same y and with predictor variables mentioned in the respective table:

| Model |            | Unstandardized Coefficients |            | Standardized Coefficients | t      | Sig. |
|-------|------------|-----------------------------|------------|---------------------------|--------|------|
|       |            | B                           | Std. Error | Beta                      |        |      |
| 1     | (Constant) | 60.106                      | 26.596     |                           | 2.260  | .050 |
|       | x1         | 4.946                       | 2.678      | .636                      | 1.847  | .098 |
|       | x2         | 2.358                       | .889       | .753                      | 2.651  | .026 |
|       | x3         | -8.230                      | 1.993      | -1.082                    | -4.129 | .003 |
|       | x4         | .095                        | 1.558      | .030                      | .061   | .953 |
|       | x5         | -8.671                      | 3.252      | -.803                     | -2.667 | .026 |

a. Dependent Variable: y

## ANOVA

| Model |            | Sum of Squares | df | Mean Square | F     | Sig.              |
|-------|------------|----------------|----|-------------|-------|-------------------|
| 1     | Regression | 1813.882       | 5  | 362.776     | 8.310 | .003 <sup>a</sup> |
|       | Residual   | 392.912        | 9  | 43.657      |       |                   |
|       | Total      | 2206.795       | 14 |             |       |                   |

a. Predictors: (Constant), x5, x2, x1, x3, x4

## ANOVA

| Model |            | Sum of Squares | df | Mean Square | F     | Sig.              |
|-------|------------|----------------|----|-------------|-------|-------------------|
| 1     | Regression | 1503.445       | 4  | 375.861     | 5.344 | .014 <sup>a</sup> |
|       | Residual   | 703.349        | 10 | 70.335      |       |                   |
|       | Total      | 2206.795       | 14 |             |       |                   |

a. Predictors: (Constant), x4, x3, x2, x1

## ANOVA

| Model |            | Sum of Squares | df | Mean Square | F     | Sig.              |
|-------|------------|----------------|----|-------------|-------|-------------------|
| 1     | Regression | 1357.668       | 3  | 452.556     | 5.863 | .012 <sup>a</sup> |
|       | Residual   | 849.127        | 11 | 77.193      |       |                   |
|       | Total      | 2206.795       | 14 |             |       |                   |

a. Predictors: (Constant), x3, x2, x1

## ANOVA

| Model |            | Sum of Squares | df | Mean Square | F     | Sig.              |
|-------|------------|----------------|----|-------------|-------|-------------------|
| 1     | Regression | 922.620        | 2  | 461.310     | 4.311 | .039 <sup>a</sup> |
|       | Residual   | 1284.175       | 12 | 107.015     |       |                   |
|       | Total      | 2206.795       | 14 |             |       |                   |

a. Predictors: (Constant), x2, x1

## ANOVA

| Model |            | Sum of Squares | df | Mean Square | F    | Sig.              |
|-------|------------|----------------|----|-------------|------|-------------------|
| 1     | Regression | 82.876         | 1  | 82.876      | .507 | .489 <sup>a</sup> |
|       | Residual   | 2123.918       | 13 | 163.378     |      |                   |
|       | Total      | 2206.795       | 14 |             |      |                   |

a. Predictors: (Constant), x1

Answer the following questions based on the above two tables:

- State the fitted regression line for the full model with five predictor variables.
- Test the goodness of fit of the model in part (a). Only state the null and alternative hypotheses, the decision and the conclusion.

- c. Calculate the coefficient of multiple determination,  $R^2$  for the model in part (a). How is it interpreted here?
- d. Do the data present sufficient evidence to indicate that  $x_2$  contributes information for the prediction of  $y$ .
- e. Test the hypothesis,  $\beta_1 = 5$  against  $\beta_1 \neq 5$ .
- f. Test whether  $X_5$  can be dropped from the regression model given that the remaining four predictor variables are retained. Use  $\alpha = 0.05$ .
- g. Test whether  $X_4$  and  $X_5$  can be dropped from the regression model that already contains  $X_1$ ,  $X_2$  and  $X_3$ . Use  $\alpha = 0.01$ .

TABLE A.2 Percentiles of the *t* Distribution

Entry is  $t(A; \nu)$  where  $P\{t(\nu) \leq t(A; \nu)\} = A$



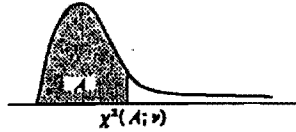
| $\nu$    | $A$   |       |       |       |       |       |        |
|----------|-------|-------|-------|-------|-------|-------|--------|
|          | .60   | .70   | .80   | .85   | .90   | .95   | .975   |
| 1        | 0.325 | 0.727 | 1.376 | 1.963 | 3.078 | 6.314 | 12.706 |
| 2        | 0.289 | 0.617 | 1.061 | 1.386 | 1.886 | 2.920 | 4.303  |
| 3        | 0.277 | 0.584 | 0.978 | 1.250 | 1.638 | 2.353 | 3.182  |
| 4        | 0.271 | 0.569 | 0.941 | 1.190 | 1.533 | 2.132 | 2.776  |
| 5        | 0.267 | 0.559 | 0.920 | 1.156 | 1.476 | 2.015 | 2.571  |
| 6        | 0.265 | 0.553 | 0.906 | 1.134 | 1.440 | 1.943 | 2.447  |
| 7        | 0.263 | 0.549 | 0.896 | 1.119 | 1.415 | 1.895 | 2.365  |
| 8        | 0.262 | 0.546 | 0.889 | 1.108 | 1.397 | 1.860 | 2.306  |
| 9        | 0.261 | 0.543 | 0.883 | 1.100 | 1.383 | 1.833 | 2.262  |
| 10       | 0.260 | 0.542 | 0.879 | 1.093 | 1.372 | 1.812 | 2.228  |
| 11       | 0.260 | 0.540 | 0.876 | 1.088 | 1.363 | 1.796 | 2.201  |
| 12       | 0.259 | 0.539 | 0.873 | 1.083 | 1.356 | 1.782 | 2.179  |
| 13       | 0.259 | 0.537 | 0.870 | 1.079 | 1.350 | 1.771 | 2.160  |
| 14       | 0.258 | 0.537 | 0.868 | 1.076 | 1.345 | 1.761 | 2.145  |
| 15       | 0.258 | 0.536 | 0.866 | 1.074 | 1.341 | 1.753 | 2.131  |
| 16       | 0.258 | 0.535 | 0.865 | 1.071 | 1.337 | 1.746 | 2.120  |
| 17       | 0.257 | 0.534 | 0.863 | 1.069 | 1.333 | 1.740 | 2.110  |
| 18       | 0.257 | 0.534 | 0.862 | 1.067 | 1.330 | 1.734 | 2.101  |
| 19       | 0.257 | 0.533 | 0.861 | 1.066 | 1.328 | 1.729 | 2.093  |
| 20       | 0.257 | 0.533 | 0.860 | 1.064 | 1.325 | 1.725 | 2.086  |
| 21       | 0.257 | 0.532 | 0.859 | 1.063 | 1.323 | 1.721 | 2.080  |
| 22       | 0.256 | 0.532 | 0.858 | 1.061 | 1.321 | 1.717 | 2.074  |
| 23       | 0.256 | 0.532 | 0.858 | 1.060 | 1.319 | 1.714 | 2.069  |
| 24       | 0.256 | 0.531 | 0.857 | 1.059 | 1.318 | 1.711 | 2.064  |
| 25       | 0.256 | 0.531 | 0.856 | 1.058 | 1.316 | 1.708 | 2.060  |
| 26       | 0.256 | 0.531 | 0.856 | 1.058 | 1.315 | 1.706 | 2.056  |
| 27       | 0.256 | 0.531 | 0.855 | 1.057 | 1.314 | 1.703 | 2.052  |
| 28       | 0.256 | 0.530 | 0.855 | 1.056 | 1.313 | 1.701 | 2.048  |
| 29       | 0.256 | 0.530 | 0.854 | 1.055 | 1.311 | 1.699 | 2.045  |
| 30       | 0.256 | 0.530 | 0.854 | 1.055 | 1.310 | 1.697 | 2.042  |
| 40       | 0.255 | 0.529 | 0.851 | 1.050 | 1.303 | 1.684 | 2.021  |
| 60       | 0.254 | 0.527 | 0.848 | 1.045 | 1.296 | 1.671 | 2.000  |
| 120      | 0.254 | 0.526 | 0.845 | 1.041 | 1.289 | 1.658 | 1.980  |
| $\infty$ | 0.253 | 0.524 | 0.842 | 1.036 | 1.282 | 1.645 | 1.960  |

TABLE A.2 (concluded) Percentiles of the *t* Distribution

| $\nu$    | $A$    |        |        |        |        |         |         |
|----------|--------|--------|--------|--------|--------|---------|---------|
|          | .98    | .985   | .99    | .9925  | .995   | .9975   | .9995   |
| 1        | 15.895 | 21.205 | 31.821 | 42.434 | 63.657 | 127.322 | 636.590 |
| 2        | 4.849  | 5.643  | 6.965  | 8.073  | 9.925  | 14.089  | 31.598  |
| 3        | 3.482  | 3.896  | 4.541  | 5.047  | 5.841  | 7.453   | 12.924  |
| 4        | 2.999  | 3.298  | 3.747  | 4.088  | 4.604  | 5.598   | 8.610   |
| 5        | 2.757  | 3.003  | 3.365  | 3.634  | 4.032  | 4.773   | 6.869   |
| 6        | 2.612  | 2.829  | 3.143  | 3.372  | 3.707  | 4.317   | 5.959   |
| 7        | 2.517  | 2.715  | 2.998  | 3.203  | 3.499  | 4.029   | 5.408   |
| 8        | 2.449  | 2.634  | 2.896  | 3.085  | 3.335  | 3.833   | 5.041   |
| 9        | 2.398  | 2.574  | 2.821  | 2.998  | 3.230  | 3.690   | 4.781   |
| 10       | 2.359  | 2.527  | 2.764  | 2.932  | 3.169  | 3.581   | 4.587   |
| 11       | 2.328  | 2.491  | 2.718  | 2.879  | 3.106  | 3.497   | 4.437   |
| 12       | 2.303  | 2.461  | 2.681  | 2.836  | 3.055  | 3.428   | 4.318   |
| 13       | 2.282  | 2.436  | 2.650  | 2.801  | 3.012  | 3.372   | 4.221   |
| 14       | 2.264  | 2.415  | 2.624  | 2.771  | 2.977  | 3.326   | 4.140   |
| 15       | 2.249  | 2.397  | 2.602  | 2.746  | 2.947  | 3.286   | 4.073   |
| 16       | 2.235  | 2.382  | 2.583  | 2.724  | 2.921  | 3.252   | 4.015   |
| 17       | 2.224  | 2.368  | 2.567  | 2.706  | 2.898  | 3.222   | 3.965   |
| 18       | 2.214  | 2.356  | 2.552  | 2.689  | 2.878  | 3.197   | 3.922   |
| 19       | 2.205  | 2.346  | 2.539  | 2.674  | 2.861  | 3.174   | 3.883   |
| 20       | 2.197  | 2.336  | 2.528  | 2.661  | 2.845  | 3.153   | 3.849   |
| 21       | 2.189  | 2.328  | 2.518  | 2.649  | 2.831  | 3.135   | 3.819   |
| 22       | 2.183  | 2.320  | 2.508  | 2.639  | 2.819  | 3.119   | 3.792   |
| 23       | 2.177  | 2.313  | 2.500  | 2.629  | 2.807  | 3.104   | 3.768   |
| 24       | 2.172  | 2.307  | 2.492  | 2.620  | 2.797  | 3.091   | 3.745   |
| 25       | 2.167  | 2.301  | 2.485  | 2.612  | 2.787  | 3.078   | 3.725   |
| 26       | 2.162  | 2.296  | 2.479  | 2.605  | 2.779  | 3.067   | 3.707   |
| 27       | 2.158  | 2.291  | 2.473  | 2.598  | 2.771  | 3.057   | 3.690   |
| 28       | 2.154  | 2.286  | 2.467  | 2.592  | 2.763  | 3.047   | 3.674   |
| 29       | 2.150  | 2.282  | 2.462  | 2.586  | 2.756  | 3.038   | 3.659   |
| 30       | 2.147  | 2.278  | 2.457  | 2.581  | 2.750  | 3.030   | 3.646   |
| 40       | 2.123  | 2.250  | 2.423  | 2.542  | 2.704  | 2.971   | 3.551   |
| 60       | 2.099  | 2.223  | 2.390  | 2.504  | 2.660  | 2.915   | 3.460   |
| 120      | 2.076  | 2.196  | 2.358  | 2.468  | 2.617  | 2.860   | 3.373   |
| $\infty$ | 2.054  | 2.170  | 2.326  | 2.432  | 2.576  | 2.807   | 3.291   |

TABLE A.3 Percentiles of the  $\chi^2$  Distribution

Entry is  $\chi^2(A; \nu)$  where  $P\{\chi^2(\nu) \leq \chi^2(A; \nu)\} = A$

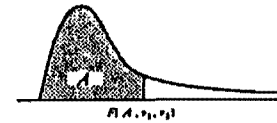


| $\nu$ | A       |        |         |         |       |       |       |       |       |       |
|-------|---------|--------|---------|---------|-------|-------|-------|-------|-------|-------|
|       | .005    | .010   | .025    | .050    | .100  | .900  | .950  | .975  | .990  | .995  |
| 1     | 0.00493 | 0.0157 | 0.03982 | 0.07393 | 0.158 | 2.71  | 3.84  | 5.02  | 6.63  | 7.88  |
| 2     | 0.0100  | 0.0201 | 0.0506  | 0.103   | 0.211 | 4.61  | 5.99  | 7.38  | 9.21  | 10.60 |
| 3     | 0.072   | 0.115  | 0.216   | 0.352   | 0.584 | 6.25  | 7.81  | 9.35  | 11.34 | 12.84 |
| 4     | 0.207   | 0.297  | 0.484   | 0.711   | 1.064 | 7.78  | 9.49  | 11.14 | 13.28 | 14.86 |
| 5     | 0.412   | 0.554  | 0.831   | 1.145   | 1.61  | 9.24  | 11.07 | 12.83 | 15.09 | 16.75 |
| 6     | 0.676   | 0.872  | 1.24    | 1.64    | 2.20  | 10.64 | 12.59 | 14.45 | 16.81 | 18.55 |
| 7     | 0.989   | 1.24   | 1.69    | 2.17    | 2.83  | 12.02 | 14.07 | 16.01 | 18.48 | 20.28 |
| 8     | 1.34    | 1.65   | 2.18    | 2.73    | 3.49  | 13.36 | 15.51 | 17.53 | 20.09 | 21.96 |
| 9     | 1.73    | 2.09   | 2.70    | 3.33    | 4.17  | 14.68 | 16.92 | 19.02 | 21.67 | 23.59 |
| 10    | 2.16    | 2.56   | 3.25    | 3.94    | 4.87  | 15.99 | 18.31 | 20.48 | 23.21 | 25.19 |
| 11    | 2.60    | 3.05   | 3.82    | 4.57    | 5.58  | 17.28 | 19.68 | 21.92 | 24.73 | 26.76 |
| 12    | 3.07    | 3.57   | 4.40    | 5.23    | 6.30  | 18.55 | 21.03 | 23.34 | 26.22 | 28.30 |
| 13    | 3.57    | 4.11   | 5.01    | 5.89    | 7.04  | 19.81 | 22.36 | 24.74 | 27.69 | 29.82 |
| 14    | 4.07    | 4.66   | 5.63    | 6.57    | 7.79  | 21.06 | 23.68 | 26.12 | 29.14 | 31.32 |
| 15    | 4.60    | 5.23   | 6.26    | 7.26    | 8.55  | 22.31 | 25.00 | 27.49 | 30.58 | 32.80 |
| 16    | 5.14    | 5.81   | 6.91    | 7.96    | 9.31  | 23.54 | 26.30 | 28.85 | 32.00 | 34.27 |
| 17    | 5.70    | 6.41   | 7.56    | 8.67    | 10.09 | 24.77 | 27.59 | 30.19 | 33.41 | 35.72 |
| 18    | 6.26    | 7.01   | 8.23    | 9.39    | 10.86 | 25.99 | 28.87 | 31.53 | 34.81 | 37.16 |
| 19    | 6.84    | 7.63   | 8.91    | 10.12   | 11.65 | 27.20 | 30.14 | 32.85 | 36.19 | 38.58 |
| 20    | 7.43    | 8.26   | 9.59    | 10.85   | 12.44 | 28.41 | 31.41 | 34.17 | 37.57 | 40.00 |
| 21    | 8.03    | 8.90   | 10.28   | 11.59   | 13.24 | 29.62 | 32.67 | 35.48 | 38.93 | 41.40 |
| 22    | 8.64    | 9.54   | 10.98   | 12.34   | 14.04 | 30.81 | 33.92 | 36.78 | 40.29 | 42.80 |
| 23    | 9.26    | 10.20  | 11.69   | 13.09   | 14.85 | 32.01 | 35.17 | 38.08 | 41.64 | 44.18 |
| 24    | 9.89    | 10.86  | 12.40   | 13.85   | 15.66 | 33.20 | 36.42 | 39.36 | 42.98 | 45.56 |
| 25    | 10.52   | 11.52  | 13.12   | 14.61   | 16.47 | 34.38 | 37.65 | 40.65 | 44.31 | 46.93 |
| 26    | 11.16   | 12.20  | 13.84   | 15.38   | 17.29 | 35.56 | 38.89 | 41.92 | 45.64 | 48.29 |
| 27    | 11.81   | 12.88  | 14.57   | 16.15   | 18.11 | 36.74 | 40.11 | 43.19 | 46.96 | 49.64 |
| 28    | 12.46   | 13.56  | 15.31   | 16.93   | 18.94 | 37.92 | 41.34 | 44.46 | 48.28 | 50.99 |
| 29    | 13.12   | 14.26  | 16.05   | 17.71   | 19.77 | 39.09 | 42.56 | 45.72 | 49.59 | 52.34 |
| 30    | 13.79   | 14.95  | 16.79   | 18.49   | 20.60 | 40.26 | 43.77 | 46.98 | 50.89 | 53.67 |
| 40    | 20.71   | 22.16  | 24.43   | 26.51   | 29.03 | 51.81 | 55.76 | 59.34 | 63.69 | 66.77 |
| 50    | 27.99   | 29.71  | 32.36   | 34.76   | 37.69 | 63.17 | 67.50 | 71.42 | 76.15 | 79.49 |
| 60    | 35.53   | 37.48  | 40.48   | 43.19   | 46.46 | 74.40 | 79.08 | 83.30 | 88.38 | 91.95 |
| 70    | 43.28   | 45.44  | 48.76   | 51.74   | 55.33 | 85.53 | 90.53 | 95.02 | 100.4 | 104.2 |
| 80    | 51.17   | 53.54  | 57.15   | 60.39   | 64.28 | 96.58 | 101.9 | 106.6 | 112.3 | 116.3 |
| 90    | 59.20   | 61.75  | 65.65   | 69.13   | 73.29 | 107.6 | 113.1 | 118.1 | 124.1 | 128.3 |
| 100   | 67.33   | 70.06  | 74.22   | 77.93   | 82.36 | 118.5 | 124.3 | 129.6 | 135.8 | 140.2 |

Source: Reprinted, with permission, from C. M. Thompson, "Table of Percentage Points of the Chi-Square Distribution," *Biometrika* 32 (1945), pp. 188-89.

TABLE A.4 Percentiles of the  $F$  Distribution

Entry is  $F(A; \nu_1, \nu_2)$  where  $P\{F(\nu_1, \nu_2) \leq F(A; \nu_1, \nu_2)\} = A$



$$F(A; \nu_1, \nu_2) = \frac{1}{F(1-A; \nu_2, \nu_1)}$$

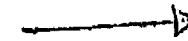


TABLE A.4 (continued) Percentiles of the F Distribution

| Den. df | A       | Numerator df |         |         |         |         |         |         |         |        |
|---------|---------|--------------|---------|---------|---------|---------|---------|---------|---------|--------|
|         |         | 1            | 2       | 3       | 4       | 5       | 6       | 7       | 8       | 9      |
| 1       | .50     | 1.00         | 1.50    | 1.71    | 1.82    | 1.89    | 1.94    | 1.98    | 2.00    | 2.03   |
|         | .90     | 39.9         | 49.3    | 53.6    | 55.8    | 57.2    | 58.2    | 58.9    | 59.4    | 59.9   |
|         | .95     | 161          | 200     | 216     | 223     | 230     | 234     | 237     | 239     | 241    |
|         | .975    | 648          | 800     | 864     | 900     | 922     | 937     | 948     | 957     | 963    |
|         | .99     | 4,032        | 5,000   | 5,403   | 5,623   | 5,764   | 5,859   | 5,928   | 5,981   | 6,022  |
|         | .995    | 16,211       | 20,000  | 21,615  | 22,500  | 23,056  | 23,437  | 23,715  | 23,925  | 24,091 |
| .999    | 405,280 | 500,000      | 540,380 | 562,300 | 576,400 | 583,940 | 592,870 | 598,140 | 602,280 |        |
| 2       | .50     | 0.667        | 1.00    | 1.13    | 1.21    | 1.25    | 1.28    | 1.30    | 1.32    | 1.33   |
|         | .90     | 8.53         | 9.00    | 9.16    | 9.24    | 9.29    | 9.33    | 9.35    | 9.37    | 9.38   |
|         | .95     | 18.5         | 19.0    | 19.2    | 19.2    | 19.3    | 19.3    | 19.4    | 19.4    | 19.4   |
|         | .975    | 38.5         | 39.0    | 39.2    | 39.2    | 39.3    | 39.3    | 39.4    | 39.4    | 39.4   |
|         | .99     | 98.5         | 99.0    | 99.2    | 99.2    | 99.3    | 99.3    | 99.4    | 99.4    | 99.4   |
|         | .995    | 199          | 199     | 199     | 199     | 199     | 199     | 199     | 199     | 199    |
| .999    | 998.5   | 999.0        | 999.2   | 999.2   | 999.3   | 999.3   | 999.4   | 999.4   | 999.4   |        |
| 3       | .50     | 0.585        | 0.881   | 1.00    | 1.06    | 1.10    | 1.13    | 1.15    | 1.16    | 1.17   |
|         | .90     | 5.34         | 5.46    | 5.39    | 5.34    | 5.31    | 5.28    | 5.27    | 5.25    | 5.24   |
|         | .95     | 10.1         | 9.53    | 9.28    | 9.12    | 9.01    | 8.94    | 8.89    | 8.85    | 8.81   |
|         | .975    | 17.4         | 16.0    | 15.4    | 15.1    | 14.9    | 14.7    | 14.6    | 14.5    | 14.5   |
|         | .99     | 34.1         | 30.8    | 29.5    | 28.7    | 28.2    | 27.9    | 27.7    | 27.5    | 27.3   |
|         | .995    | 55.6         | 49.8    | 47.5    | 46.2    | 45.4    | 44.8    | 44.4    | 44.1    | 43.9   |
| .999    | 167.0   | 148.5        | 141.1   | 137.1   | 134.6   | 132.8   | 131.6   | 130.6   | 129.9   |        |
| 4       | .50     | 0.549        | 0.828   | 0.941   | 1.00    | 1.04    | 1.06    | 1.08    | 1.09    | 1.10   |
|         | .90     | 4.34         | 4.32    | 4.19    | 4.11    | 4.05    | 4.01    | 3.98    | 3.95    | 3.94   |
|         | .95     | 7.71         | 6.94    | 6.39    | 6.39    | 6.26    | 6.16    | 6.09    | 6.04    | 6.00   |
|         | .975    | 12.2         | 10.6    | 9.98    | 9.60    | 9.36    | 9.20    | 9.07    | 8.98    | 8.90   |
|         | .99     | 21.2         | 18.0    | 16.7    | 16.0    | 15.5    | 15.2    | 15.0    | 14.8    | 14.7   |
|         | .995    | 31.3         | 26.3    | 24.3    | 23.2    | 22.5    | 22.0    | 21.6    | 21.4    | 21.1   |
| .999    | 74.1    | 61.2         | 56.2    | 53.4    | 51.7    | 50.5    | 49.7    | 49.0    | 48.5    |        |
| 5       | .50     | 0.528        | 0.799   | 0.907   | 0.965   | 1.00    | 1.02    | 1.04    | 1.05    | 1.06   |
|         | .90     | 4.06         | 3.78    | 3.62    | 3.52    | 3.45    | 3.40    | 3.37    | 3.34    | 3.32   |
|         | .95     | 6.61         | 5.79    | 5.41    | 5.19    | 5.05    | 4.95    | 4.88    | 4.82    | 4.77   |
|         | .975    | 10.0         | 8.43    | 7.76    | 7.39    | 7.15    | 6.98    | 6.85    | 6.76    | 6.68   |
|         | .99     | 16.3         | 13.3    | 12.1    | 11.4    | 11.0    | 10.7    | 10.5    | 10.3    | 10.2   |
|         | .995    | 22.8         | 18.3    | 16.5    | 15.6    | 14.9    | 14.5    | 14.2    | 14.0    | 13.8   |
| .999    | 47.2    | 37.1         | 33.2    | 31.1    | 29.8    | 28.8    | 28.2    | 27.6    | 27.2    |        |
| 6       | .50     | 0.515        | 0.780   | 0.886   | 0.942   | 0.977   | 1.00    | 1.02    | 1.03    | 1.04   |
|         | .90     | 3.78         | 3.46    | 3.29    | 3.18    | 3.11    | 3.05    | 3.01    | 2.98    | 2.96   |
|         | .95     | 5.99         | 5.14    | 4.76    | 4.33    | 4.39    | 4.28    | 4.21    | 4.15    | 4.10   |
|         | .975    | 8.81         | 7.26    | 6.60    | 6.23    | 5.99    | 5.82    | 5.70    | 5.60    | 5.52   |
|         | .99     | 13.7         | 10.9    | 9.78    | 9.15    | 8.75    | 8.47    | 8.26    | 8.10    | 7.98   |
|         | .995    | 18.6         | 14.5    | 12.9    | 12.0    | 11.5    | 11.1    | 10.8    | 10.6    | 10.4   |
| .999    | 35.5    | 27.0         | 23.7    | 21.9    | 20.8    | 20.0    | 19.5    | 19.0    | 18.7    |        |
| 7       | .50     | 0.506        | 0.767   | 0.871   | 0.926   | 0.960   | 0.983   | 1.00    | 1.01    | 1.02   |
|         | .90     | 3.59         | 3.26    | 3.07    | 2.96    | 2.88    | 2.83    | 2.78    | 2.75    | 2.72   |
|         | .95     | 5.59         | 4.74    | 4.35    | 4.12    | 3.97    | 3.87    | 3.79    | 3.73    | 3.68   |
|         | .975    | 8.07         | 6.54    | 5.89    | 5.52    | 5.29    | 5.12    | 4.99    | 4.90    | 4.82   |
|         | .99     | 12.2         | 9.55    | 8.45    | 7.85    | 7.46    | 7.19    | 6.99    | 6.84    | 6.72   |
|         | .995    | 16.2         | 12.4    | 10.9    | 10.1    | 9.52    | 9.16    | 8.89    | 8.68    | 8.51   |
| .999    | 29.2    | 21.7         | 18.8    | 17.2    | 16.2    | 15.5    | 15.0    | 14.6    | 14.3    |        |

TABLE A.4 (continued) Percentiles of the F Distribution

| Den. df | A       | Numerator df |         |         |         |         |         |         |         |          |  |
|---------|---------|--------------|---------|---------|---------|---------|---------|---------|---------|----------|--|
|         |         | 10           | 12      | 15      | 20      | 24      | 30      | 60      | 120     | $\infty$ |  |
| 1       | .50     | 2.04         | 2.07    | 2.09    | 2.12    | 2.13    | 2.15    | 2.17    | 2.18    | 2.20     |  |
|         | .90     | 60.2         | 60.7    | 61.2    | 61.7    | 62.0    | 62.3    | 62.8    | 63.1    | 63.3     |  |
|         | .95     | 242          | 244     | 246     | 248     | 249     | 250     | 252     | 253     | 254      |  |
|         | .975    | 969          | 977     | 985     | 993     | 997     | 1,001   | 1,010   | 1,014   | 1,018    |  |
|         | .99     | 6,056        | 6,106   | 6,157   | 6,209   | 6,235   | 6,261   | 6,313   | 6,339   | 6,366    |  |
|         | .995    | 24,234       | 24,426  | 24,630  | 24,836  | 24,940  | 25,044  | 25,253  | 25,359  | 25,464   |  |
| .999    | 605,620 | 610,670      | 615,760 | 620,910 | 623,500 | 626,100 | 631,340 | 633,970 | 636,620 |          |  |
| 2       | .50     | 1.34         | 1.36    | 1.38    | 1.39    | 1.40    | 1.41    | 1.43    | 1.43    | 1.44     |  |
|         | .90     | 9.39         | 9.41    | 9.42    | 9.44    | 9.45    | 9.46    | 9.47    | 9.48    | 9.49     |  |
|         | .95     | 19.4         | 19.4    | 19.4    | 19.4    | 19.5    | 19.5    | 19.5    | 19.5    | 19.5     |  |
|         | .975    | 39.4         | 39.4    | 39.4    | 39.4    | 39.5    | 39.5    | 39.5    | 39.5    | 39.5     |  |
|         | .99     | 99.4         | 99.4    | 99.4    | 99.4    | 99.5    | 99.5    | 99.5    | 99.5    | 99.5     |  |
|         | .995    | 199          | 199     | 199     | 199     | 199     | 199     | 199     | 199     | 200      |  |
| .999    | 999.4   | 999.4        | 999.4   | 999.4   | 999.5   | 999.5   | 999.5   | 999.5   | 999.5   |          |  |
| 3       | .50     | 1.18         | 1.20    | 1.21    | 1.23    | 1.23    | 1.24    | 1.25    | 1.26    | 1.27     |  |
|         | .90     | 5.23         | 5.22    | 5.20    | 5.18    | 5.18    | 5.17    | 5.15    | 5.14    | 5.13     |  |
|         | .95     | 8.79         | 8.74    | 8.70    | 8.66    | 8.64    | 8.62    | 8.57    | 8.55    | 8.53     |  |
|         | .975    | 14.4         | 14.3    | 14.3    | 14.2    | 14.1    | 14.1    | 14.0    | 13.9    | 13.9     |  |
|         | .99     | 27.2         | 27.1    | 26.9    | 26.7    | 26.6    | 26.5    | 26.3    | 26.2    | 26.1     |  |
|         | .995    | 43.7         | 43.4    | 43.1    | 42.8    | 42.6    | 42.5    | 42.1    | 42.0    | 41.8     |  |
| .999    | 129.2   | 128.3        | 127.4   | 126.4   | 125.9   | 125.4   | 124.5   | 124.0   | 123.5   |          |  |
| 4       | .50     | 1.11         | 1.13    | 1.14    | 1.15    | 1.16    | 1.16    | 1.18    | 1.18    | 1.19     |  |
|         | .90     | 3.92         | 3.90    | 3.87    | 3.84    | 3.83    | 3.82    | 3.79    | 3.78    | 3.76     |  |
|         | .95     | 5.96         | 5.91    | 5.86    | 5.80    | 5.77    | 5.75    | 5.69    | 5.66    | 5.63     |  |
|         | .975    | 8.84         | 8.75    | 8.66    | 8.56    | 8.51    | 8.46    | 8.36    | 8.31    | 8.26     |  |
|         | .99     | 14.5         | 14.4    | 14.2    | 14.0    | 13.9    | 13.8    | 13.7    | 13.6    | 13.5     |  |
|         | .995    | 21.0         | 20.7    | 20.4    | 20.2    | 20.0    | 19.9    | 19.6    | 19.5    | 19.3     |  |
| .999    | 48.1    | 47.4         | 46.8    | 46.1    | 45.8    | 45.4    | 44.7    | 44.4    | 44.1    |          |  |
| 5       | .50     | 1.07         | 1.09    | 1.10    | 1.11    | 1.12    | 1.12    | 1.14    | 1.14    | 1.15     |  |
|         | .90     | 3.30         | 3.27    | 3.24    | 3.21    | 3.19    | 3.17    | 3.14    | 3.12    | 3.11     |  |
|         | .95     | 4.74         | 4.68    | 4.62    | 4.56    | 4.53    | 4.50    | 4.43    | 4.40    | 4.37     |  |
|         | .975    | 6.62         | 6.52    | 6.43    | 6.33    | 6.28    | 6.23    | 6.12    | 6.07    | 6.02     |  |
|         | .99     | 10.1         | 9.89    | 9.72    | 9.55    | 9.47    | 9.38    | 9.20    | 9.11    | 9.02     |  |
|         | .995    | 13.6         | 13.4    | 13.1    | 12.9    | 12.8    | 12.7    | 12.4    | 12.3    | 12.1     |  |
| .999    | 26.9    | 26.4         | 25.9    | 25.4    | 25.1    | 24.9    | 24.3    | 24.1    | 23.8    |          |  |
| 6       | .50     | 1.05         | 1.06    | 1.07    | 1.08    | 1.09    | 1.10    | 1.11    | 1.12    | 1.12     |  |
|         | .90     | 2.94         | 2.90    | 2.87    | 2.84    | 2.82    | 2.80    | 2.76    | 2.74    | 2.72     |  |
|         | .95     | 4.06         | 4.00    | 3.94    | 3.87    | 3.84    | 3.81    | 3.74    | 3.70    | 3.67     |  |
|         | .975    | 5.46         | 5.37    | 5.27    | 5.17    | 5.12    | 5.07    | 4.96    | 4.90    | 4.85     |  |
|         | .99     | 7.87         | 7.72    | 7.56    | 7.40    | 7.31    | 7.23    | 7.06    | 6.97    | 6.88     |  |
|         | .995    | 10.2         | 10.0    | 9.81    | 9.59    | 9.47    | 9.36    | 9.12    | 9.00    | 8.88     |  |
| .999    | 18.4    | 18.0         | 17.6    | 17.1    | 16.9    | 16.7    | 16.2    | 16.0    | 15.7    |          |  |
| 7       | .50     | 1.03         | 1.04    | 1.05    | 1.07    | 1.07    | 1.08    | 1.09    | 1.10    | 1.10     |  |
|         | .90     | 2.70         | 2.67    | 2.63    | 2.59    | 2.58    | 2.56    | 2.51    | 2.49    | 2.47     |  |
|         | .95     | 3.64         | 3.57    | 3.51    | 3.44    | 3.41    | 3.38    | 3.30    | 3.27    | 3.23     |  |
|         | .975    | 4.76         | 4.67    | 4.57    | 4.47    | 4.42    | 4.36    | 4.23    | 4.20    | 4.14     |  |
|         | .99     | 6.62         | 6.47    | 6.31    | 6.16    | 6.07    | 5.99    | 5.82    | 5.74    | 5.65     |  |
|         | .995    | 8.38         | 8.18    | 7.97    | 7.75    | 7.65    | 7.53    | 7.31    | 7.19    | 7.08     |  |
| .999    | 14.1    | 13.7         | 13.3    | 12.9    | 12.7    | 12.5    | 12.1    | 11.9    | 11.7    |          |  |



TABLE A.4 (continued) Percentiles of the F Distribution

| Den. df | 4    | Numerator df |       |       |       |       |       |       |       |       |
|---------|------|--------------|-------|-------|-------|-------|-------|-------|-------|-------|
|         |      | 1            | 2     | 3     | 4     | 5     | 6     | 7     | 8     | 9     |
| 8       | .50  | 0.499        | 0.757 | 0.860 | 0.915 | 0.948 | 0.971 | 0.988 | 1.00  | 1.01  |
|         | .90  | 3.46         | 3.11  | 2.92  | 2.81  | 2.73  | 2.67  | 2.62  | 2.59  | 2.56  |
|         | .95  | 5.32         | 4.46  | 4.07  | 3.84  | 3.69  | 3.58  | 3.50  | 3.44  | 3.39  |
|         | .975 | 7.57         | 6.06  | 5.42  | 5.05  | 4.82  | 4.65  | 4.53  | 4.43  | 4.36  |
|         | .99  | 11.3         | 8.65  | 7.59  | 7.01  | 6.63  | 6.37  | 6.18  | 6.03  | 5.91  |
|         | .995 | 14.7         | 11.0  | 9.60  | 8.81  | 8.30  | 7.95  | 7.69  | 7.50  | 7.34  |
| .999    | 25.4 | 18.5         | 15.8  | 14.4  | 13.5  | 12.9  | 12.4  | 12.0  | 11.8  |       |
| 9       | .50  | 0.494        | 0.749 | 0.852 | 0.906 | 0.939 | 0.962 | 0.978 | 0.990 | 1.00  |
|         | .90  | 3.36         | 3.01  | 2.81  | 2.69  | 2.61  | 2.55  | 2.51  | 2.47  | 2.44  |
|         | .95  | 5.12         | 4.26  | 3.86  | 3.63  | 3.48  | 3.37  | 3.29  | 3.23  | 3.18  |
|         | .975 | 7.21         | 5.71  | 5.08  | 4.72  | 4.48  | 4.32  | 4.20  | 4.10  | 4.03  |
|         | .99  | 10.6         | 8.02  | 6.99  | 6.42  | 6.06  | 5.80  | 5.61  | 5.47  | 5.35  |
|         | .995 | 13.6         | 10.1  | 8.72  | 7.96  | 7.47  | 7.13  | 6.88  | 6.69  | 6.54  |
| .999    | 22.9 | 16.4         | 13.9  | 12.6  | 11.7  | 11.1  | 10.7  | 10.4  | 10.1  |       |
| 10      | .50  | 0.490        | 0.743 | 0.845 | 0.899 | 0.932 | 0.954 | 0.971 | 0.983 | 0.992 |
|         | .90  | 3.29         | 2.92  | 2.73  | 2.61  | 2.52  | 2.46  | 2.41  | 2.38  | 2.35  |
|         | .95  | 4.96         | 4.10  | 3.71  | 3.48  | 3.33  | 3.22  | 3.14  | 3.07  | 3.02  |
|         | .975 | 6.94         | 5.46  | 4.83  | 4.47  | 4.24  | 4.07  | 3.95  | 3.85  | 3.78  |
|         | .99  | 10.0         | 7.56  | 6.55  | 5.99  | 5.64  | 5.39  | 5.20  | 5.06  | 4.94  |
|         | .995 | 12.8         | 9.43  | 8.08  | 7.34  | 6.87  | 6.54  | 6.30  | 6.12  | 5.97  |
| .999    | 21.0 | 14.9         | 12.6  | 11.3  | 10.5  | 9.93  | 9.52  | 9.20  | 8.96  |       |
| 12      | .50  | 0.484        | 0.735 | 0.835 | 0.888 | 0.921 | 0.943 | 0.959 | 0.972 | 0.981 |
|         | .90  | 3.18         | 2.81  | 2.61  | 2.48  | 2.39  | 2.33  | 2.28  | 2.24  | 2.21  |
|         | .95  | 4.75         | 3.89  | 3.49  | 3.26  | 3.11  | 3.00  | 2.91  | 2.85  | 2.80  |
|         | .975 | 6.55         | 5.10  | 4.47  | 4.12  | 3.89  | 3.73  | 3.61  | 3.51  | 3.44  |
|         | .99  | 9.33         | 6.93  | 5.95  | 5.41  | 5.06  | 4.82  | 4.64  | 4.50  | 4.39  |
|         | .995 | 11.8         | 8.51  | 7.23  | 6.52  | 6.07  | 5.76  | 5.52  | 5.35  | 5.20  |
| .999    | 18.6 | 13.0         | 10.8  | 9.63  | 8.89  | 8.38  | 8.00  | 7.71  | 7.48  |       |
| 15      | .50  | 0.478        | 0.726 | 0.826 | 0.878 | 0.911 | 0.933 | 0.949 | 0.960 | 0.970 |
|         | .90  | 3.07         | 2.70  | 2.49  | 2.36  | 2.27  | 2.21  | 2.16  | 2.12  | 2.09  |
|         | .95  | 4.54         | 3.68  | 3.29  | 3.06  | 2.90  | 2.79  | 2.71  | 2.64  | 2.59  |
|         | .975 | 6.20         | 4.77  | 4.15  | 3.80  | 3.58  | 3.41  | 3.29  | 3.20  | 3.12  |
|         | .99  | 8.68         | 6.36  | 5.42  | 4.89  | 4.56  | 4.32  | 4.14  | 4.00  | 3.89  |
|         | .995 | 10.8         | 7.70  | 6.48  | 5.50  | 5.37  | 5.07  | 4.85  | 4.67  | 4.54  |
| .999    | 16.6 | 11.3         | 9.34  | 8.25  | 7.57  | 7.09  | 6.74  | 6.47  | 6.26  |       |
| 20      | .50  | 0.472        | 0.718 | 0.816 | 0.868 | 0.900 | 0.922 | 0.938 | 0.950 | 0.959 |
|         | .90  | 2.97         | 2.59  | 2.38  | 2.25  | 2.16  | 2.09  | 2.04  | 2.00  | 1.96  |
|         | .95  | 4.35         | 3.49  | 3.10  | 2.87  | 2.71  | 2.60  | 2.51  | 2.45  | 2.39  |
|         | .975 | 5.87         | 4.46  | 3.86  | 3.51  | 3.29  | 3.13  | 3.01  | 2.91  | 2.84  |
|         | .99  | 8.10         | 5.85  | 4.94  | 4.43  | 4.10  | 3.87  | 3.70  | 3.56  | 3.46  |
|         | .995 | 9.94         | 6.99  | 5.82  | 5.17  | 4.76  | 4.47  | 4.26  | 4.09  | 3.96  |
| .999    | 14.8 | 9.95         | 8.10  | 7.10  | 6.46  | 6.02  | 5.69  | 5.44  | 5.24  |       |
| 24      | .50  | 0.469        | 0.714 | 0.812 | 0.863 | 0.895 | 0.917 | 0.932 | 0.944 | 0.953 |
|         | .90  | 2.93         | 2.54  | 2.33  | 2.19  | 2.10  | 2.04  | 1.98  | 1.94  | 1.91  |
|         | .95  | 4.26         | 3.40  | 3.01  | 2.78  | 2.62  | 2.51  | 2.42  | 2.36  | 2.30  |
|         | .975 | 5.72         | 4.32  | 3.72  | 3.38  | 3.15  | 2.99  | 2.87  | 2.78  | 2.70  |
|         | .99  | 7.82         | 5.61  | 4.72  | 4.22  | 3.90  | 3.67  | 3.50  | 3.36  | 3.26  |
|         | .995 | 9.55         | 6.66  | 5.52  | 4.89  | 4.49  | 4.20  | 3.99  | 3.83  | 3.69  |
| .999    | 14.0 | 9.34         | 7.55  | 6.59  | 5.98  | 5.55  | 5.23  | 4.99  | 4.80  |       |

TABLE A.4 (continued) Percentiles of the F Distribution

| Den. df | 4    | Numerator df |       |       |       |      |      |      |      |      |
|---------|------|--------------|-------|-------|-------|------|------|------|------|------|
|         |      | 10           | 12    | 15    | 20    | 24   | 30   | 60   | 120  | ∞    |
| 8       | .50  | 1.02         | 1.03  | 1.04  | 1.05  | 1.06 | 1.07 | 1.08 | 1.08 | 1.09 |
|         | .90  | 2.34         | 2.50  | 2.46  | 2.42  | 2.40 | 2.38 | 2.34 | 2.32 | 2.29 |
|         | .95  | 3.35         | 3.28  | 3.22  | 3.15  | 3.12 | 3.08 | 3.01 | 2.97 | 2.93 |
|         | .975 | 4.30         | 4.20  | 4.10  | 4.00  | 3.95 | 3.89 | 3.78 | 3.73 | 3.67 |
|         | .99  | 5.81         | 5.67  | 5.52  | 5.36  | 5.28 | 5.20 | 5.03 | 4.95 | 4.86 |
|         | .995 | 7.21         | 7.01  | 6.81  | 6.61  | 6.50 | 6.40 | 6.18 | 6.06 | 5.95 |
| .999    | 11.5 | 11.2         | 10.8  | 10.5  | 10.3  | 10.1 | 9.73 | 9.53 | 9.33 |      |
| 9       | .50  | 1.01         | 1.02  | 1.03  | 1.04  | 1.05 | 1.05 | 1.07 | 1.07 | 1.08 |
|         | .90  | 2.42         | 2.38  | 2.34  | 2.30  | 2.28 | 2.25 | 2.21 | 2.18 | 2.16 |
|         | .95  | 3.14         | 3.07  | 3.01  | 2.94  | 2.90 | 2.86 | 2.79 | 2.75 | 2.71 |
|         | .975 | 3.96         | 3.87  | 3.77  | 3.67  | 3.61 | 3.56 | 3.45 | 3.39 | 3.33 |
|         | .99  | 5.26         | 5.11  | 4.96  | 4.81  | 4.73 | 4.65 | 4.48 | 4.40 | 4.31 |
|         | .995 | 6.42         | 6.23  | 6.03  | 5.83  | 5.73 | 5.62 | 5.41 | 5.30 | 5.19 |
| .999    | 9.89 | 9.57         | 9.24  | 8.90  | 8.72  | 8.55 | 8.19 | 8.00 | 7.81 |      |
| 10      | .50  | 1.00         | 1.01  | 1.02  | 1.03  | 1.04 | 1.05 | 1.06 | 1.06 | 1.07 |
|         | .90  | 2.32         | 2.28  | 2.24  | 2.20  | 2.18 | 2.16 | 2.11 | 2.08 | 2.06 |
|         | .95  | 2.98         | 2.91  | 2.84  | 2.77  | 2.74 | 2.70 | 2.62 | 2.58 | 2.54 |
|         | .975 | 3.72         | 3.62  | 3.52  | 3.42  | 3.37 | 3.31 | 3.20 | 3.14 | 3.08 |
|         | .99  | 4.85         | 4.71  | 4.56  | 4.41  | 4.33 | 4.25 | 4.08 | 4.00 | 3.91 |
|         | .995 | 5.85         | 5.66  | 5.47  | 5.27  | 5.17 | 5.07 | 4.86 | 4.75 | 4.64 |
| .999    | 8.75 | 8.45         | 8.15  | 7.80  | 7.64  | 7.47 | 7.12 | 6.94 | 6.76 |      |
| 12      | .50  | 0.989        | 1.00  | 1.01  | 1.02  | 1.03 | 1.03 | 1.05 | 1.05 | 1.06 |
|         | .90  | 2.19         | 2.15  | 2.10  | 2.06  | 2.04 | 2.01 | 1.96 | 1.93 | 1.90 |
|         | .95  | 2.75         | 2.69  | 2.62  | 2.54  | 2.51 | 2.47 | 2.38 | 2.34 | 2.30 |
|         | .975 | 3.37         | 3.28  | 3.18  | 3.07  | 3.02 | 2.96 | 2.85 | 2.79 | 2.72 |
|         | .99  | 4.30         | 4.16  | 4.01  | 3.86  | 3.78 | 3.70 | 3.54 | 3.45 | 3.36 |
|         | .995 | 5.09         | 4.91  | 4.72  | 4.53  | 4.43 | 4.33 | 4.12 | 4.01 | 3.90 |
| .999    | 7.29 | 7.00         | 6.71  | 6.40  | 6.25  | 6.09 | 5.76 | 5.59 | 5.42 |      |
| 15      | .50  | 0.977        | 0.989 | 1.00  | 1.01  | 1.02 | 1.02 | 1.03 | 1.04 | 1.05 |
|         | .90  | 2.06         | 2.02  | 1.97  | 1.92  | 1.90 | 1.87 | 1.82 | 1.79 | 1.76 |
|         | .95  | 2.54         | 2.48  | 2.40  | 2.33  | 2.29 | 2.25 | 2.16 | 2.11 | 2.07 |
|         | .975 | 3.06         | 2.96  | 2.86  | 2.76  | 2.70 | 2.64 | 2.52 | 2.46 | 2.40 |
|         | .99  | 3.80         | 3.67  | 3.52  | 3.37  | 3.29 | 3.21 | 3.05 | 2.96 | 2.87 |
|         | .995 | 4.42         | 4.25  | 4.07  | 3.88  | 3.79 | 3.69 | 3.48 | 3.37 | 3.26 |
| .999    | 6.08 | 5.81         | 5.54  | 5.25  | 5.10  | 4.95 | 4.64 | 4.48 | 4.31 |      |
| 20      | .50  | 0.966        | 0.977 | 0.989 | 1.00  | 1.01 | 1.01 | 1.02 | 1.03 | 1.03 |
|         | .90  | 1.94         | 1.89  | 1.84  | 1.79  | 1.77 | 1.74 | 1.68 | 1.64 | 1.61 |
|         | .95  | 2.35         | 2.28  | 2.20  | 2.12  | 2.08 | 2.04 | 1.95 | 1.90 | 1.84 |
|         | .975 | 2.77         | 2.68  | 2.57  | 2.46  | 2.41 | 2.35 | 2.22 | 2.16 | 2.09 |
|         | .99  | 3.37         | 3.23  | 3.09  | 2.94  | 2.86 | 2.78 | 2.61 | 2.52 | 2.42 |
|         | .995 | 3.85         | 3.68  | 3.50  | 3.32  | 3.22 | 3.12 | 2.92 | 2.81 | 2.69 |
| .999    | 5.08 | 4.82         | 4.56  | 4.29  | 4.15  | 4.00 | 3.70 | 3.54 | 3.38 |      |
| 24      | .50  | 0.961        | 0.972 | 0.983 | 0.994 | 1.00 | 1.01 | 1.02 | 1.02 | 1.03 |
|         | .90  | 1.88         | 1.83  | 1.78  | 1.73  | 1.70 | 1.67 | 1.61 | 1.57 | 1.53 |
|         | .95  | 2.25         | 2.18  | 2.11  | 2.03  | 1.98 | 1.94 | 1.84 | 1.79 | 1.73 |
|         | .975 | 2.64         | 2.54  | 2.44  | 2.33  | 2.27 | 2.21 | 2.08 | 2.01 | 1.94 |
|         | .99  | 3.17         | 3.03  | 2.89  | 2.74  | 2.66 | 2.58 | 2.40 | 2.31 | 2.21 |
|         | .995 | 3.59         | 3.42  | 3.25  | 3.06  | 2.97 | 2.87 | 2.66 | 2.55 | 2.43 |
| .999    | 4.64 | 4.39         | 4.14  | 3.87  | 3.74  | 3.59 | 3.29 | 3.14 | 2.97 |      |

TABLE A.4 (continued) Percentiles of the F Distribution

| Den. df | A    | Numerator df |       |       |       |       |       |       |       |       |       |
|---------|------|--------------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
|         |      | 1            | 2     | 3     | 4     | 5     | 6     | 7     | 8     | 9     |       |
| 30      | .50  | 0.466        | 0.709 | 0.807 | 0.858 | 0.890 | 0.912 | 0.927 | 0.939 | 0.948 |       |
|         | .90  | 2.88         | 2.49  | 2.28  | 2.14  | 2.05  | 1.98  | 1.93  | 1.88  | 1.85  |       |
|         | .95  | 4.17         | 3.32  | 2.92  | 2.69  | 2.53  | 2.42  | 2.33  | 2.27  | 2.21  |       |
|         | .975 | 5.57         | 4.18  | 3.59  | 3.25  | 3.03  | 2.87  | 2.75  | 2.65  | 2.57  |       |
|         | .99  | 7.56         | 5.39  | 4.51  | 4.02  | 3.70  | 3.47  | 3.30  | 3.17  | 3.07  |       |
|         | .995 | 9.18         | 6.35  | 5.24  | 4.62  | 4.23  | 3.95  | 3.74  | 3.58  | 3.45  |       |
|         | .999 | 13.3         | 8.77  | 7.05  | 6.12  | 5.53  | 5.12  | 4.82  | 4.58  | 4.39  |       |
|         | 60   | .50          | 0.461 | 0.701 | 0.798 | 0.849 | 0.880 | 0.901 | 0.917 | 0.928 | 0.937 |
|         |      | .90          | 2.79  | 2.39  | 2.18  | 2.04  | 1.95  | 1.87  | 1.82  | 1.77  | 1.74  |
|         |      | .95          | 4.00  | 3.15  | 2.76  | 2.53  | 2.37  | 2.25  | 2.17  | 2.10  | 2.04  |
| .975    |      | 5.29         | 3.93  | 3.34  | 3.01  | 2.79  | 2.63  | 2.51  | 2.41  | 2.33  |       |
| .99     |      | 7.08         | 4.98  | 4.13  | 3.65  | 3.34  | 3.12  | 2.95  | 2.82  | 2.72  |       |
| .995    |      | 8.49         | 5.80  | 4.73  | 4.14  | 3.76  | 3.49  | 3.29  | 3.13  | 3.01  |       |
| .999    |      | 12.0         | 7.77  | 6.17  | 5.31  | 4.76  | 4.37  | 4.09  | 3.86  | 3.69  |       |
| 120     |      | .50          | 0.458 | 0.697 | 0.793 | 0.844 | 0.875 | 0.896 | 0.912 | 0.923 | 0.932 |
|         |      | .90          | 2.75  | 2.35  | 2.13  | 1.99  | 1.90  | 1.82  | 1.77  | 1.72  | 1.68  |
|         |      | .95          | 3.92  | 3.07  | 2.68  | 2.45  | 2.29  | 2.18  | 2.09  | 2.02  | 1.96  |
|         | .975 | 5.15         | 3.80  | 3.23  | 2.89  | 2.67  | 2.52  | 2.39  | 2.30  | 2.22  |       |
|         | .99  | 6.85         | 4.79  | 3.95  | 3.48  | 3.17  | 2.96  | 2.79  | 2.66  | 2.56  |       |
|         | .995 | 8.18         | 5.54  | 4.50  | 3.92  | 3.55  | 3.28  | 3.09  | 2.93  | 2.81  |       |
|         | .999 | 11.4         | 7.32  | 5.78  | 4.95  | 4.42  | 4.04  | 3.77  | 3.55  | 3.38  |       |
|         | ∞    | .50          | 0.455 | 0.693 | 0.789 | 0.839 | 0.870 | 0.891 | 0.907 | 0.918 | 0.927 |
|         |      | .90          | 2.71  | 2.30  | 2.08  | 1.94  | 1.85  | 1.77  | 1.72  | 1.67  | 1.63  |
|         |      | .95          | 3.84  | 3.00  | 2.60  | 2.37  | 2.21  | 2.10  | 2.01  | 1.94  | 1.88  |
| .975    |      | 5.02         | 3.69  | 3.12  | 2.79  | 2.57  | 2.41  | 2.29  | 2.19  | 2.11  |       |
| .99     |      | 6.63         | 4.61  | 3.78  | 3.32  | 3.02  | 2.80  | 2.64  | 2.51  | 2.41  |       |
| .995    |      | 7.88         | 5.30  | 4.28  | 3.72  | 3.35  | 3.09  | 2.90  | 2.74  | 2.62  |       |
| .999    |      | 10.8         | 6.91  | 5.42  | 4.62  | 4.10  | 3.74  | 3.47  | 3.27  | 3.10  |       |

TABLE A.4 (concluded) Percentiles of the F Distribution

| Den. df | A    | Numerator df |       |       |       |       |       |       |       |       |      |
|---------|------|--------------|-------|-------|-------|-------|-------|-------|-------|-------|------|
|         |      | 10           | 12    | 15    | 20    | 24    | 30    | 60    | 120   | ∞     |      |
| 30      | .50  | 0.955        | 0.966 | 0.978 | 0.989 | 0.994 | 1.00  | 1.01  | 1.02  | 1.02  |      |
|         | .90  | 1.82         | 1.77  | 1.72  | 1.67  | 1.64  | 1.61  | 1.54  | 1.50  | 1.46  |      |
|         | .95  | 2.16         | 2.09  | 2.01  | 1.93  | 1.89  | 1.84  | 1.74  | 1.68  | 1.62  |      |
|         | .975 | 2.51         | 2.41  | 2.31  | 2.20  | 2.14  | 2.07  | 1.94  | 1.87  | 1.79  |      |
|         | .99  | 2.98         | 2.84  | 2.70  | 2.55  | 2.47  | 2.39  | 2.21  | 2.11  | 2.01  |      |
|         | .995 | 3.34         | 3.18  | 3.01  | 2.82  | 2.73  | 2.63  | 2.42  | 2.30  | 2.18  |      |
|         | .999 | 4.24         | 4.00  | 3.75  | 3.49  | 3.36  | 3.22  | 2.92  | 2.76  | 2.59  |      |
|         | 60   | .50          | 0.945 | 0.956 | 0.967 | 0.978 | 0.983 | 0.989 | 1.00  | 1.01  | 1.01 |
|         |      | .90          | 1.71  | 1.66  | 1.60  | 1.54  | 1.51  | 1.48  | 1.40  | 1.35  | 1.29 |
|         |      | .95          | 1.99  | 1.92  | 1.84  | 1.75  | 1.70  | 1.65  | 1.53  | 1.47  | 1.39 |
| .975    |      | 2.27         | 2.17  | 2.06  | 1.94  | 1.88  | 1.82  | 1.67  | 1.58  | 1.48  |      |
| .99     |      | 2.63         | 2.50  | 2.35  | 2.20  | 2.12  | 2.03  | 1.84  | 1.73  | 1.60  |      |
| .995    |      | 2.90         | 2.74  | 2.57  | 2.39  | 2.29  | 2.19  | 1.96  | 1.83  | 1.69  |      |
| .999    |      | 3.54         | 3.32  | 3.08  | 2.83  | 2.69  | 2.55  | 2.25  | 2.08  | 1.89  |      |
| 120     |      | .50          | 0.939 | 0.950 | 0.961 | 0.972 | 0.978 | 0.983 | 0.994 | 1.00  | 1.01 |
|         |      | .90          | 1.65  | 1.60  | 1.55  | 1.48  | 1.45  | 1.41  | 1.32  | 1.26  | 1.19 |
|         |      | .95          | 1.91  | 1.83  | 1.75  | 1.66  | 1.61  | 1.55  | 1.43  | 1.35  | 1.25 |
|         | .975 | 2.16         | 2.05  | 1.95  | 1.82  | 1.76  | 1.69  | 1.53  | 1.43  | 1.31  |      |
|         | .99  | 2.47         | 2.34  | 2.19  | 2.03  | 1.95  | 1.86  | 1.66  | 1.53  | 1.38  |      |
|         | .995 | 2.71         | 2.54  | 2.37  | 2.19  | 2.09  | 1.98  | 1.75  | 1.61  | 1.43  |      |
|         | .999 | 3.24         | 3.02  | 2.78  | 2.53  | 2.40  | 2.26  | 1.95  | 1.77  | 1.54  |      |
|         | ∞    | .50          | 0.934 | 0.945 | 0.956 | 0.967 | 0.972 | 0.978 | 0.989 | 0.994 | 1.00 |
|         |      | .90          | 1.60  | 1.55  | 1.49  | 1.42  | 1.38  | 1.34  | 1.24  | 1.17  | 1.00 |
|         |      | .95          | 1.83  | 1.75  | 1.67  | 1.57  | 1.52  | 1.46  | 1.32  | 1.22  | 1.00 |
| .975    |      | 2.05         | 1.94  | 1.83  | 1.71  | 1.64  | 1.57  | 1.39  | 1.27  | 1.00  |      |
| .99     |      | 2.32         | 2.18  | 2.04  | 1.88  | 1.79  | 1.70  | 1.47  | 1.32  | 1.00  |      |
| .995    |      | 2.52         | 2.36  | 2.19  | 2.00  | 1.90  | 1.79  | 1.53  | 1.36  | 1.00  |      |
| .999    |      | 2.96         | 2.74  | 2.51  | 2.27  | 2.13  | 1.99  | 1.66  | 1.45  | 1.00  |      |

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TABLE B.7  
Durbin-Watson  
Test Bounds.

| Level of Significance $\alpha = .05$ |       |       |       |       |       |       |       |       |       |       |
|--------------------------------------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| n                                    | p-1=1 |       | p-1=2 |       | p-1=3 |       | p-1=4 |       | p-1=5 |       |
|                                      | $d_L$ | $d_U$ | $d_L$ | $d_U$ | $d_L$ | $d_U$ | $d_L$ | $d_U$ | $d_L$ | $d_U$ |
| 15                                   | 1.08  | 1.36  | 0.95  | 1.54  | 0.82  | 1.75  | 0.69  | 1.97  | 0.56  | 2.21  |
| 16                                   | 1.10  | 1.37  | 0.98  | 1.54  | 0.86  | 1.73  | 0.74  | 1.93  | 0.62  | 2.15  |
| 17                                   | 1.13  | 1.38  | 1.02  | 1.54  | 0.90  | 1.71  | 0.78  | 1.90  | 0.67  | 2.10  |
| 18                                   | 1.16  | 1.39  | 1.05  | 1.53  | 0.93  | 1.69  | 0.82  | 1.87  | 0.71  | 2.06  |
| 19                                   | 1.18  | 1.40  | 1.08  | 1.53  | 0.97  | 1.68  | 0.86  | 1.85  | 0.75  | 2.02  |
| 20                                   | 1.20  | 1.41  | 1.10  | 1.54  | 1.00  | 1.68  | 0.90  | 1.83  | 0.79  | 1.99  |
| 21                                   | 1.22  | 1.42  | 1.13  | 1.54  | 1.03  | 1.67  | 0.93  | 1.81  | 0.83  | 1.96  |
| 22                                   | 1.24  | 1.43  | 1.15  | 1.54  | 1.05  | 1.66  | 0.96  | 1.80  | 0.86  | 1.94  |
| 23                                   | 1.26  | 1.44  | 1.17  | 1.54  | 1.08  | 1.66  | 0.99  | 1.79  | 0.90  | 1.92  |
| 24                                   | 1.27  | 1.45  | 1.19  | 1.55  | 1.10  | 1.66  | 1.01  | 1.78  | 0.93  | 1.90  |
| 25                                   | 1.29  | 1.45  | 1.21  | 1.55  | 1.12  | 1.66  | 1.04  | 1.77  | 0.95  | 1.89  |
| 26                                   | 1.30  | 1.46  | 1.22  | 1.55  | 1.14  | 1.65  | 1.06  | 1.76  | 0.98  | 1.88  |
| 27                                   | 1.32  | 1.47  | 1.24  | 1.56  | 1.16  | 1.65  | 1.08  | 1.76  | 1.01  | 1.86  |
| 28                                   | 1.33  | 1.48  | 1.26  | 1.56  | 1.18  | 1.65  | 1.10  | 1.75  | 1.03  | 1.85  |
| 29                                   | 1.34  | 1.48  | 1.27  | 1.56  | 1.20  | 1.65  | 1.12  | 1.74  | 1.05  | 1.84  |
| 30                                   | 1.35  | 1.49  | 1.28  | 1.57  | 1.21  | 1.65  | 1.14  | 1.74  | 1.07  | 1.83  |
| 31                                   | 1.36  | 1.50  | 1.30  | 1.57  | 1.23  | 1.65  | 1.16  | 1.74  | 1.09  | 1.83  |
| 32                                   | 1.37  | 1.50  | 1.31  | 1.57  | 1.24  | 1.65  | 1.18  | 1.73  | 1.11  | 1.82  |
| 33                                   | 1.38  | 1.51  | 1.32  | 1.58  | 1.26  | 1.65  | 1.19  | 1.73  | 1.13  | 1.81  |
| 34                                   | 1.39  | 1.51  | 1.33  | 1.58  | 1.27  | 1.65  | 1.21  | 1.73  | 1.15  | 1.81  |
| 35                                   | 1.40  | 1.52  | 1.34  | 1.58  | 1.28  | 1.65  | 1.22  | 1.73  | 1.16  | 1.80  |
| 36                                   | 1.41  | 1.52  | 1.35  | 1.59  | 1.29  | 1.65  | 1.24  | 1.73  | 1.18  | 1.80  |
| 37                                   | 1.42  | 1.53  | 1.36  | 1.59  | 1.31  | 1.66  | 1.25  | 1.72  | 1.19  | 1.80  |
| 38                                   | 1.43  | 1.54  | 1.37  | 1.59  | 1.32  | 1.66  | 1.26  | 1.72  | 1.21  | 1.79  |
| 39                                   | 1.43  | 1.54  | 1.38  | 1.60  | 1.33  | 1.66  | 1.27  | 1.72  | 1.22  | 1.79  |
| 40                                   | 1.44  | 1.54  | 1.39  | 1.60  | 1.34  | 1.66  | 1.29  | 1.72  | 1.23  | 1.79  |
| 45                                   | 1.48  | 1.57  | 1.43  | 1.62  | 1.38  | 1.67  | 1.34  | 1.72  | 1.29  | 1.78  |
| 50                                   | 1.50  | 1.59  | 1.46  | 1.63  | 1.42  | 1.67  | 1.38  | 1.72  | 1.34  | 1.77  |
| 55                                   | 1.53  | 1.60  | 1.49  | 1.64  | 1.45  | 1.68  | 1.41  | 1.72  | 1.38  | 1.77  |
| 60                                   | 1.55  | 1.62  | 1.51  | 1.65  | 1.48  | 1.69  | 1.44  | 1.73  | 1.41  | 1.77  |
| 65                                   | 1.57  | 1.63  | 1.54  | 1.66  | 1.50  | 1.70  | 1.47  | 1.73  | 1.44  | 1.77  |
| 70                                   | 1.58  | 1.64  | 1.55  | 1.67  | 1.52  | 1.70  | 1.49  | 1.74  | 1.46  | 1.77  |
| 75                                   | 1.60  | 1.65  | 1.57  | 1.68  | 1.54  | 1.71  | 1.51  | 1.74  | 1.49  | 1.77  |
| 80                                   | 1.61  | 1.66  | 1.59  | 1.69  | 1.56  | 1.72  | 1.53  | 1.74  | 1.51  | 1.77  |
| 85                                   | 1.62  | 1.67  | 1.60  | 1.70  | 1.57  | 1.72  | 1.55  | 1.75  | 1.52  | 1.77  |
| 90                                   | 1.63  | 1.68  | 1.61  | 1.70  | 1.59  | 1.73  | 1.57  | 1.75  | 1.54  | 1.78  |
| 95                                   | 1.64  | 1.69  | 1.62  | 1.71  | 1.60  | 1.73  | 1.58  | 1.75  | 1.56  | 1.78  |
| 100                                  | 1.65  | 1.69  | 1.63  | 1.72  | 1.61  | 1.74  | 1.59  | 1.76  | 1.57  | 1.78  |

TABLE B.7  
(concluded)  
Durbin-Watson  
Test Bounds.

| Level of Significance $\alpha = .01$ |       |       |       |       |       |       |       |       |       |       |
|--------------------------------------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| n                                    | p-1=1 |       | p-1=2 |       | p-1=3 |       | p-1=4 |       | p-1=5 |       |
|                                      | $d_L$ | $d_U$ | $d_L$ | $d_U$ | $d_L$ | $d_U$ | $d_L$ | $d_U$ | $d_L$ | $d_U$ |
| 15                                   | 0.81  | 1.07  | 0.70  | 1.25  | 0.59  | 1.46  | 0.49  | 1.70  | 0.39  | 1.96  |
| 16                                   | 0.84  | 1.09  | 0.74  | 1.25  | 0.63  | 1.44  | 0.53  | 1.66  | 0.44  | 1.90  |
| 17                                   | 0.87  | 1.10  | 0.77  | 1.25  | 0.67  | 1.43  | 0.57  | 1.63  | 0.48  | 1.85  |
| 18                                   | 0.90  | 1.12  | 0.80  | 1.26  | 0.71  | 1.42  | 0.61  | 1.60  | 0.52  | 1.80  |
| 19                                   | 0.93  | 1.13  | 0.83  | 1.26  | 0.74  | 1.41  | 0.65  | 1.58  | 0.56  | 1.77  |
| 20                                   | 0.95  | 1.15  | 0.86  | 1.27  | 0.77  | 1.41  | 0.68  | 1.57  | 0.60  | 1.74  |
| 21                                   | 0.97  | 1.16  | 0.89  | 1.27  | 0.80  | 1.41  | 0.72  | 1.55  | 0.63  | 1.71  |
| 22                                   | 1.00  | 1.17  | 0.91  | 1.28  | 0.83  | 1.40  | 0.75  | 1.54  | 0.66  | 1.69  |
| 23                                   | 1.02  | 1.19  | 0.94  | 1.29  | 0.86  | 1.40  | 0.77  | 1.53  | 0.70  | 1.67  |
| 24                                   | 1.04  | 1.20  | 0.96  | 1.30  | 0.88  | 1.41  | 0.80  | 1.53  | 0.72  | 1.66  |
| 25                                   | 1.05  | 1.21  | 0.98  | 1.30  | 0.90  | 1.41  | 0.83  | 1.52  | 0.75  | 1.65  |
| 26                                   | 1.07  | 1.22  | 1.00  | 1.31  | 0.93  | 1.41  | 0.85  | 1.52  | 0.78  | 1.64  |
| 27                                   | 1.09  | 1.23  | 1.02  | 1.32  | 0.95  | 1.41  | 0.88  | 1.51  | 0.81  | 1.63  |
| 28                                   | 1.10  | 1.24  | 1.04  | 1.32  | 0.97  | 1.41  | 0.90  | 1.51  | 0.83  | 1.62  |
| 29                                   | 1.12  | 1.25  | 1.05  | 1.33  | 0.99  | 1.42  | 0.92  | 1.51  | 0.85  | 1.61  |
| 30                                   | 1.13  | 1.26  | 1.07  | 1.34  | 1.01  | 1.42  | 0.94  | 1.51  | 0.88  | 1.61  |
| 31                                   | 1.15  | 1.27  | 1.08  | 1.34  | 1.02  | 1.42  | 0.96  | 1.51  | 0.90  | 1.60  |
| 32                                   | 1.16  | 1.28  | 1.10  | 1.35  | 1.04  | 1.43  | 0.98  | 1.51  | 0.92  | 1.60  |
| 33                                   | 1.17  | 1.29  | 1.11  | 1.36  | 1.05  | 1.43  | 1.00  | 1.51  | 0.94  | 1.59  |
| 34                                   | 1.18  | 1.30  | 1.13  | 1.36  | 1.07  | 1.43  | 1.01  | 1.51  | 0.95  | 1.59  |
| 35                                   | 1.19  | 1.31  | 1.14  | 1.37  | 1.08  | 1.44  | 1.03  | 1.51  | 0.97  | 1.59  |
| 36                                   | 1.21  | 1.32  | 1.15  | 1.38  | 1.10  | 1.44  | 1.04  | 1.51  | 0.99  | 1.59  |
| 37                                   | 1.22  | 1.32  | 1.16  | 1.38  | 1.11  | 1.45  | 1.06  | 1.51  | 1.00  | 1.59  |
| 38                                   | 1.23  | 1.33  | 1.18  | 1.39  | 1.12  | 1.45  | 1.07  | 1.52  | 1.02  | 1.58  |
| 39                                   | 1.24  | 1.34  | 1.19  | 1.39  | 1.14  | 1.45  | 1.09  | 1.52  | 1.03  | 1.58  |
| 40                                   | 1.25  | 1.34  | 1.20  | 1.40  | 1.15  | 1.46  | 1.10  | 1.52  | 1.05  | 1.58  |
| 45                                   | 1.29  | 1.38  | 1.24  | 1.42  | 1.20  | 1.48  | 1.16  | 1.53  | 1.11  | 1.58  |
| 50                                   | 1.32  | 1.40  | 1.28  | 1.45  | 1.24  | 1.49  | 1.20  | 1.54  | 1.16  | 1.59  |
| 55                                   | 1.36  | 1.43  | 1.32  | 1.47  | 1.28  | 1.51  | 1.25  | 1.55  | 1.21  | 1.59  |
| 60                                   | 1.38  | 1.45  | 1.35  | 1.48  | 1.32  | 1.52  | 1.28  | 1.56  | 1.25  | 1.60  |
| 65                                   | 1.41  | 1.47  | 1.38  | 1.50  | 1.35  | 1.53  | 1.31  | 1.57  | 1.28  | 1.61  |
| 70                                   | 1.43  | 1.49  | 1.40  | 1.52  | 1.37  | 1.55  | 1.34  | 1.58  | 1.31  | 1.61  |
| 75                                   | 1.45  | 1.50  | 1.42  | 1.53  | 1.39  | 1.56  | 1.37  | 1.59  | 1.34  | 1.62  |
| 80                                   | 1.47  | 1.52  | 1.44  | 1.54  | 1.42  | 1.57  | 1.39  | 1.60  | 1.36  | 1.62  |
| 85                                   | 1.48  | 1.53  | 1.46  | 1.55  | 1.43  | 1.58  | 1.41  | 1.60  | 1.39  | 1.63  |
| 90                                   | 1.50  | 1.54  | 1.47  | 1.56  | 1.45  | 1.59  | 1.43  | 1.61  | 1.41  | 1.64  |
| 95                                   | 1.51  | 1.55  | 1.49  | 1.57  | 1.47  | 1.60  | 1.45  | 1.62  | 1.42  | 1.64  |
| 100                                  | 1.52  | 1.56  | 1.50  | 1.58  | 1.48  | 1.60  | 1.46  | 1.63  | 1.44  | 1.65  |

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