

UNIVERSITY OF SWAZILAND

SUPPLEMENTARY EXAMINATION PAPER 2014

TITLE OF PAPER : INTRODUCTION TO REGRESSION ANALYSIS

COURSE CODE : ST304

TIME ALLOWED : 2 (TWO) HOURS

**REQUIRMENTS : STATISTICAL TABLES
AND CALCULATOR**

**INSTRUCTIONS : ANSWER QUESTION ONE AND ANY
THREE (3) QUESTIONS. ALL QUESTIONS
CARRY MARKS AS INDICATED WITHIN
THE PARENTHESIS.**

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GRANTED BY THE INVIGILATOR**

QUESTION ONE.

[5 + 5 + 5 + 5 + 5 + 5 + 5 + 5 + 5 marks]

1.1 The following results were computed from a multiple regression model with a sample of 15:

$$(X'X)^{-1} = \begin{bmatrix} 37.232491 & -0.0225082 & 1.336707 \\ -0.0225082 & 0.0000137 & -0.0008319 \\ 1.336707 & -0.0008319 & .054034 \end{bmatrix}, X'Y = \begin{bmatrix} 29135 \\ 62905821 \\ 247934 \end{bmatrix}, \text{ and } MSE = 164.73797$$

- Find the estimated regression function
- Estimate the variance-covariance matrix.
- Construct a 99% confidence interval for β_0 .
- Construct a 95% joint confidence interval for β_1 and β_2 .
- Test the hypothesis, $\beta_2 = 10$ against $\beta_2 \neq 10$.
- Test the hypothesis, $\beta_1 = 0$ against $\beta_1 < 0$.

1.2 Consider the following ANOVA table with decomposition of SSR for three predictor variables:

Source of Variation	SS	df	MS
Regression	246.58	3	
X_1	198.27	1	
$X_2 X_1$	33.17	1	
$X_3 X_2, X_1$	15.14	1	
Error	98.41	15	
Total		18	

- Test whether X_3 can be dropped from the regression model given that X_1 and X_2 are retained. Use $\alpha = 0.05$.
- Test whether X_2 and X_3 can be dropped from the regression model that already contains X_1 . Use $\alpha = 0.01$.

QUESTION TWO.

[8 + 6 + 6 marks]

- Define Heteroscedasticity. Explain how one can use the Generalized Least Squares (GLS) method to obtain estimates of regression coefficients when heteroscedasticity is present.
- Carry out the Spearman's rank correlation test of Heteroscedasticity with $\alpha = 0.01$ if you are given $n = 15$ and rank correlation, $r_s = 0.46$.
 - Carry out the Goldfeld-Quandt test of Heteroscedasticity with $\alpha = 0.05$ if you are given the following information:

$$N = 40 \quad c = 6 \quad k = 2$$

$$RSS_1 \text{ based on the first 17 observations} = 413.15$$

$$RSS_2 \text{ based on the last 17 observations} = 1636.40$$

QUESTION THREE.

[4 + 5 + 1 + 5 + 5 marks]

The following tables are the partial SPSS output of a multiple regression problem:

Model		Sum of Squares	df	Mean Square	F	Sig.
1	Regression	126.714	3	42.238	5.171	.054
	Residual	40.842	5	8.168		
	Total	167.556	8			

Model		Unstandardized Coefficients		Standardized Coefficients	t	Sig.
		B	Std. Error	Beta		
1	(Constant)	35.626	13.418		2.655	.045
	X1	3.675	1.639	.745	2.243	.075
	X2	2.892	1.055	.614	2.740	.041
	X3	-.110	1.625	-.022	-.068	.949

Answer the following questions based on the above two tables:

- State the fitted regression line.
- Test the goodness of fitted model. You must state the null and alternative hypotheses and clearly state the conclusion.
- What is the estimated value of σ^2 ?
- Calculate the coefficient of multiple determination R^2 . How is it interpreted here?
- Do the data present sufficient evidence to indicate that the third predictor variable contributes information for the prediction of the response variable?

QUESTION FOUR.

[10 + 5 + 5 marks]

- Define Multicollinearity. Discuss the effects of multicollinearity, when it is present in the model, (i) on the Regression Coefficients, (ii) on the Standard Errors of the estimated regression coefficients, $s\{b_k\}$, and (iii) on the F -test.
- Discuss the five informal diagnostics for multicollinearity.
- Discuss five remedial measures for serious multicollinearity.

QUESTION FIVE.

[10 + 5 + 5 marks]

- 5.1 What is Autocorrelation? Discuss briefly, the problems of using ordinary least squares procedure when the error terms in the regression model are positively correlated.
- 5.2 The following residuals were found running a multiple regression model with 3 predictor variables:

-1.0	-2.7	4.1	5.7
-0.9	-1.8	0.0	-2.0
-4.0	-2.8	6.5	-1.4
-2.2	0.5	-0.7	2.8

Use Durbin-Watson test statistic to test the existence of autocorrelation at any suitable α .

- 5.3 Given a sample of 30 observations and 3 predictor variables with 5% level of significance, what can you say about autocorrelation (positive or negative) if
- $D = 1.06?$
 - $D = 1.49?$
 - $D = 2.05?$
 - $D = 2.42?$
 - $D = 3.58?$

[You must show all your work regarding your conclusions]

TABLE A.2 Percentiles of the *t* Distribution

Entry is $t(A; \nu)$ where $P\{t(\nu) \leq t(A; \nu)\} = A$



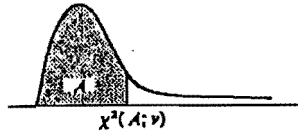
ν	A						
	.60	.70	.80	.85	.90	.95	.975
1	0.325	0.727	1.376	1.963	3.078	6.314	12.706
2	0.289	0.617	1.061	1.386	1.886	2.920	4.303
3	0.277	0.584	0.978	1.250	1.638	2.353	3.182
4	0.271	0.569	0.941	1.190	1.533	2.132	2.776
5	0.267	0.559	0.920	1.156	1.476	2.015	2.571
6	0.265	0.553	0.906	1.134	1.440	1.943	2.447
7	0.263	0.549	0.896	1.119	1.415	1.895	2.365
8	0.262	0.546	0.889	1.108	1.397	1.860	2.306
9	0.261	0.543	0.883	1.100	1.383	1.833	2.262
10	0.260	0.542	0.879	1.093	1.372	1.812	2.228
11	0.260	0.540	0.876	1.088	1.363	1.796	2.201
12	0.259	0.539	0.873	1.083	1.356	1.782	2.179
13	0.259	0.537	0.870	1.079	1.350	1.771	2.160
14	0.258	0.537	0.868	1.076	1.345	1.761	2.145
15	0.258	0.536	0.866	1.074	1.341	1.753	2.131
16	0.258	0.535	0.865	1.071	1.337	1.746	2.120
17	0.257	0.534	0.863	1.069	1.333	1.740	2.110
18	0.257	0.534	0.862	1.067	1.330	1.734	2.101
19	0.257	0.533	0.861	1.066	1.328	1.729	2.093
20	0.257	0.533	0.860	1.064	1.325	1.725	2.086
21	0.257	0.532	0.859	1.063	1.323	1.721	2.080
22	0.256	0.532	0.858	1.061	1.321	1.717	2.074
23	0.256	0.532	0.858	1.060	1.319	1.714	2.069
24	0.256	0.531	0.857	1.059	1.318	1.711	2.064
25	0.256	0.531	0.856	1.058	1.316	1.708	2.060
26	0.256	0.531	0.856	1.058	1.315	1.706	2.056
27	0.256	0.531	0.855	1.057	1.314	1.703	2.052
28	0.256	0.530	0.855	1.056	1.313	1.701	2.048
29	0.256	0.530	0.854	1.055	1.311	1.699	2.045
30	0.256	0.530	0.854	1.055	1.310	1.697	2.042
40	0.255	0.529	0.851	1.050	1.303	1.684	2.021
60	0.254	0.527	0.848	1.045	1.296	1.671	2.000
120	0.254	0.526	0.845	1.041	1.289	1.658	1.980
∞	0.253	0.524	0.842	1.036	1.282	1.645	1.960

TABLE A.2 (concluded) Percentiles of the *t* Distribution

ν	A						
	.98	.985	.99	.9925	.995	.9975	.9995
1	15.895	21.205	31.821	42.434	63.657	127.322	636.590
2	4.849	5.643	6.965	8.073	9.925	14.089	31.598
3	3.482	3.896	4.541	5.047	5.841	7.453	12.924
4	2.999	3.298	3.747	4.088	4.604	5.598	8.610
5	2.757	3.003	3.365	3.634	4.032	4.773	6.869
6	2.612	2.829	3.143	3.372	3.707	4.317	5.959
7	2.517	2.715	2.998	3.203	3.499	4.029	5.408
8	2.449	2.634	2.896	3.085	3.355	3.833	5.041
9	2.398	2.574	2.821	2.998	3.250	3.690	4.781
10	2.359	2.527	2.764	2.932	3.169	3.581	4.587
11	2.328	2.491	2.718	2.879	3.106	3.497	4.437
12	2.303	2.461	2.681	2.836	3.055	3.428	4.318
13	2.282	2.436	2.650	2.801	3.012	3.372	4.221
14	2.264	2.415	2.624	2.771	2.977	3.326	4.140
15	2.249	2.397	2.602	2.746	2.947	3.286	4.073
16	2.235	2.382	2.583	2.724	2.921	3.252	4.015
17	2.224	2.368	2.567	2.706	2.898	3.222	3.965
18	2.214	2.356	2.552	2.689	2.878	3.197	3.922
19	2.205	2.346	2.539	2.674	2.861	3.174	3.883
20	2.197	2.336	2.528	2.661	2.845	3.153	3.849
21	2.189	2.328	2.518	2.649	2.831	3.135	3.819
22	2.183	2.320	2.508	2.639	2.819	3.119	3.792
23	2.177	2.313	2.500	2.629	2.807	3.104	3.768
24	2.172	2.307	2.492	2.620	2.797	3.091	3.745
25	2.167	2.301	2.485	2.612	2.787	3.078	3.725
26	2.162	2.296	2.479	2.605	2.779	3.067	3.707
27	2.158	2.291	2.473	2.598	2.771	3.057	3.690
28	2.154	2.286	2.467	2.592	2.763	3.047	3.674
29	2.150	2.282	2.462	2.586	2.756	3.038	3.659
30	2.147	2.278	2.457	2.581	2.750	3.030	3.646
40	2.123	2.250	2.423	2.542	2.704	2.971	3.551
60	2.099	2.223	2.390	2.504	2.660	2.915	3.460
120	2.076	2.196	2.358	2.468	2.617	2.860	3.373
∞	2.054	2.170	2.326	2.432	2.576	2.807	3.291

TABLE A.3 Percentiles of the χ^2 Distribution

Entry is $\chi^2(A; \nu)$ where $P\{\chi^2(\nu) \leq \chi^2(A; \nu)\} = A$

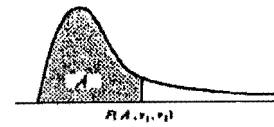


ν	A									
	.005	.010	.025	.050	.100	.900	.950	.975	.990	.995
1	0.00393	0.0157	0.03982	0.07393	0.0158	2.71	3.84	5.02	6.63	7.88
2	0.0100	0.0201	0.0506	0.103	0.211	4.61	5.99	7.38	9.21	10.60
3	0.072	0.115	0.216	0.352	0.584	6.25	7.81	9.35	11.34	12.84
4	0.207	0.297	0.484	0.711	1.064	7.78	9.49	11.14	13.28	14.86
5	0.412	0.554	0.831	1.145	1.61	9.24	11.07	12.83	15.09	16.75
6	0.676	0.872	1.24	1.64	2.20	10.64	12.59	14.45	16.81	18.55
7	0.989	1.24	1.69	2.17	2.83	12.02	14.07	16.01	18.48	20.28
8	1.34	1.65	2.18	2.73	3.49	13.36	15.51	17.53	20.09	21.96
9	1.73	2.09	2.70	3.33	4.17	14.68	16.92	19.02	21.67	23.59
10	2.16	2.56	3.25	3.94	4.87	15.99	18.31	20.48	23.21	25.19
11	2.60	3.05	3.82	4.57	5.58	17.28	19.68	21.92	24.73	26.76
12	3.07	3.57	4.40	5.23	6.30	18.55	21.03	23.34	26.22	28.30
13	3.57	4.11	5.01	5.89	7.04	19.81	22.36	24.74	27.69	29.82
14	4.07	4.66	5.63	6.57	7.79	21.06	23.68	26.12	29.14	31.32
15	4.60	5.23	6.26	7.26	8.55	22.31	25.00	27.49	30.58	32.80
16	5.14	5.81	6.91	7.96	9.31	23.54	26.30	28.85	32.00	34.27
17	5.70	6.41	7.56	8.67	10.09	24.77	27.59	30.19	33.41	35.72
18	6.26	7.01	8.23	9.39	10.86	25.99	28.87	31.53	34.81	37.16
19	6.84	7.63	8.91	10.12	11.65	27.20	30.14	32.85	36.19	38.58
20	7.43	8.26	9.59	10.85	12.44	28.41	31.41	34.17	37.57	40.00
21	8.03	8.90	10.28	11.59	13.24	29.62	32.67	35.48	38.93	41.40
22	8.64	9.54	10.98	12.34	14.04	30.81	33.92	36.78	40.29	42.80
23	9.26	10.20	11.69	13.09	14.85	32.01	35.17	38.08	41.64	44.18
24	9.89	10.86	12.40	13.85	15.66	33.20	36.42	39.36	42.98	45.56
25	10.52	11.52	13.12	14.61	16.47	34.38	37.65	40.65	44.31	46.93
26	11.16	12.20	13.84	15.38	17.29	35.56	38.89	41.92	45.64	48.29
27	11.81	12.88	14.57	16.15	18.11	36.74	40.11	43.19	46.96	49.64
28	12.46	13.56	15.31	16.93	18.94	37.92	41.34	44.46	48.28	50.99
29	13.12	14.26	16.05	17.71	19.77	39.09	42.56	45.72	49.59	52.34
30	13.79	14.95	16.79	18.49	20.60	40.26	43.77	46.98	50.89	53.67
40	20.71	22.16	24.43	26.51	29.05	51.81	55.76	59.34	63.69	66.77
50	27.99	29.71	32.36	34.76	37.69	63.17	67.50	71.42	76.15	79.49
60	35.33	37.48	40.48	43.19	46.46	74.40	79.08	83.30	88.38	91.95
70	43.28	45.44	48.76	51.74	55.33	85.53	90.53	95.02	100.4	104.2
80	51.17	53.54	57.15	60.39	64.28	96.58	101.9	106.6	112.3	116.3
90	59.20	61.75	65.65	69.13	73.29	107.6	113.1	118.1	124.1	128.3
100	67.33	70.06	74.22	77.93	82.36	118.5	124.3	129.6	135.8	140.2

Source: Reprinted, with permission, from C. M. Thompson, "Table of Percentage Points of the Chi-Square Distribution," *Biometrika* 32 (1941), pp. 188-89.

TABLE A.4 Percentiles of the F Distribution

Entry is $F(A; \nu_1, \nu_2)$ where $P\{F(\nu_1, \nu_2) \leq F(A; \nu_1, \nu_2)\} = A$



$$F(A; \nu_1, \nu_2) = \frac{1}{F(1-A; \nu_2, \nu_1)}$$

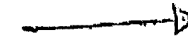


TABLE A.4 (continued) Percentiles of the F Distribution

Den. df A	Numerator df								
	1	2	3	4	5	6	7	8	9
1 .50	1.00	1.50	1.71	1.82	1.89	1.94	1.98	2.00	2.03
.90	39.9	49.5	53.6	55.8	57.2	58.2	58.9	59.4	59.9
.95	161	200	216	225	230	234	237	239	241
.975	648	800	864	900	922	937	948	957	963
.99	4,052	5,000	5,403	5,625	5,764	5,859	5,928	5,981	6,022
.995	16,211	20,000	21,615	22,500	23,056	23,437	23,715	23,925	24,091
.999	405,280	500,000	540,380	562,500	576,400	585,940	592,870	598,140	602,280
2 .50	0.667	1.00	1.13	1.21	1.25	1.28	1.30	1.32	1.33
.90	8.53	9.00	9.16	9.24	9.29	9.33	9.35	9.37	9.38
.95	18.5	19.0	19.2	19.2	19.3	19.3	19.4	19.4	19.4
.975	38.5	39.0	39.2	39.2	39.3	39.3	39.4	39.4	39.4
.99	98.5	99.0	99.2	99.2	99.3	99.3	99.4	99.4	99.4
.995	199	199	199	199	199	199	199	199	199
.999	998.5	999.0	999.2	999.2	999.3	999.3	999.4	999.4	999.4
3 .50	0.585	0.881	1.00	1.06	1.10	1.13	1.15	1.16	1.17
.90	5.54	5.46	5.39	5.34	5.31	5.28	5.27	5.25	5.24
.95	10.1	9.55	9.28	9.12	9.01	8.94	8.89	8.85	8.81
.975	17.4	16.0	15.4	15.1	14.9	14.7	14.6	14.5	14.5
.99	34.1	30.8	29.5	28.7	28.2	27.9	27.7	27.5	27.3
.995	55.6	49.8	47.5	46.2	45.4	44.8	44.4	44.1	43.9
.999	167.0	148.5	141.1	137.1	134.6	132.8	131.6	130.6	129.9
4 .50	0.549	0.828	0.941	1.00	1.04	1.06	1.08	1.09	1.10
.90	4.54	4.32	4.19	4.11	4.05	4.01	3.98	3.95	3.94
.95	7.71	6.94	6.59	6.39	6.26	6.16	6.09	6.04	6.00
.975	12.2	10.6	9.98	9.60	9.36	9.20	9.07	8.98	8.90
.99	21.2	18.0	16.7	16.0	15.5	15.2	15.0	14.8	14.7
.995	31.3	26.3	24.3	23.2	22.5	22.0	21.6	21.4	21.1
.999	74.1	61.2	56.2	53.4	51.7	50.5	49.7	49.0	48.5
5 .50	0.528	0.799	0.907	0.965	1.00	1.02	1.04	1.05	1.06
.90	4.06	3.78	3.62	3.52	3.45	3.40	3.37	3.34	3.32
.95	6.61	5.79	5.41	5.19	5.05	4.95	4.88	4.82	4.77
.975	10.0	8.43	7.76	7.39	7.15	6.98	6.85	6.76	6.68
.99	16.3	13.3	12.1	11.4	11.0	10.7	10.5	10.3	10.2
.995	22.8	18.3	16.5	15.6	14.9	14.5	14.2	14.0	13.8
.999	47.2	37.1	33.2	31.1	29.8	28.8	28.2	27.6	27.2
6 .50	0.515	0.780	0.886	0.942	0.977	1.00	1.02	1.03	1.04
.90	3.78	3.46	3.29	3.18	3.11	3.05	3.01	2.98	2.96
.95	5.99	5.14	4.76	4.53	4.39	4.28	4.21	4.15	4.10
.975	8.81	7.26	6.60	6.23	5.99	5.82	5.70	5.60	5.52
.99	13.7	10.9	9.78	9.15	8.75	8.47	8.26	8.10	7.98
.995	18.6	14.5	12.9	12.0	11.5	11.1	10.8	10.6	10.4
.999	35.5	27.0	23.7	21.9	20.8	20.0	19.5	19.0	18.7
7 .50	0.506	0.767	0.871	0.926	0.960	0.983	1.00	1.01	1.02
.90	3.59	3.26	3.07	2.96	2.88	2.83	2.78	2.75	2.72
.95	5.59	4.74	4.35	4.12	3.97	3.87	3.79	3.73	3.68
.975	8.07	6.54	5.89	5.52	5.29	5.12	4.99	4.90	4.82
.99	12.2	9.55	8.45	7.85	7.46	7.19	6.99	6.84	6.72
.995	16.2	12.4	10.9	10.1	9.52	9.16	8.89	8.68	8.51
.999	29.2	21.7	18.8	17.2	16.2	15.5	15.0	14.6	14.3

TABLE A.4 (continued) Percentiles of the F Distribution

Den. df A	Numerator df								
	10	12	15	20	24	30	60	120	∞
1 .50	2.04	2.07	2.09	2.12	2.13	2.15	2.17	2.18	2.20
.90	60.2	60.7	61.2	61.7	62.0	62.3	62.8	63.1	63.3
.95	242	244	246	248	249	250	252	253	254
.975	969	977	985	993	997	1,001	1,010	1,014	1,018
.99	6,056	6,106	6,157	6,209	6,235	6,261	6,313	6,339	6,366
.995	24,234	24,426	24,630	24,836	24,940	25,044	25,253	25,359	25,464
.999	605,620	610,670	615,760	620,910	623,500	626,100	631,340	633,970	636,620
2 .50	1.34	1.36	1.38	1.39	1.40	1.41	1.43	1.43	1.44
.90	9.39	9.41	9.42	9.44	9.45	9.46	9.47	9.48	9.49
.95	19.4	19.4	19.4	19.4	19.5	19.5	19.5	19.5	19.5
.975	39.4	39.4	39.4	39.4	39.5	39.5	39.5	39.5	39.5
.99	99.4	99.4	99.4	99.4	99.5	99.5	99.5	99.5	99.5
.995	199	199	199	199	199	199	199	199	200
.999	999.4	999.4	999.4	999.4	999.5	999.5	999.5	999.5	999.5
3 .50	1.18	1.20	1.21	1.23	1.23	1.24	1.25	1.26	1.27
.90	5.23	5.22	5.20	5.18	5.18	5.17	5.15	5.14	5.13
.95	8.79	8.74	8.70	8.66	8.64	8.62	8.57	8.55	8.53
.975	14.4	14.3	14.3	14.2	14.1	14.1	14.0	13.9	13.9
.99	27.2	27.1	26.9	26.7	26.6	26.5	26.3	26.2	26.1
.995	43.7	43.4	43.1	42.8	42.6	42.5	42.1	42.0	41.8
.999	129.2	128.3	127.4	126.4	125.9	125.4	124.5	124.0	123.5
4 .50	1.11	1.13	1.14	1.15	1.16	1.16	1.18	1.18	1.19
.90	3.92	3.90	3.87	3.84	3.83	3.82	3.79	3.78	3.76
.95	5.96	5.91	5.86	5.80	5.77	5.75	5.69	5.66	5.63
.975	8.84	8.75	8.66	8.56	8.51	8.46	8.36	8.31	8.26
.99	14.5	14.4	14.2	14.0	13.9	13.8	13.7	13.6	13.5
.995	21.0	20.7	20.4	20.2	20.0	19.9	19.6	19.5	19.3
.999	48.1	47.4	46.8	46.1	45.8	45.4	44.7	44.4	44.1
5 .50	1.07	1.09	1.10	1.11	1.12	1.12	1.14	1.14	1.15
.90	3.30	3.27	3.24	3.21	3.19	3.17	3.14	3.12	3.11
.95	4.74	4.68	4.62	4.56	4.53	4.50	4.43	4.40	4.37
.975	6.62	6.52	6.43	6.33	6.28	6.23	6.12	6.07	6.02
.99	10.1	9.89	9.72	9.55	9.47	9.38	9.20	9.11	9.02
.995	13.6	13.4	13.1	12.9	12.8	12.7	12.4	12.3	12.1
.999	26.9	26.4	25.9	25.4	25.1	24.9	24.3	24.1	23.8
6 .50	1.05	1.06	1.07	1.08	1.09	1.10	1.11	1.12	1.12
.90	2.94	2.90	2.87	2.84	2.82	2.80	2.76	2.74	2.72
.95	4.06	4.00	3.94	3.87	3.84	3.81	3.74	3.70	3.67
.975	5.46	5.37	5.27	5.17	5.12	5.07	4.96	4.90	4.85
.99	7.87	7.72	7.56	7.40	7.31	7.23	7.06	6.97	6.88
.995	10.2	10.0	9.81	9.59	9.47	9.36	9.12	9.00	8.88
.999	18.4	18.0	17.6	17.1	16.9	16.7	16.2	16.0	15.7
7 .50	1.03	1.04	1.05	1.07	1.07	1.08	1.09	1.10	1.10
.90	2.70	2.67	2.63	2.59	2.58	2.56	2.51	2.49	2.47
.95	3.64	3.57	3.51	3.44	3.41	3.38	3.30	3.27	3.23
.975	4.76	4.67	4.57	4.47	4.42	4.36	4.25	4.20	4.14
.99	6.62	6.47	6.31	6.16	6.07	5.99	5.82	5.74	5.65
.995	8.38	8.18	7.97	7.75	7.65	7.53	7.31	7.19	7.08
.999	14.1	13.7	13.3	12.9	12.7	12.5	12.1	11.9	11.7

TABLE A.4 (continued) Percentiles of the F Distribution

Den. df	Numerator df								
	1	2	3	4	5	6	7	8	9
8 .50	0.499	0.737	0.860	0.915	0.948	0.971	0.988	1.00	1.01
.90	3.46	3.11	2.92	2.81	2.73	2.67	2.62	2.59	2.56
.95	5.32	4.46	4.07	3.84	3.69	3.58	3.50	3.44	3.39
.975	7.57	6.06	5.42	5.05	4.82	4.65	4.53	4.43	4.36
.99	11.3	8.65	7.59	7.01	6.63	6.37	6.18	6.03	5.91
.995	14.7	11.0	9.60	8.81	8.30	7.95	7.69	7.50	7.34
.999	25.4	18.5	15.8	14.4	13.5	12.9	12.4	12.0	11.8
9 .50	0.494	0.749	0.852	0.906	0.939	0.962	0.978	0.990	1.00
.90	3.36	3.01	2.81	2.69	2.61	2.55	2.51	2.47	2.44
.95	5.12	4.26	3.86	3.63	3.48	3.37	3.29	3.23	3.18
.975	7.21	5.71	5.08	4.72	4.48	4.32	4.20	4.10	4.03
.99	10.6	8.02	6.99	6.42	6.06	5.80	5.61	5.47	5.35
.995	13.6	10.1	8.72	7.96	7.47	7.13	6.88	6.69	6.54
.999	22.9	16.4	13.9	12.6	11.7	11.1	10.7	10.4	10.1
10 .50	0.490	0.743	0.845	0.899	0.932	0.954	0.971	0.983	0.992
.90	3.29	2.92	2.73	2.61	2.52	2.46	2.41	2.38	2.35
.95	4.96	4.10	3.71	3.48	3.33	3.22	3.14	3.07	3.02
.975	6.94	5.46	4.83	4.47	4.24	4.07	3.95	3.85	3.78
.99	10.0	7.36	6.35	5.99	5.64	5.39	5.20	5.06	4.94
.995	12.8	9.43	8.08	7.34	6.87	6.34	6.30	6.12	5.97
.999	21.0	14.9	12.6	11.3	10.5	9.93	9.52	9.20	8.96
12 .50	0.484	0.735	0.835	0.888	0.921	0.943	0.959	0.972	0.981
.90	3.18	2.81	2.61	2.48	2.39	2.33	2.28	2.24	2.21
.95	4.75	3.89	3.49	3.26	3.11	3.00	2.91	2.85	2.80
.975	6.55	5.10	4.47	4.12	3.89	3.73	3.61	3.51	3.44
.99	9.33	6.93	5.95	5.41	5.06	4.82	4.64	4.50	4.39
.995	11.8	8.51	7.23	6.52	6.07	5.76	5.52	5.35	5.20
.999	18.6	13.0	10.8	9.63	8.89	8.38	8.00	7.71	7.48
15 .50	0.478	0.726	0.826	0.878	0.911	0.933	0.949	0.960	0.970
.90	3.07	2.70	2.49	2.36	2.27	2.21	2.16	2.12	2.09
.95	4.54	3.68	3.29	3.06	2.90	2.79	2.71	2.64	2.59
.975	6.20	4.77	4.15	3.80	3.58	3.41	3.29	3.20	3.12
.99	8.68	6.36	5.42	4.89	4.56	4.32	4.14	4.00	3.89
.995	10.8	7.70	6.48	5.30	5.37	5.07	4.85	4.67	4.54
.999	16.6	11.3	9.34	8.25	7.57	7.09	6.74	6.47	6.26
20 .50	0.472	0.718	0.816	0.868	0.900	0.922	0.938	0.950	0.959
.90	2.97	2.59	2.38	2.25	2.16	2.09	2.04	2.00	1.96
.95	4.35	3.49	3.10	2.87	2.71	2.60	2.51	2.45	2.39
.975	5.87	4.46	3.86	3.51	3.29	3.13	3.01	2.91	2.84
.99	8.10	5.85	4.94	4.43	4.10	3.87	3.70	3.56	3.46
.995	9.94	6.99	5.82	5.17	4.76	4.47	4.26	4.09	3.96
.999	14.8	9.95	8.10	7.10	6.46	6.02	5.69	5.44	5.24
24 .50	0.469	0.714	0.812	0.863	0.895	0.917	0.932	0.944	0.953
.90	2.93	2.54	2.33	2.19	2.10	2.04	1.98	1.94	1.91
.95	4.26	3.40	3.01	2.78	2.62	2.51	2.42	2.36	2.30
.975	5.72	4.32	3.72	3.38	3.15	2.99	2.87	2.78	2.70
.99	7.82	5.61	4.72	4.22	3.90	3.67	3.50	3.36	3.26
.995	9.55	6.66	5.52	4.89	4.49	4.20	3.99	3.83	3.69
.999	14.0	9.34	7.55	6.59	5.98	5.55	5.23	4.99	4.80

TABLE A.4 (continued) Percentiles of the F Distribution

Den. df	Numerator df								
	10	12	15	20	24	30	60	120	∞
8 .50	1.02	1.03	1.04	1.05	1.06	1.07	1.08	1.08	1.09
.90	2.54	2.50	2.46	2.42	2.40	2.38	2.34	2.32	2.29
.95	3.35	3.28	3.22	3.15	3.12	3.08	3.01	2.97	2.93
.975	4.30	4.20	4.10	4.00	3.95	3.89	3.78	3.73	3.67
.99	5.81	5.67	5.52	5.36	5.28	5.20	5.03	4.95	4.86
.995	7.21	7.01	6.81	6.61	6.50	6.40	6.18	6.06	5.95
.999	11.5	11.2	10.8	10.5	10.3	10.1	9.73	9.53	9.33
9 .50	1.01	1.02	1.03	1.04	1.05	1.05	1.07	1.07	1.08
.90	2.42	2.38	2.34	2.30	2.28	2.25	2.21	2.18	2.16
.95	3.14	3.07	3.01	2.94	2.90	2.86	2.79	2.75	2.71
.975	3.96	3.87	3.77	3.67	3.61	3.56	3.45	3.39	3.33
.99	5.26	5.11	4.96	4.81	4.73	4.65	4.48	4.40	4.31
.995	6.42	6.23	6.03	5.83	5.73	5.62	5.41	5.30	5.19
.999	9.89	9.57	9.24	8.90	8.72	8.55	8.19	8.00	7.81
10 .50	1.00	1.01	1.02	1.03	1.04	1.05	1.06	1.06	1.07
.90	2.32	2.28	2.24	2.20	2.18	2.16	2.11	2.08	2.06
.95	2.98	2.91	2.84	2.77	2.74	2.70	2.62	2.58	2.54
.975	3.72	3.62	3.52	3.42	3.37	3.31	3.20	3.14	3.08
.99	4.85	4.71	4.56	4.41	4.33	4.25	4.08	4.00	3.91
.995	5.85	5.66	5.47	5.27	5.17	5.07	4.86	4.75	4.64
.999	8.75	8.45	8.13	7.80	7.64	7.47	7.12	6.94	6.76
12 .50	0.989	1.00	1.01	1.02	1.03	1.03	1.05	1.05	1.06
.90	2.19	2.15	2.10	2.06	2.04	2.01	1.96	1.93	1.90
.95	2.75	2.69	2.62	2.54	2.51	2.47	2.38	2.34	2.30
.975	3.37	3.28	3.18	3.07	3.02	2.96	2.85	2.79	2.72
.99	4.30	4.16	4.01	3.86	3.78	3.70	3.54	3.45	3.36
.995	5.09	4.91	4.72	4.53	4.43	4.33	4.12	4.01	3.90
.999	7.29	7.00	6.71	6.40	6.25	6.09	5.76	5.59	5.42
15 .50	0.977	0.989	1.00	1.01	1.02	1.02	1.03	1.04	1.05
.90	2.06	2.02	1.97	1.92	1.90	1.87	1.82	1.79	1.76
.95	2.54	2.48	2.40	2.33	2.29	2.25	2.16	2.11	2.07
.975	3.06	2.96	2.86	2.76	2.70	2.64	2.52	2.46	2.40
.99	3.80	3.67	3.52	3.37	3.29	3.21	3.05	2.96	2.87
.995	4.42	4.25	4.07	3.88	3.79	3.69	3.48	3.37	3.26
.999	6.08	5.81	5.54	5.25	5.10	4.95	4.64	4.48	4.31
20 .50	0.966	0.977	0.989	1.00	1.01	1.01	1.02	1.03	1.03
.90	1.94	1.89	1.84	1.79	1.77	1.74	1.68	1.64	1.61
.95	2.35	2.28	2.20	2.12	2.08	2.04	1.95	1.90	1.84
.975	2.77	2.68	2.57	2.46	2.41	2.35	2.22	2.16	2.09
.99	3.37	3.23	3.09	2.94	2.86	2.78	2.61	2.52	2.42
.995	3.85	3.68	3.50	3.32	3.22	3.12	2.92	2.81	2.69
.999	5.08	4.82	4.56	4.29	4.15	4.00	3.70	3.54	3.38
24 .50	0.961	0.972	0.983	0.994	1.00	1.01	1.02	1.02	1.03
.90	1.88	1.83	1.78	1.73	1.70	1.67	1.61	1.57	1.53
.95	2.25	2.18	2.11	2.03	1.98	1.94	1.84	1.79	1.73
.975	2.64	2.54	2.44	2.33	2.27	2.21	2.08	2.01	1.94
.99	3.17	3.03	2.89	2.74	2.66	2.58	2.40	2.31	2.21
.995	3.59	3.42	3.25	3.06	2.97	2.87	2.66	2.55	2.43
.999	4.64	4.39	4.14	3.87	3.74	3.59	3.29	3.14	2.97

TABLE A.3 (continued) Percentiles of the F Distribution

Den. df	A	Numerator df									
		1	2	3	4	5	6	7	8	9	
30	.50	0.466	0.709	0.807	0.858	0.890	0.912	0.927	0.939	0.948	
	.90	2.88	2.49	2.28	2.14	2.05	1.98	1.93	1.88	1.85	
	.95	4.17	3.32	2.92	2.69	2.53	2.42	2.33	2.27	2.21	
	.975	5.57	4.18	3.59	3.25	3.03	2.87	2.75	2.65	2.57	
	.99	7.56	5.39	4.51	4.02	3.70	3.47	3.30	3.17	3.07	
	.995	9.18	6.35	5.24	4.62	4.23	3.95	3.74	3.58	3.45	
	.999	13.3	8.77	7.05	6.12	5.53	5.12	4.82	4.58	4.39	
	60	.50	0.461	0.701	0.798	0.849	0.880	0.901	0.917	0.928	0.937
		.90	2.79	2.39	2.18	2.04	1.95	1.87	1.82	1.77	1.74
		.95	4.00	3.15	2.76	2.53	2.37	2.25	2.17	2.10	2.04
.975		5.29	3.93	3.34	3.01	2.79	2.63	2.51	2.41	2.33	
.99		7.08	4.98	4.13	3.65	3.34	3.12	2.95	2.82	2.72	
.995		8.49	5.80	4.73	4.14	3.76	3.49	3.29	3.13	3.01	
.999		12.0	7.77	6.17	5.31	4.76	4.37	4.09	3.86	3.69	
120		.50	0.458	0.697	0.793	0.844	0.875	0.896	0.912	0.923	0.932
		.90	2.75	2.35	2.13	1.99	1.90	1.82	1.77	1.72	1.68
		.95	3.92	3.07	2.68	2.45	2.29	2.18	2.09	2.02	1.96
	.975	5.15	3.80	3.23	2.89	2.67	2.52	2.39	2.30	2.22	
	.99	6.85	4.79	3.95	3.48	3.17	2.96	2.79	2.66	2.56	
	.995	8.18	5.54	4.50	3.92	3.55	3.28	3.09	2.93	2.81	
	.999	11.4	7.32	5.78	4.95	4.42	4.04	3.77	3.55	3.38	
	∞	.50	0.455	0.693	0.789	0.839	0.870	0.891	0.907	0.918	0.927
		.90	2.71	2.30	2.08	1.94	1.85	1.77	1.72	1.67	1.63
		.95	3.84	3.00	2.60	2.37	2.21	2.10	2.01	1.94	1.88
.975		5.02	3.69	3.12	2.79	2.57	2.41	2.29	2.19	2.11	
.99		6.63	4.61	3.78	3.32	3.02	2.80	2.64	2.51	2.41	
.995		7.88	5.30	4.28	3.72	3.35	3.09	2.90	2.74	2.62	
.999		10.8	6.91	5.42	4.62	4.10	3.74	3.47	3.27	3.10	

TABLE A.4 (concluded) Percentiles of the F Distribution

Den. df	A	Numerator df									
		10	12	15	20	24	30	60	120	∞	
30	.50	0.955	0.966	0.978	0.989	0.994	1.00	1.01	1.02	1.02	
	.90	1.82	1.77	1.72	1.67	1.64	1.61	1.54	1.50	1.46	
	.95	2.16	2.09	2.01	1.93	1.89	1.84	1.74	1.68	1.62	
	.975	2.51	2.41	2.31	2.20	2.14	2.07	1.94	1.87	1.79	
	.99	2.98	2.84	2.70	2.55	2.47	2.39	2.21	2.11	2.01	
	.995	3.34	3.18	3.01	2.82	2.73	2.63	2.42	2.30	2.18	
	.999	4.24	4.00	3.75	3.49	3.36	3.22	2.92	2.76	2.59	
	60	.50	0.945	0.956	0.967	0.978	0.983	0.989	1.00	1.01	1.01
		.90	1.71	1.66	1.60	1.54	1.51	1.48	1.40	1.35	1.29
		.95	1.99	1.92	1.84	1.75	1.70	1.65	1.53	1.47	1.39
.975		2.27	2.17	2.06	1.94	1.88	1.82	1.67	1.58	1.48	
.99		2.63	2.50	2.35	2.20	2.12	2.03	1.84	1.73	1.60	
.995		2.90	2.74	2.57	2.39	2.29	2.19	1.96	1.83	1.69	
.999		3.54	3.32	3.08	2.83	2.69	2.55	2.25	2.08	1.89	
120		.50	0.939	0.950	0.961	0.972	0.978	0.983	0.994	1.00	1.01
		.90	1.65	1.60	1.55	1.48	1.45	1.41	1.32	1.26	1.19
		.95	1.91	1.83	1.75	1.66	1.61	1.55	1.43	1.35	1.25
	.975	2.16	2.05	1.95	1.82	1.76	1.69	1.53	1.43	1.31	
	.99	2.47	2.34	2.19	2.03	1.95	1.86	1.66	1.53	1.38	
	.995	2.71	2.54	2.37	2.19	2.09	1.98	1.75	1.61	1.43	
	.999	3.24	3.02	2.78	2.53	2.40	2.26	1.95	1.77	1.54	
	∞	.50	0.934	0.945	0.956	0.967	0.972	0.978	0.989	0.994	1.00
		.90	1.60	1.55	1.49	1.42	1.38	1.34	1.24	1.17	1.00
		.95	1.83	1.75	1.67	1.57	1.52	1.46	1.32	1.22	1.00
.975		2.05	1.94	1.83	1.71	1.64	1.57	1.39	1.27	1.00	
.99		2.32	2.18	2.04	1.88	1.79	1.70	1.47	1.32	1.00	
.995		2.52	2.36	2.19	2.00	1.90	1.79	1.53	1.36	1.00	
.999		2.96	2.74	2.51	2.27	2.13	1.99	1.66	1.45	1.00	

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TABLE B.7
Durbin-Watson
Test Bounds.

Level of Significance $\alpha = .05$										
n	p-1=1		p-1=2		p-1=3		p-1=4		p-1=5	
	d_L	d_U	d_L	d_U	d_L	d_U	d_L	d_U	d_L	d_U
15	1.08	1.36	0.95	1.54	0.82	1.75	0.69	1.97	0.56	2.21
16	1.10	1.37	0.98	1.54	0.86	1.73	0.74	1.93	0.62	2.15
17	1.13	1.38	1.02	1.54	0.90	1.71	0.78	1.90	0.67	2.10
18	1.16	1.39	1.05	1.53	0.93	1.69	0.82	1.87	0.71	2.06
19	1.18	1.40	1.08	1.53	0.97	1.68	0.86	1.85	0.75	2.02
20	1.20	1.41	1.10	1.54	1.00	1.68	0.90	1.83	0.79	1.99
21	1.22	1.42	1.13	1.54	1.03	1.67	0.93	1.81	0.83	1.96
22	1.24	1.43	1.15	1.54	1.05	1.66	0.96	1.80	0.86	1.94
23	1.26	1.44	1.17	1.54	1.08	1.66	0.99	1.79	0.90	1.92
24	1.27	1.45	1.19	1.55	1.10	1.66	1.01	1.78	0.93	1.90
25	1.29	1.45	1.21	1.55	1.12	1.66	1.04	1.77	0.95	1.89
26	1.30	1.46	1.22	1.55	1.14	1.65	1.06	1.76	0.98	1.88
27	1.32	1.47	1.24	1.56	1.16	1.65	1.08	1.76	1.01	1.86
28	1.33	1.48	1.26	1.56	1.18	1.65	1.10	1.75	1.03	1.85
29	1.34	1.48	1.27	1.56	1.20	1.65	1.12	1.74	1.05	1.84
30	1.35	1.49	1.28	1.57	1.21	1.65	1.14	1.74	1.07	1.83
31	1.36	1.50	1.30	1.57	1.23	1.65	1.16	1.74	1.09	1.83
32	1.37	1.50	1.31	1.57	1.24	1.65	1.18	1.73	1.11	1.82
33	1.38	1.51	1.32	1.58	1.26	1.65	1.19	1.73	1.13	1.81
34	1.39	1.51	1.33	1.58	1.27	1.65	1.21	1.73	1.15	1.81
35	1.40	1.52	1.34	1.58	1.28	1.65	1.22	1.73	1.16	1.80
36	1.41	1.52	1.35	1.59	1.29	1.65	1.24	1.73	1.18	1.80
37	1.42	1.53	1.36	1.59	1.31	1.66	1.25	1.72	1.19	1.80
38	1.43	1.54	1.37	1.59	1.32	1.66	1.26	1.72	1.21	1.79
39	1.43	1.54	1.38	1.60	1.33	1.66	1.27	1.72	1.22	1.79
40	1.44	1.54	1.39	1.60	1.34	1.66	1.29	1.72	1.23	1.79
45	1.48	1.57	1.43	1.62	1.38	1.67	1.34	1.72	1.29	1.78
50	1.50	1.59	1.46	1.63	1.42	1.67	1.38	1.72	1.34	1.77
55	1.53	1.60	1.49	1.64	1.45	1.68	1.41	1.72	1.38	1.77
60	1.55	1.62	1.51	1.65	1.48	1.69	1.44	1.73	1.41	1.77
65	1.57	1.63	1.54	1.66	1.50	1.70	1.47	1.73	1.44	1.77
70	1.58	1.64	1.55	1.67	1.52	1.70	1.49	1.74	1.46	1.77
75	1.60	1.65	1.57	1.68	1.54	1.71	1.51	1.74	1.49	1.77
80	1.61	1.66	1.59	1.69	1.56	1.72	1.53	1.74	1.51	1.77
85	1.62	1.67	1.60	1.70	1.57	1.72	1.55	1.75	1.52	1.77
90	1.63	1.68	1.61	1.70	1.59	1.73	1.57	1.75	1.54	1.78
95	1.64	1.69	1.62	1.71	1.60	1.73	1.58	1.75	1.56	1.78
100	1.65	1.69	1.63	1.72	1.61	1.74	1.59	1.76	1.57	1.78

TABLE B.7
(concluded)
Durbin-Watson
Test Bounds.

Level of Significance $\alpha = .01$										
n	p-1=1		p-1=2		p-1=3		p-1=4		p-1=5	
	d_L	d_U	d_L	d_U	d_L	d_U	d_L	d_U	d_L	d_U
15	0.81	1.07	0.70	1.25	0.59	1.46	0.49	1.70	0.39	1.96
16	0.84	1.09	0.74	1.25	0.63	1.44	0.53	1.66	0.44	1.90
17	0.87	1.10	0.77	1.25	0.67	1.43	0.57	1.63	0.48	1.85
18	0.90	1.12	0.80	1.26	0.71	1.42	0.61	1.60	0.52	1.80
19	0.93	1.13	0.83	1.26	0.74	1.41	0.65	1.58	0.56	1.77
20	0.95	1.15	0.86	1.27	0.77	1.41	0.68	1.57	0.60	1.74
21	0.97	1.16	0.89	1.27	0.80	1.41	0.72	1.55	0.63	1.71
22	1.00	1.17	0.91	1.28	0.83	1.40	0.75	1.54	0.66	1.69
23	1.02	1.19	0.94	1.29	0.86	1.40	0.77	1.53	0.70	1.67
24	1.04	1.20	0.96	1.30	0.88	1.41	0.80	1.53	0.72	1.66
25	1.05	1.21	0.98	1.30	0.90	1.41	0.83	1.52	0.75	1.65
26	1.07	1.22	1.00	1.31	0.93	1.41	0.85	1.52	0.78	1.64
27	1.09	1.23	1.02	1.32	0.95	1.41	0.88	1.51	0.81	1.63
28	1.10	1.24	1.04	1.32	0.97	1.41	0.90	1.51	0.83	1.62
29	1.12	1.25	1.05	1.33	0.99	1.42	0.92	1.51	0.85	1.61
30	1.13	1.26	1.07	1.34	1.01	1.42	0.94	1.51	0.88	1.61
31	1.15	1.27	1.08	1.34	1.02	1.42	0.96	1.51	0.90	1.60
32	1.16	1.28	1.10	1.35	1.04	1.43	0.98	1.51	0.92	1.60
33	1.17	1.29	1.11	1.36	1.05	1.43	1.00	1.51	0.94	1.59
34	1.18	1.30	1.13	1.36	1.07	1.43	1.01	1.51	0.95	1.59
35	1.19	1.31	1.14	1.37	1.08	1.44	1.03	1.51	0.97	1.59
36	1.21	1.32	1.15	1.38	1.10	1.44	1.04	1.51	0.99	1.59
37	1.22	1.32	1.16	1.38	1.11	1.45	1.06	1.51	1.00	1.59
38	1.23	1.33	1.18	1.39	1.12	1.45	1.07	1.52	1.02	1.58
39	1.24	1.34	1.19	1.39	1.14	1.45	1.09	1.52	1.03	1.58
40	1.25	1.34	1.20	1.40	1.15	1.46	1.10	1.52	1.05	1.58
45	1.29	1.38	1.24	1.42	1.20	1.48	1.16	1.53	1.11	1.58
50	1.32	1.40	1.28	1.45	1.24	1.49	1.20	1.54	1.16	1.59
55	1.36	1.43	1.32	1.47	1.28	1.51	1.25	1.55	1.21	1.59
60	1.38	1.45	1.35	1.48	1.32	1.52	1.28	1.56	1.25	1.60
65	1.41	1.47	1.38	1.50	1.35	1.53	1.31	1.57	1.28	1.61
70	1.43	1.49	1.40	1.52	1.37	1.55	1.34	1.58	1.31	1.61
75	1.45	1.50	1.42	1.53	1.39	1.56	1.37	1.59	1.34	1.62
80	1.47	1.52	1.44	1.54	1.42	1.57	1.39	1.60	1.36	1.62
85	1.48	1.53	1.46	1.55	1.43	1.58	1.41	1.60	1.39	1.63
90	1.50	1.54	1.47	1.56	1.45	1.59	1.43	1.61	1.41	1.64
95	1.51	1.55	1.49	1.57	1.47	1.60	1.45	1.62	1.42	1.64
100	1.52	1.56	1.50	1.58	1.48	1.60	1.46	1.63	1.44	1.65

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