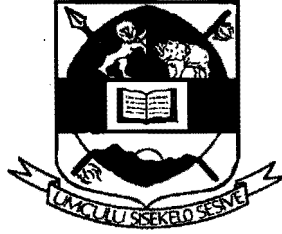


UNIVERSITY OF SWAZILAND



SUPPLEMENTARY EXAMINATION PAPER 2015

TITLE OF PAPER : PROBABILITY THEORY

COURSE CODE : ST 201

TIME ALLOWED : THREE (3) HOURS

INSTRUCTIONS : ANSWER ANY FIVE QUESTIONS.

**REQUIREMENTS : SCIENTIFIC CALCULATOR AND
STATISTICAL TABLES.**

Question 1

A Personal Identification Number (PIN) consists of four digits in order, each of which may be any one of 0, 1, 2, ..., 9.

- a) Find the number of PINs satisfying each of the following requirements.
- (i) All four digits are different.
 - (ii) There are exactly three different digits.
 - (iii) There are two different digits, each of which occurs twice.
 - (iv) There are exactly three digits the same.
- (9 Marks)
- b) Two PINs are chosen independently and at random, and you are given that each PIN consists of four different digits. Let X be the random variable denoting the number of digits that the two PINs have in common.

(i) Explain clearly why $P(X = k) = \frac{\binom{4}{k} \binom{6}{4-k}}{\binom{10}{4}}$, for $k = 0, 1, 2, 3, 4$.

(4 Marks)

- (ii) Hence write down the values of the probability mass function of X , and find its mean and variance.

(7 Marks)

Question 2

The continuous random variable X has probability density function given by

$$f_X(x) = c(1 - x^2), \quad -1 \leq x \leq 1.$$

where c is a suitable constant.

- a) Show that $c = \frac{3}{4}$ and plot the graph of $f_X(x)$ against x .
- b) Show that the cumulative distribution function of X is given by

(6 Marks)

$$F_X(x) = \begin{cases} 0 & , x < -1, \\ \frac{2 + 3x + x^3}{4} & , -1 \leq x < 1, \\ 1 & , x \geq 1. \end{cases}$$

Also find $P(-1/2 \leq X \leq 1/2)$

(6 Marks)

- c) Obtain the mean and variance of X , giving your answer correct to 3 significant figures.

(8 Marks)

Question 3

- a) Consider the sample space

$$\{(a, b, c), (a, c, b), (b, a, c), (b, c, a), (c, a, b), (c, b, a), (a, a, a), (b, b, b), (c, c, c)\}$$

Assign the probability of $1/9$ to each sample point. Let A_i be the event that the i^{th} place in a sample point is occupied by the letter a. Show that the events A_1, A_2, A_3 are pairwise independent but not completely independent.

(8 Marks)

- b) Suppose a rare disease occurs by chance in 1 per 10,000 people. Suppose there is a diagnostic test with the following properties : if a person has the disease, the test will diagnose this correctly with probability 0.95; if a person does not have the disease, the test will diagnose this correctly with probability 0.995.

If the test says that a person has the disease, calculate the probability that this is a correct diagnosis.

(12 Marks)

Question 4

- a) Consider a study in which scientists are interested in finding out the number of children with side effects after a vaccine, out of 200 children. Assume that the probability of having side effect is 5%. Suppose that scientists consider the vaccine "successful" if the number of children affected by side effects is no more than 15. What is the (approximate) probability that the vaccine will be successful?

(4 Marks)

- b) Researchers in Great Britain studied the incidence of childhood leukaemia over a 16-year period and determined that the rate was (approximately) 1.50 children per square mile area and that the numbers followed an approximate Poisson process. What is the probability that more than 3 leukaemia cases will be observed?

(4 Marks)

- c) A firm that produces certain toys and has a demand represented by the function

$$X = \alpha I/P,$$

where P is the unit price, α is a preference parameter equal to 0.5, and I stands for the consumer's income which is uniformly distributed between E1,000 and E2,000 per week.

- i) If total costs (TC) are fixed and equal to E1,000, find the expected profits of this firm.

(5 Marks)

- ii) Find the standard deviation of the Firm's profits.

(7 Marks)

Question 5

- a) The continuous random variable X has probability density function

$$f_X(x) = \frac{1}{\sqrt{2\pi x}} \exp\left\{-\frac{x}{2}\right\}, \quad x > 0.$$

Show that X has moment generating function (mgf)

$$M_X(t) = \frac{1}{\sqrt{(1-2t)}}, \quad t < \frac{1}{2}.$$

Hence find the expected value and variance of X .

(10 Marks)

- b) Suppose that the discrete random variable X has the probability function

$$P(X=x) = (1-\theta)^{x-1} \theta, \quad x = 1, 2, \dots$$

Show that X has moment generating function (mgf)

$$M_X(t) = \frac{e^{t\theta}}{1 - e^{t(1-\theta)}}, \quad t < -\ln(1-\theta).$$

Hence find the expected value and variance of X .

(10 Marks)

Question 6

Suppose the random variables X and Y are independent and are Gamma distributed with parameters (α, λ) and (β, λ) respectively

$$\text{i.e. } f_X(x) = \frac{\lambda^\alpha x^{\alpha-1} \exp(-\lambda x)}{\Gamma(\alpha)}, \quad x \geq 0; \quad \alpha, \lambda > 0,$$

with a similar expression for $f_Y(y)$.

By calculating the joint probability density function of $X+Y$ and X/Y , show that these random variables are independent and that $X+Y$ has the Gamma $(\alpha + \beta, \lambda)$ distribution. Find the probability density function of X/Y . Why are $X+Y$ and Y/X independent? What is the probability density function of Y/X ?

(20 marks)

Question 7

The continuous random variables X and Y have the joint probability density function

$$\frac{\Gamma(\alpha + \beta + \gamma)}{\Gamma(\alpha)\Gamma(\beta)\Gamma(\gamma)} x^{\alpha-1} y^{\beta-1} (1-x-y)^{\gamma-1}, \quad 0 < x < 1, 0 < y < 1, 0 < x+y < 1.$$

where $\alpha > 0, \beta > 0, \gamma > 0$ and $\Gamma(\cdot)$ is the gamma function.

- a) Let r and s be non-negative integers. Show that the expected value of $X^r Y^s$ is

$$E(X^r Y^s) = \frac{\Gamma(\alpha + r)}{\Gamma(\alpha)} \cdot \frac{\Gamma(\beta + s)}{\Gamma(\beta)} \cdot \frac{\Gamma(\alpha + \beta + \gamma)}{\Gamma(\alpha + \beta + \gamma + r + s)}$$

(8 Marks)

- b) Hence determine the expected value and variance of X..

(6 Marks)

- c) Find the correlation between X and Y.

(6 Marks)

Appendix

C

STATISTICAL TABLES

Normal Distribution

Table C-1. Cumulative Probabilities of the Standard Normal Distribution.

Entry is area A under the standard normal curve from $-\infty$ to $z(A)$

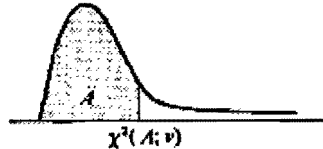


z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998

Chi-Square Distribution

Table C-2. Percentiles of the χ^2 Distribution

Entry is $\chi^2(A; \nu)$ where $P\{\chi^2(\nu) \leq \chi^2(A; \nu)\} = A$



ν	A									
	.005	.010	.025	.050	.100	.900	.950	.975	.990	.995
1	0.00393	0.0157	0.03982	0.07393	0.1538	2.71	3.84	5.02	6.63	7.88
2	0.0100	0.0201	0.0506	0.103	0.211	4.61	5.99	7.38	9.21	10.60
3	0.072	0.115	0.216	0.352	0.584	6.25	7.81	9.35	11.34	12.84
4	0.207	0.297	0.484	0.711	1.064	7.78	9.49	11.14	13.28	14.86
5	0.412	0.554	0.831	1.145	1.61	9.24	11.07	12.83	15.09	16.75
6	0.676	0.872	1.24	1.64	2.20	10.64	12.59	14.45	16.81	18.55
7	0.989	1.24	1.69	2.17	2.83	12.02	14.07	16.01	18.48	20.28
8	1.34	1.65	2.18	2.73	3.49	13.36	15.51	17.53	20.09	21.96
9	1.73	2.09	2.70	3.33	4.17	14.68	16.92	19.02	21.67	23.59
10	2.16	2.56	3.25	3.94	4.87	15.99	18.31	20.48	23.21	25.19
11	2.60	3.05	3.82	4.57	5.58	17.28	19.68	21.92	24.73	26.76
12	3.07	3.57	4.40	5.23	6.30	18.55	21.03	23.34	26.22	28.30
13	3.57	4.11	5.01	5.89	7.04	19.81	22.36	24.74	27.69	29.82
14	4.07	4.66	5.63	6.57	7.79	21.06	23.68	26.12	29.14	31.32
15	4.60	5.23	6.26	7.26	8.55	22.31	25.00	27.49	30.58	32.80
16	5.14	5.81	6.91	7.96	9.31	23.54	26.30	28.85	32.00	34.27
17	5.70	6.41	7.56	8.67	10.09	24.77	27.59	30.19	33.41	35.72
18	6.26	7.01	8.23	9.39	10.86	25.99	28.87	31.53	34.81	37.16
19	6.84	7.63	8.91	10.12	11.65	27.20	30.14	32.85	36.19	38.58
20	7.43	8.26	9.59	10.85	12.44	28.41	31.41	34.17	37.57	40.00
21	8.03	8.90	10.28	11.59	13.24	29.62	32.67	35.48	38.93	41.40
22	8.64	9.54	10.98	12.34	14.04	30.81	33.92	36.78	40.29	42.80
23	9.26	10.20	11.69	13.09	14.85	32.01	35.17	38.08	41.64	44.18
24	9.89	10.86	12.40	13.85	15.66	33.20	36.42	39.36	42.98	45.56
25	10.52	11.52	13.12	14.61	16.47	34.38	37.65	40.65	44.31	46.93
26	11.16	12.20	13.84	15.38	17.29	35.56	38.89	41.92	45.64	48.29
27	11.81	12.88	14.57	16.15	18.11	36.74	40.11	43.19	46.96	49.64
28	12.46	13.56	15.31	16.93	18.94	37.92	41.34	44.46	48.28	50.99
29	13.12	14.26	16.05	17.71	19.77	39.09	42.56	45.72	49.59	52.34
30	13.79	14.95	16.79	18.49	20.60	40.26	43.77	46.98	50.89	53.67
40	20.71	22.16	24.43	26.51	29.05	51.81	55.76	59.34	63.69	66.77
50	27.99	29.71	32.36	34.76	37.69	63.17	67.50	71.42	76.15	79.49
60	35.53	37.48	40.48	43.19	46.46	74.40	79.08	83.30	88.38	91.95
70	43.28	45.44	48.76	51.74	55.33	85.53	90.53	95.02	100.4	104.2
80	51.17	53.54	57.15	60.39	64.28	96.58	101.9	106.6	112.3	116.3
90	59.20	61.75	65.65	69.13	73.29	107.6	113.1	118.1	124.1	128.3
100	67.33	70.06	74.22	77.93	82.36	118.5	124.3	129.6	135.8	140.2

Student's Distribution (t Distribution)

Table C-4 Percentiles of the t Distribution

Entry is $t(A; \nu)$ where $P\{t(\nu) \leq t(A; \nu)\} = A$



ν	A						
	.60	.70	.80	.85	.90	.95	.975
1	0.325	0.727	1.376	1.963	3.078	5.314	12.706
2	0.289	0.617	1.061	1.386	1.886	2.920	4.303
3	0.277	0.584	0.978	1.250	1.638	2.353	3.182
4	0.271	0.569	0.941	1.190	1.533	2.132	2.776
5	0.267	0.559	0.920	1.156	1.476	2.015	2.571
6	0.265	0.553	0.906	1.134	1.440	1.943	2.447
7	0.263	0.549	0.896	1.119	1.415	1.895	2.365
8	0.262	0.546	0.889	1.108	1.397	1.860	2.306
9	0.261	0.543	0.883	1.100	1.383	1.833	2.262
10	0.260	0.542	0.879	1.093	1.372	1.812	2.228
11	0.260	0.540	0.876	1.088	1.363	1.796	2.201
12	0.259	0.539	0.873	1.083	1.356	1.782	2.179
13	0.259	0.537	0.870	1.079	1.350	1.771	2.160
14	0.258	0.537	0.868	1.076	1.345	1.761	2.145
15	0.258	0.536	0.866	1.074	1.341	1.753	2.131
16	0.258	0.535	0.865	1.071	1.337	1.746	2.120
17	0.257	0.534	0.863	1.069	1.333	1.740	2.110
18	0.257	0.534	0.862	1.067	1.330	1.734	2.101
19	0.257	0.533	0.861	1.066	1.328	1.729	2.093
20	0.257	0.533	0.860	1.064	1.325	1.725	2.086
21	0.257	0.532	0.859	1.063	1.323	1.721	2.080
22	0.256	0.532	0.858	1.061	1.321	1.717	2.074
23	0.256	0.532	0.858	1.060	1.319	1.714	2.069
24	0.256	0.531	0.857	1.059	1.318	1.711	2.064
25	0.256	0.531	0.856	1.058	1.316	1.708	2.060
26	0.256	0.531	0.856	1.058	1.315	1.706	2.056
27	0.256	0.531	0.855	1.057	1.314	1.703	2.052
28	0.256	0.530	0.855	1.056	1.313	1.701	2.048
29	0.256	0.530	0.854	1.055	1.311	1.699	2.045
30	0.256	0.530	0.854	1.055	1.310	1.697	2.042
40	0.255	0.529	0.851	1.050	1.303	1.684	2.021
60	0.254	0.527	0.848	1.045	1.296	1.671	2.000
120	0.254	0.526	0.845	1.041	1.289	1.658	1.980
∞	0.253	0.524	0.842	1.036	1.282	1.645	1.960

Table C-4 (Continued) Percentiles of the t Distribution

ν	A						
	.98	.985	.99	.9925	.995	.9975	.9995
1	15.895	21.205	31.821	42.434	63.657	127.322	636.590
2	4.849	5.643	6.965	8.073	9.925	14.089	31.598
3	3.482	3.896	4.541	5.047	5.841	7.453	12.924
4	2.999	3.298	3.747	4.088	4.604	5.598	8.610
5	2.757	3.003	3.365	3.634	4.032	4.773	6.869
6	2.612	2.829	3.143	3.372	3.707	4.317	5.959
7	2.517	2.715	2.998	3.203	3.499	4.029	5.408
8	2.449	2.634	2.896	3.085	3.355	3.833	5.041
9	2.398	2.574	2.821	2.998	3.250	3.690	4.781
10	2.359	2.527	2.764	2.932	3.169	3.581	4.587
11	2.328	2.491	2.718	2.879	3.106	3.497	4.437
12	2.303	2.461	2.681	2.836	3.055	3.428	4.318
13	2.282	2.436	2.650	2.801	3.012	3.372	4.221
14	2.264	2.415	2.624	2.771	2.977	3.326	4.140
15	2.249	2.397	2.602	2.746	2.947	3.286	4.073
16	2.235	2.382	2.583	2.724	2.921	3.252	4.015
17	2.224	2.368	2.567	2.706	2.898	3.222	3.965
18	2.214	2.356	2.552	2.689	2.878	3.197	3.922
19	2.205	2.346	2.539	2.674	2.861	3.174	3.883
20	2.197	2.336	2.528	2.661	2.845	3.153	3.849
21	2.189	2.328	2.518	2.649	2.831	3.135	3.819
22	2.183	2.320	2.508	2.639	2.819	3.119	3.792
23	2.177	2.313	2.500	2.629	2.807	3.104	3.768
24	2.172	2.307	2.492	2.620	2.797	3.091	3.745
25	2.167	2.301	2.485	2.612	2.787	3.078	3.725
26	2.162	2.296	2.479	2.605	2.779	3.067	3.707
27	2.158	2.291	2.473	2.598	2.771	3.057	3.690
28	2.154	2.286	2.467	2.592	2.763	3.047	3.674
29	2.150	2.282	2.462	2.586	2.756	3.038	3.659
30	2.147	2.278	2.457	2.581	2.750	3.030	3.646
40	2.123	2.250	2.423	2.542	2.704	2.971	3.551
60	2.099	2.223	2.390	2.504	2.660	2.915	3.460
120	2.076	2.196	2.358	2.468	2.617	2.860	3.373
∞	2.054	2.170	2.326	2.432	2.576	2.807	3.291

F Distribution

Table C-5 Percentiles of the F Distribution

Entry is $F(A; \nu_1, \nu_2)$ where $P\{F(\nu_1, \nu_2) \leq F(A; \nu_1, \nu_2)\} = A$

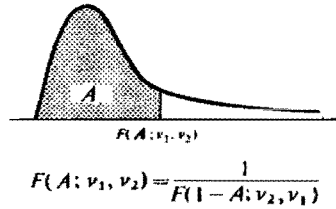


Table C-5 (Continued) Percentiles of the F Distribution

Den. df	A	Numerator df								
		1	2	3	4	5	6	7	8	9
1	.50	1.00	1.50	1.71	1.82	1.89	1.94	1.98	2.00	2.03
	.90	39.9	49.5	53.6	55.8	57.2	58.2	58.9	59.4	59.9
	.95	161	200	216	225	230	234	237	239	241
	.975	648	800	864	900	922	937	948	957	963
	.99	4,052	5,000	5,403	5,625	5,764	5,859	5,928	5,981	6,022
	.995	16,211	20,000	21,615	22,500	23,056	23,437	23,715	23,925	24,091
.999	405,280	500,000	540,380	562,500	576,400	585,940	592,870	598,140	602,280	
2	.50	0.667	1.00	1.13	1.21	1.25	1.28	1.30	1.32	1.33
	.90	8.53	9.00	9.16	9.24	9.29	9.33	9.35	9.37	9.38
	.95	18.5	19.0	19.2	19.2	19.3	19.3	19.4	19.4	19.4
	.975	38.5	39.0	39.2	39.2	39.3	39.3	39.4	39.4	39.4
	.99	98.5	99.0	99.2	99.2	99.3	99.3	99.4	99.4	99.4
	.995	199	199	199	199	199	199	199	199	199
.999	998.5	999.0	999.2	999.2	999.3	999.3	999.4	999.4	999.4	
3	.50	0.585	0.881	1.00	1.06	1.10	1.13	1.15	1.16	1.17
	.90	5.54	5.46	5.39	5.34	5.31	5.28	5.27	5.25	5.24
	.95	10.1	9.55	9.28	9.12	9.01	8.94	8.89	8.85	8.81
	.975	17.4	16.0	15.4	15.1	14.9	14.7	14.6	14.5	14.5
	.99	34.1	30.8	29.5	28.7	28.2	27.9	27.7	27.5	27.3
	.995	55.6	49.8	47.5	46.2	45.4	44.8	44.4	44.1	43.9
.999	167.0	148.5	141.1	137.1	134.6	132.8	131.6	130.6	129.8	
4	.50	0.549	0.828	0.941	1.00	1.04	1.06	1.08	1.09	1.10
	.90	4.54	4.32	4.19	4.11	4.05	4.01	3.98	3.95	3.94
	.95	7.71	6.94	6.59	6.39	6.26	6.16	6.09	6.04	6.00
	.975	12.2	10.6	9.98	9.60	9.36	9.20	9.07	8.98	8.90
	.99	21.2	18.0	16.7	16.0	15.5	15.2	15.0	14.8	14.7
	.995	31.3	26.3	24.3	23.2	22.5	22.0	21.6	21.4	21.1
.999	74.1	61.2	56.2	53.4	51.7	50.5	49.7	49.0	48.5	
5	.50	0.528	0.799	0.907	0.965	1.00	1.02	1.04	1.05	1.06
	.90	4.06	3.78	3.62	3.52	3.45	3.40	3.37	3.34	3.32
	.95	6.61	5.79	5.41	5.19	5.05	4.95	4.88	4.82	4.77
	.975	10.0	8.43	7.76	7.39	7.15	6.98	6.83	6.76	6.68
	.99	16.3	13.3	12.1	11.4	11.0	10.7	10.5	10.3	10.2
	.995	22.8	18.3	16.5	15.6	14.9	14.5	14.2	14.0	13.8
.999	47.2	37.1	33.2	31.1	29.8	28.8	28.2	27.6	27.2	
6	.50	0.515	0.780	0.886	0.942	0.977	1.00	1.02	1.03	1.04
	.90	3.78	3.46	3.29	3.18	3.11	3.05	3.01	2.98	2.96
	.95	5.99	5.14	4.76	4.53	4.39	4.28	4.21	4.15	4.10
	.975	8.81	7.26	6.60	6.23	5.99	5.82	5.70	5.60	5.52
	.99	13.7	10.9	9.78	9.15	8.75	8.47	8.26	8.10	7.98
	.995	18.6	14.5	12.9	12.0	11.5	11.1	10.8	10.6	10.4
.999	35.5	27.0	23.7	21.9	20.8	20.0	19.5	19.0	18.7	
7	.50	0.506	0.767	0.871	0.926	0.960	0.983	1.00	1.01	1.02
	.90	3.59	3.26	3.07	2.96	2.88	2.83	2.78	2.75	2.72
	.95	5.59	4.74	4.35	4.12	3.97	3.87	3.79	3.73	3.68
	.975	8.07	6.54	5.89	5.52	5.29	5.12	4.99	4.90	4.82
	.99	12.2	9.55	8.45	7.85	7.46	7.19	6.99	6.84	6.72
	.995	16.2	12.4	10.9	10.1	9.52	9.16	8.89	8.68	8.51
.999	29.2	21.7	18.8	17.2	16.2	15.5	15.0	14.6	14.3	

Table C-5 (Continued) Percentiles of the F Distribution

Den. df	A	Numerator df								
		10	12	15	20	24	30	60	120	∞
1	.50	2.04	2.07	2.09	2.12	2.13	2.15	2.17	2.18	2.20
	.90	60.2	60.7	61.2	61.7	62.0	62.3	62.8	63.1	63.3
	.95	242	244	246	248	249	250	252	253	254
	.975	969	977	985	993	997	1,001	1,010	1,014	1,018
	.99	6,056	6,106	6,157	6,209	6,235	6,261	6,313	6,339	6,366
	.995	24,224	24,426	24,630	24,836	24,940	25,044	25,253	25,359	25,464
.999	605,620	610,670	615,760	620,910	623,500	626,100	631,340	633,970	636,620	
2	.50	1.34	1.36	1.38	1.39	1.40	1.41	1.43	1.43	1.44
	.90	9.39	9.41	9.42	9.44	9.45	9.46	9.47	9.48	9.49
	.95	19.4	19.4	19.4	19.4	19.5	19.5	19.5	19.5	19.5
	.975	39.4	39.4	39.4	39.4	39.5	39.5	39.5	39.5	39.5
	.99	99.4	99.4	99.4	99.4	99.5	99.5	99.5	99.5	99.5
	.995	199	199	199	199	199	199	199	199	200
.999	999.4	999.4	999.4	999.4	999.5	999.5	999.5	999.5	999.5	
3	.50	1.18	1.20	1.21	1.23	1.23	1.24	1.25	1.26	1.27
	.90	5.23	5.22	5.20	5.18	5.18	5.17	5.15	5.14	5.13
	.95	8.79	8.74	8.70	8.66	8.64	8.62	8.57	8.55	8.53
	.975	14.4	14.3	14.3	14.2	14.1	14.1	14.0	13.9	13.9
	.99	27.2	27.1	26.9	26.7	26.6	26.5	26.3	26.2	26.1
	.995	43.7	43.4	43.1	42.8	42.6	42.5	42.1	42.0	41.8
.999	129.2	128.3	127.4	126.4	125.9	125.4	124.5	124.0	123.5	
4	.50	1.11	1.13	1.14	1.15	1.16	1.16	1.18	1.18	1.19
	.90	3.92	3.90	3.87	3.84	3.83	3.82	3.79	3.78	3.76
	.95	5.96	5.91	5.86	5.80	5.77	5.75	5.69	5.66	5.63
	.975	8.84	8.75	8.66	8.56	8.51	8.46	8.36	8.31	8.26
	.99	14.5	14.4	14.2	14.0	13.9	13.8	13.7	13.6	13.5
	.995	21.0	20.7	20.4	20.2	20.0	19.9	19.6	19.5	19.3
.999	48.1	47.4	46.8	46.1	45.8	45.4	44.7	44.4	44.1	
5	.50	1.07	1.09	1.10	1.11	1.12	1.12	1.14	1.14	1.15
	.90	3.30	3.27	3.24	3.21	3.19	3.17	3.14	3.12	3.11
	.95	4.74	4.68	4.62	4.56	4.53	4.50	4.43	4.40	4.37
	.975	6.62	6.52	6.43	6.33	6.28	6.23	6.12	6.07	6.02
	.99	10.1	9.89	9.72	9.55	9.47	9.38	9.20	9.11	9.02
	.995	13.6	13.4	13.1	12.9	12.8	12.7	12.4	12.3	12.1
.999	26.9	26.4	25.9	25.4	25.1	24.9	24.3	24.1	23.8	
6	.50	1.05	1.06	1.07	1.08	1.09	1.10	1.11	1.12	1.12
	.90	2.94	2.90	2.87	2.84	2.82	2.80	2.76	2.74	2.72
	.95	4.06	4.00	3.94	3.87	3.84	3.81	3.74	3.70	3.67
	.975	5.46	5.37	5.27	5.17	5.12	5.07	4.96	4.90	4.85
	.99	7.87	7.72	7.56	7.40	7.31	7.23	7.06	6.97	6.88
	.995	10.2	10.0	9.81	9.59	9.47	9.36	9.12	9.00	8.88
.999	18.4	18.0	17.6	17.1	16.9	16.7	16.2	16.0	15.7	
7	.50	1.03	1.04	1.05	1.07	1.07	1.08	1.09	1.10	1.10
	.90	2.70	2.67	2.63	2.59	2.58	2.56	2.51	2.49	2.47
	.95	3.64	3.57	3.51	3.44	3.41	3.38	3.30	3.27	3.23
	.975	4.76	4.67	4.57	4.47	4.42	4.36	4.25	4.20	4.14
	.99	6.62	6.47	6.31	6.16	6.07	5.99	5.82	5.74	5.65
	.995	8.38	8.18	7.97	7.75	7.65	7.53	7.31	7.19	7.08
.999	14.1	13.7	13.3	12.9	12.7	12.5	12.1	11.9	11.7	

Table C-5 (Continued) Percentiles of the F Distribution

Den. df	A	Numerator df								
		1	2	3	4	5	6	7	8	9
8	.50	0.499	0.757	0.860	0.915	0.948	0.971	0.988	1.00	1.01
	.90	3.46	3.11	2.92	2.81	2.73	2.67	2.62	2.59	2.56
	.95	5.32	4.46	4.07	3.84	3.69	3.58	3.50	3.44	3.39
	.975	7.57	6.06	5.42	5.05	4.82	4.65	4.53	4.43	4.36
	.99	11.3	8.65	7.59	7.01	6.65	6.37	6.18	6.03	5.91
	.995	14.7	11.0	9.60	8.81	8.30	7.95	7.69	7.50	7.34
.999	25.4	18.5	15.8	14.4	13.5	12.9	12.4	12.0	11.8	
9	.50	0.494	0.749	0.852	0.906	0.939	0.962	0.978	0.990	1.00
	.90	3.36	3.01	2.81	2.69	2.61	2.55	2.51	2.47	2.44
	.95	5.12	4.26	3.86	3.63	3.48	3.37	3.29	3.23	3.18
	.975	7.21	5.71	5.08	4.72	4.48	4.32	4.20	4.10	4.03
	.99	10.6	8.02	6.99	6.42	6.06	5.80	5.61	5.47	5.35
	.995	13.6	10.1	8.72	7.96	7.47	7.13	6.88	6.69	6.54
.999	22.9	16.4	13.9	12.6	11.7	11.1	10.7	10.4	10.1	
10	.50	0.490	0.743	0.845	0.899	0.932	0.954	0.971	0.983	0.992
	.90	3.29	2.92	2.73	2.61	2.52	2.46	2.41	2.38	2.35
	.95	4.96	4.10	3.71	3.48	3.33	3.22	3.14	3.07	3.02
	.975	6.94	5.46	4.83	4.47	4.24	4.07	3.95	3.85	3.78
	.99	10.0	7.56	6.55	5.99	5.64	5.39	5.20	5.06	4.94
	.995	12.8	9.43	8.08	7.34	6.87	6.54	6.30	6.12	5.97
.999	21.0	14.9	12.6	11.3	10.5	9.93	9.52	9.20	8.96	
12	.50	0.484	0.735	0.835	0.888	0.921	0.943	0.959	0.972	0.981
	.90	3.18	2.81	2.61	2.48	2.39	2.33	2.28	2.24	2.21
	.95	4.75	3.89	3.49	3.26	3.11	3.00	2.91	2.85	2.80
	.975	6.55	5.10	4.47	4.12	3.89	3.73	3.61	3.51	3.44
	.99	9.33	6.93	5.95	5.41	5.06	4.82	4.64	4.50	4.39
	.995	11.8	8.51	7.23	6.52	6.07	5.76	5.52	5.35	5.20
.999	18.6	13.0	10.8	9.63	8.89	8.38	8.00	7.71	7.48	
15	.50	0.478	0.726	0.826	0.878	0.911	0.933	0.949	0.960	0.970
	.90	3.07	2.70	2.49	2.36	2.27	2.21	2.16	2.12	2.09
	.95	4.54	3.68	3.29	3.06	2.90	2.79	2.71	2.64	2.59
	.975	6.20	4.77	4.15	3.80	3.58	3.41	3.29	3.20	3.12
	.99	8.68	6.36	5.42	4.89	4.56	4.32	4.14	4.00	3.89
	.995	10.8	7.70	6.48	5.80	5.37	5.07	4.85	4.67	4.54
.999	16.6	11.3	9.34	8.25	7.57	7.09	6.74	6.47	6.26	
20	.50	0.472	0.718	0.816	0.868	0.900	0.922	0.938	0.950	0.959
	.90	2.97	2.59	2.38	2.25	2.16	2.09	2.04	2.00	1.96
	.95	4.35	3.49	3.10	2.87	2.71	2.60	2.51	2.45	2.39
	.975	5.87	4.46	3.86	3.51	3.29	3.13	3.01	2.91	2.84
	.99	8.10	5.85	4.94	4.43	4.10	3.87	3.70	3.56	3.46
	.995	9.94	6.99	5.82	5.17	4.76	4.47	4.26	4.09	3.96
.999	14.8	9.95	8.10	7.10	6.46	6.02	5.69	5.44	5.24	
24	.50	0.469	0.714	0.812	0.863	0.895	0.917	0.932	0.944	0.953
	.90	2.93	2.54	2.33	2.19	2.10	2.04	1.98	1.94	1.91
	.95	4.26	3.40	3.01	2.78	2.62	2.51	2.42	2.36	2.30
	.975	5.72	4.32	3.72	3.38	3.15	2.99	2.87	2.78	2.70
	.99	7.82	5.61	4.72	4.22	3.90	3.67	3.50	3.36	3.26
	.995	9.55	6.66	5.52	4.89	4.49	4.20	3.99	3.83	3.69
.999	14.0	9.34	7.55	6.39	5.98	5.55	5.23	4.99	4.80	

Table C-5 (Continued) Percentiles of the F Distribution

Den. df	Numerator df								
	10	12	15	20	24	30	60	120	∞
8 .50	1.02	1.03	1.04	1.05	1.06	1.07	1.08	1.08	1.09
.90	2.54	2.50	2.46	2.42	2.40	2.38	2.34	2.32	2.29
.95	3.35	3.28	3.22	3.15	3.12	3.08	3.01	2.97	2.93
.975	4.30	4.20	4.10	4.00	3.95	3.89	3.78	3.73	3.67
.99	5.81	5.67	5.52	5.36	5.28	5.20	5.03	4.95	4.86
.995	7.21	7.01	6.81	6.61	6.50	6.40	6.18	6.06	5.95
.999	11.5	11.2	10.8	10.5	10.3	10.1	9.73	9.53	9.33
9 .50	1.01	1.02	1.03	1.04	1.05	1.05	1.07	1.07	1.08
.90	2.42	2.38	2.34	2.30	2.28	2.25	2.21	2.18	2.16
.95	3.14	3.07	3.01	2.94	2.90	2.86	2.79	2.75	2.71
.975	3.96	3.87	3.77	3.67	3.61	3.56	3.45	3.39	3.33
.99	5.26	5.11	4.96	4.81	4.73	4.65	4.48	4.40	4.31
.995	6.42	6.23	6.03	5.83	5.73	5.62	5.41	5.30	5.19
.999	9.89	9.57	9.24	8.90	8.72	8.55	8.19	8.00	7.81
10 .50	1.00	1.01	1.02	1.03	1.04	1.05	1.06	1.06	1.07
.90	2.32	2.28	2.24	2.20	2.18	2.16	2.11	2.08	2.06
.95	2.98	2.91	2.84	2.77	2.74	2.70	2.62	2.58	2.54
.975	3.72	3.62	3.52	3.42	3.37	3.31	3.20	3.14	3.08
.99	4.85	4.71	4.56	4.41	4.33	4.25	4.08	4.00	3.91
.995	5.85	5.66	5.47	5.27	5.17	5.07	4.86	4.75	4.64
.999	8.75	8.45	8.13	7.80	7.64	7.47	7.12	6.94	6.76
12 .50	0.989	1.00	1.01	1.02	1.03	1.03	1.05	1.05	1.06
.90	2.19	2.15	2.10	2.06	2.04	2.01	1.96	1.93	1.90
.95	2.75	2.69	2.62	2.54	2.51	2.47	2.38	2.34	2.30
.975	3.37	3.28	3.18	3.07	3.02	2.96	2.85	2.79	2.72
.99	4.30	4.16	4.01	3.86	3.78	3.70	3.54	3.45	3.36
.995	5.09	4.91	4.72	4.53	4.43	4.33	4.12	4.01	3.90
.999	7.29	7.00	6.71	6.40	6.25	6.09	5.76	5.59	5.42
15 .50	0.977	0.989	1.00	1.01	1.02	1.02	1.03	1.04	1.05
.90	2.06	2.02	1.97	1.92	1.90	1.87	1.82	1.79	1.76
.95	2.54	2.48	2.40	2.33	2.29	2.25	2.16	2.11	2.07
.975	3.06	2.96	2.86	2.76	2.70	2.64	2.52	2.46	2.40
.99	3.80	3.67	3.52	3.37	3.29	3.21	3.05	2.96	2.87
.995	4.42	4.25	4.07	3.88	3.79	3.69	3.48	3.37	3.26
.999	6.08	5.81	5.54	5.25	5.10	4.95	4.64	4.48	4.31
20 .50	0.966	0.977	0.989	1.00	1.01	1.01	1.02	1.03	1.03
.90	1.94	1.89	1.84	1.79	1.77	1.74	1.68	1.64	1.61
.95	2.35	2.28	2.20	2.12	2.08	2.04	1.95	1.90	1.84
.975	2.77	2.68	2.57	2.46	2.41	2.35	2.22	2.16	2.09
.99	3.37	3.23	3.09	2.94	2.86	2.78	2.61	2.52	2.42
.995	3.85	3.68	3.50	3.32	3.22	3.12	2.92	2.81	2.69
.999	5.08	4.82	4.56	4.29	4.15	4.00	3.70	3.54	3.38
24 .50	0.961	0.972	0.983	0.994	1.00	1.01	1.02	1.02	1.03
.90	1.88	1.83	1.78	1.73	1.70	1.67	1.61	1.57	1.53
.95	2.25	2.18	2.11	2.03	1.98	1.94	1.84	1.79	1.73
.975	2.64	2.54	2.44	2.33	2.27	2.21	2.08	2.01	1.94
.99	3.17	3.03	2.89	2.74	2.66	2.58	2.40	2.31	2.21
.995	3.59	3.42	3.25	3.06	2.97	2.87	2.66	2.55	2.43
.999	4.64	4.39	4.14	3.87	3.74	3.59	3.29	3.14	2.97

Table C-5 (Continued) Percentiles of the F Distribution

Den. df	A	Numerator df								
		1	2	3	4	5	6	7	8	9
30 .50	0.466	0.709	0.807	0.858	0.890	0.912	0.927	0.939	0.948	
.90	2.88	2.49	2.28	2.14	2.05	1.98	1.93	1.88	1.85	
.95	4.17	3.32	2.92	2.69	2.53	2.42	2.33	2.27	2.21	
.975	5.57	4.18	3.59	3.25	3.03	2.87	2.75	2.65	2.57	
.99	7.56	5.39	4.51	4.02	3.70	3.47	3.30	3.17	3.07	
.995	9.18	6.35	5.24	4.62	4.23	3.95	3.74	3.58	3.45	
.999	13.3	8.77	7.05	6.12	5.53	5.12	4.82	4.58	4.39	
60 .50	0.461	0.701	0.798	0.849	0.880	0.901	0.917	0.928	0.937	
.90	2.79	2.39	2.18	2.04	1.95	1.87	1.82	1.77	1.74	
.95	4.00	3.15	2.76	2.53	2.37	2.25	2.17	2.10	2.04	
.975	5.29	3.93	3.34	3.01	2.79	2.63	2.51	2.41	2.33	
.99	7.08	4.98	4.13	3.65	3.34	3.12	2.95	2.82	2.72	
.995	8.49	5.80	4.73	4.14	3.76	3.49	3.29	3.13	3.01	
.999	12.0	7.77	6.17	5.31	4.76	4.37	4.09	3.86	3.69	
120 .50	0.458	0.697	0.793	0.844	0.875	0.896	0.912	0.923	0.932	
.90	2.75	2.35	2.13	1.99	1.90	1.82	1.77	1.72	1.68	
.95	3.92	3.07	2.68	2.45	2.29	2.18	2.09	2.02	1.96	
.975	5.15	3.80	3.23	2.89	2.67	2.52	2.39	2.30	2.22	
.99	6.85	4.79	3.95	3.48	3.17	2.96	2.79	2.66	2.56	
.995	8.18	5.54	4.50	3.92	3.55	3.28	3.09	2.93	2.81	
.999	11.4	7.32	5.78	4.95	4.42	4.04	3.77	3.55	3.38	
∞ .50	0.455	0.693	0.789	0.839	0.870	0.891	0.907	0.918	0.927	
.90	2.71	2.30	2.08	1.94	1.85	1.77	1.72	1.67	1.63	
.95	3.84	3.00	2.60	2.37	2.21	2.10	2.01	1.94	1.88	
.975	5.02	3.69	3.12	2.79	2.57	2.41	2.29	2.19	2.11	
.99	6.63	4.61	3.78	3.32	3.02	2.80	2.64	2.51	2.41	
.995	7.88	5.30	4.28	3.72	3.35	3.09	2.90	2.74	2.62	
.999	10.8	6.91	5.42	4.62	4.10	3.74	3.47	3.27	3.10	

Table C-5 (Continued) Percentiles of the F Distribution

Den. df	i	Numerator df								
		10	12	15	20	24	30	60	120	∞
30	.50	0.955	0.966	0.978	0.989	0.994	1.00	1.01	1.02	1.02
	.90	1.82	1.77	1.72	1.67	1.64	1.61	1.54	1.50	1.46
	.95	2.16	2.09	2.01	1.93	1.89	1.84	1.74	1.68	1.62
	.975	2.51	2.41	2.31	2.20	2.14	2.07	1.94	1.87	1.79
	.99	2.98	2.84	2.70	2.55	2.47	2.39	2.21	2.11	2.01
	.995	3.34	3.18	3.01	2.82	2.73	2.63	2.42	2.30	2.18
.999	4.24	4.00	3.75	3.49	3.36	3.22	2.92	2.76	2.59	
60	.50	0.945	0.956	0.967	0.978	0.983	0.989	1.00	1.01	1.01
	.90	1.71	1.66	1.60	1.54	1.51	1.48	1.40	1.35	1.29
	.95	1.99	1.92	1.84	1.75	1.70	1.65	1.53	1.47	1.39
	.975	2.27	2.17	2.06	1.94	1.88	1.82	1.67	1.58	1.48
	.99	2.63	2.50	2.35	2.20	2.12	2.03	1.84	1.73	1.60
	.995	2.90	2.74	2.57	2.39	2.29	2.19	1.96	1.83	1.69
.999	3.54	3.32	3.08	2.83	2.69	2.55	2.25	2.08	1.89	
120	.50	0.939	0.950	0.961	0.972	0.978	0.983	0.994	1.00	1.01
	.90	1.65	1.60	1.55	1.48	1.45	1.41	1.32	1.26	1.19
	.95	1.91	1.83	1.75	1.66	1.61	1.55	1.43	1.35	1.25
	.975	2.16	2.05	1.95	1.82	1.76	1.69	1.53	1.43	1.31
	.99	2.47	2.34	2.19	2.03	1.95	1.86	1.66	1.53	1.38
	.995	2.71	2.54	2.37	2.19	2.09	1.98	1.75	1.61	1.43
.999	3.24	3.02	2.78	2.53	2.40	2.26	1.95	1.77	1.54	
∞	.50	0.934	0.945	0.956	0.967	0.972	0.978	0.989	0.994	1.00
	.90	1.60	1.55	1.49	1.42	1.38	1.34	1.24	1.17	1.00
	.95	1.83	1.75	1.67	1.57	1.52	1.46	1.32	1.22	1.00
	.975	2.05	1.94	1.83	1.71	1.64	1.57	1.39	1.27	1.00
	.99	2.32	2.18	2.04	1.88	1.79	1.70	1.47	1.32	1.00
	.995	2.52	2.36	2.19	2.00	1.90	1.79	1.53	1.36	1.00
.999	2.96	2.74	2.51	2.27	2.13	1.99	1.66	1.45	1.00	