UNIVERSITY OF SWAZILAND

DEPARTMENT OF STATISTICS AND DEMOGRAPHY

MAIN EXAMINATION, 2014/15

COURSE TITLE: MATHETHEMATICS FOR STATISTICS

COURSE CODE: ST 202

TIME ALLOWED: TWO (2) HOURS

INSTRUCTION: ANSWER ANY THREE QUESTIONS
ALL QUESTIONS CARRY EQUAL MARKS (20 MARKS)

SPECIAL REQUIREMENTS: SCIENTIFIC CALCULATORS

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Question 1

The likelihood function of β, given a sample of size n=1 from a Gamma (α, β) distribution (with $\alpha = 2$), is written as

$$L(\beta) = \beta^2 xe^{-\beta x}.$$

(a) What value of β (in terms of x) will maximize $L(\beta)$? **(10 marks)**
[Hint: Keep in mind that the differentiation of $L(\beta)$ is **with respect to** β and that x should be treated as a constant as far as β is concerned.]

(b) The natural logarithm of the likelihood function is called the log-likelihood function. What is the log-likelihood function of β for the Gamma (α, β) density? (Simplify the logarithm as much as possible.) **(5 marks)**

$\ln[L(\beta] = ?$

(c) Now, differentiate $\ln[L(\beta)]$ with respect to β to show that the same value of β that maximizes $L(\beta)$ in part (a) also maximizes $\ln[L(\beta)]$. **(5 marks)**

Question 2

(a) A national toy distributor determines the cost and revenue models for one of its games as:

$$C = 2.4x - 0.0002x^2, 0 \le x \le 6000$$
$$R = 7.2x - 0.001x^2, 0 \le x \le 6000$$

Determine the interval on which the profit function is increasing. **(6 marks)**

(b) Simplify the logarithmic expressions:
(i) $3\ln 2 - 2\ln(x-1)$,
(ii) $2\ln x + \ln y - 3\ln(z+4)$, **(3 marks)**

(c) Find the derivative of the functions:
(i) $y = e^{-3x} + 5$,
(ii) $y = \ln\frac{5x}{x+2}$, **(4 marks)**

(c) Find the second derivative of the function

$$f(x) = x \ln\sqrt{x} + 2x$$ **(3 marks)**

(d) Find the first partial derivatives of $f(x, y) = xe^{xy}$, and evaluate it at the point (1, ln2) **(4 marks)**

Question 3

(a) Evaluate the function:

$$\int xe^{x^2}\,dx$$ **(3 marks)**

(b)Let x and y be two continuous random variables having the joint probability density function given by:

$$f(x,y)=\begin{cases}24xy, 0<x<1, 0<y<1, x+y<1\\ 0, elsewhere\end{cases},$$

Find $P(x>\frac{1}{2}, y<\frac{3}{4})$, **(5 marks)**

(c) If X is Gamma distributed with α=2 and β=3, the probability density function for x will be given by:

$$f(x)=\frac{1}{9}xe^{-x/3}, for x>0$$

(i)Determine the expected value and standard deviation of the distribution. **(8 marks)**
(ii) Find $P(x>4)$ **(4 marks)**

Question 4

(a) Find A^{-1} for the following matrix:

$$A=\begin{bmatrix}1 & 0 & 4\\ 4 & 1 & -2\\ 3 & 1 & -1\end{bmatrix}$$ **(7 marks)**

(b) Solve the following linear system of equations using the method of determinants:

$$\begin{aligned} x + \quad 4z &= 4\\ 4x+y-2z &= 0\\ 3x+y-z &= 2 \end{aligned}$$ **(7 marks)**

(c) Suppose $x=\begin{bmatrix}2\\0\\-4\end{bmatrix}$ and $y=\begin{bmatrix}0\\-1\\-3\end{bmatrix}$

Use vector algebra to find the least squares regression line through the set of points determined by vectors x and y. **(6 marks)**

Question 5

(a) Find eigenvalues and eigenvectors of matrix $A = \begin{bmatrix} 0 & 0 & -2 \\ 1 & 2 & 1 \\ 1 & 0 & 3 \end{bmatrix}$

(10 marks)

(b) Solve the following system of equations using the Gauss-Jordan elimination method:

$$\begin{aligned} x + y + 2z &= 9 \\ 2x + 4y - 3z &= 1 \\ 3x + 6y - 5z &= 0 \end{aligned}$$

(10 marks)