UNIVERSITY OF SWAZILAND

DEPARTMENT OF STATISTICS AND DEMOGRAPHY

SUPPLEMENTARY EXAMINATION, 2014/15

COURSE TITLE: MATHETHEMATICS FOR STATISTICS

COURSE CODE: ST 202

TIME ALLOWED: TWO (2) HOURS

INSTRUCTION: ANSWER ANY THREE QUESTIONS
ALL QUESTIONS CARRY EQUAL MARKS (20 MARKS)

SPECIAL REQUIREMENTS: SCIENTIFIC CALCULATORS

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Question 1

Integrate the following functions: **(5+5+5+5 marks)**

(a) $\int x^3(3x^4+1)^2dx$

(b) $\int x\sqrt{x^2+4}dx$

(c) $\int \frac{3x^2}{\sqrt{1-x^3}}dx$

(d) $\int \frac{3(3x^2+4x)}{x^3+2x^2}dx$

Question 2

If the joint probability density function of X and Y is given by:

$$f(x,y)=\begin{cases}\frac{1}{3}(x+y), for 0<x<1, 0<y<2 \\ 0, elsewhere\end{cases}$$

(a) Find the mean and variance of X. **(10 marks)**

(b) Find the mean and variance of Y. **(10 marks)**

Question 3

(a) Suppose that the service life in hours of a semiconductor is a random variable having a Weibull distribution with α=0.025 and β= 0.50. The probability density function is given by:

$$f(x,y)=\begin{cases}0.0125x^{-0.5}e^{-0.025x^{0.5}}\ for x>0\\ 0, elsewhere\end{cases}$$

Find $P(x>4{,}000)$ **(10 marks)**

(a) Suppose that a random variable X has a Gamma distribution with α=3 and β=4. The probability density function for this random variable X is given by:

$$f(x)=\begin{cases}\frac{1}{128}x^2e^{-x/4},\ for x>0\\ 0, elsewhere\end{cases}$$

Find the probability that the value of the random variable will exceed 4. **(10 marks)**

Question 4

(a) The probability density function of a random variable X is given by:

$$f(x)=\begin{cases}\frac{c}{\sqrt{x}},\ for 0<x<4\\ 0, elsewhere\end{cases}$$

Find

(i) the value of c;

(ii) $P(X<\frac{1}{4}) and P(X>1)$ **(10 marks)**

(b) **State** the Mean value Theorem. Determine the intervals on the x-axis on which the function:

$f(x)=3x^4-4x^3-12x^2+5$ is increasing and the intervals on which it is decreasing

(10 marks)

Question 5

(a) Find eigenvalues and eigenvectors of matrix $A = \begin{bmatrix} 0 & 0 & -2 \\ 1 & 2 & 1 \\ 1 & 0 & 3 \end{bmatrix}$

(10 marks)

(b) Solve the following system of equations using the Gauss-Jordan elimination method:

$$\begin{aligned} x + 2y &= 1 \\ 2x + y + 2z &= -1 \\ -x + 3y &= 4 \end{aligned}$$

(10 marks)