

**UNIVERSITY OF SWAZILAND**

**FINAL EXAMINATION PAPER 2015**

**TITLE OF PAPER : LINEAR STATISTICAL METHODS**

**COURSE CODE : ST204**

**TIME ALLOWED : 2 (TWO) HOURS**

**REQUIRMENTS : STATISTICAL TABLES  
AND CALCULATOR**

**INSTRUCTIONS : ANSWER ANY 4 (FOUR) QUESTIONS.  
ALL QUESTIONS CARRY EQUAL MARKS.**

**THIS PAPER IS NOT TO BE OPENED UNTIL PERMISSION HAS  
BEEN GRANTED BY THE INVIGILATOR**

**QUESTION ONE.**

[ 6 + 4 + 3 + 6 + 5 + 1 marks ]

- 1.1 State and discuss the important features of the single factor ANOVA model.
- 1.2 State the hypothesis used in the above model and the hypothesis used in a regression model analysed by ANOVA. Discuss the purpose of the tests in these two cases.
- 1.3 A researcher wishes to see whether there is any difference in the weight gains of athletes following one of three special diets. Athletes are randomly assigned to three groups and placed on the diet for six weeks. The weight gains (in pounds) are shown below:

<u>Diet A</u>	<u>Diet B</u>	<u>Diet C</u>
3	10	8
6	12	3
7	11	2
4	14	5
	8	
	6	

- a. Identify the dependent variable, factor studied and factor levels.
- b. Complete the computation of the ANOVA table.
- c. Can the researcher conclude that there is a difference in the diets? Clearly state all the steps in the test including the conclusion. Use 5% level of significance.
- d. Which diet would you prefer for the athletes? Explain.

**QUESTION TWO.**

[ 7 + 6 + 6 + 3 + 3 marks ]

- 2.1 State the Simple Linear Regression Model with distribution of error terms unspecified. There are three assumptions in the correct statement, identify those one by one.
- 2.2 Discuss the important features of the model.
- 2.3 Show that the least square estimate of the regression coefficient,  $\beta_1$  as follows:

$$\hat{\beta}_1 = \frac{n \sum X_i Y_i - \sum X_i \sum Y_i}{n \sum X_i^2 - (\sum X_i)^2}$$

- 2.4 Assume that  $X = 0$  is within the scope of the model defined in part 2.1. What is the implication for the regression function if  $\beta_0 = 0$ ?
- 2.5 What is the implication for the regression function if  $\beta_1 = 0$  in the model defined in part 2.1?

**QUESTION THREE.**

[ 1 + 5 + 2 + 1 + 3 + 5 + 5 + 3 marks ]

A study was conducted at a large hospital to determine whether there was a relationship between the number of years nurses were employed and the number of nurses who voluntarily resign. Twenty-nine nurses resigned voluntarily in year 2014; the study results were classified by the year employed as shown below:

<b>Years employed</b>	5	6	3	7	8	2
<b>No. of resignations</b>	7	4	7	1	2	8

- Identify the response variable and the predictor variable.
- Fit the regression line,  $Y_i = \beta_0 + \beta_1 X_i$ .
- Interpret the estimated values of  $\beta_0$  and  $\beta_1$ .
- How many nurses may voluntarily resign after four years of employment?
- Estimate  $\sigma^2$ .
- Construct a 98% confidence interval for  $\beta_1$ .
- Test  $\beta_1 = 0$  against  $\beta_1 < 0$  at  $\alpha = 0.05$
- Compute the coefficient of correlation and interpret the value.

**QUESTION FOUR.**

[ 5 + 4 + 3 + 2 + 5 + 3 + 3 marks ]

- State the *Cell Means Model* and the *Factor Effects Model* for two-factor studies.
- List, both,  $H_0$  and  $H_1$  of the F-test when using ANOVA table for each of those two models. Also mention the purpose of those tests.
- A researcher wishes to see whether the type of gasoline used and the type of automobile driven have any effect on gasoline consumption. The data (in miles per gallon) are shown below:

Type of Gasoline	Type of Automobile	
	Two-Wheel	Four-Wheel
Regular	26.7	28.6
	25.2	29.3
High-Octane	32.3	26.1
	32.8	24.2

- Identify the dependent variable, factor A and factor B studied and their factor levels.
- How many treatments are there in this experiment? List all.
- Given SS due to Gasoline = 3.920, SS due to Automobile = 9.680, SS due to interaction = 54.080. Compute only total SS and complete the ANOVA table.
- Use the 0.05 level in determining whether the treatments have any effect on the gasoline consumption.
- Use the 0.05 level in determining whether the combination of type of gasoline and type of automobile have any effect on the gasoline consumption.

**QUESTION FIVE.**

[ 2 + 1 + 4 + 5 + 5 + 6 + 2 marks ]

The director of admissions of a small college administered a newly designed entrance test to 16 students selected at random from the new year I class in a study to determine whether a student's grade point average (GPA) at the end of the year I can be predicted from the entrance test score. The following two tables are a part of the output; obtained by running the model,  $Y_i = \beta_0 + \beta_1 X_i + \varepsilon_i$  using SPSS:

ANOVA

Model		Sum of Squares	df	Mean Square	F	Sig.
1	Regression	5.346	1	5.346	16.188	.001 <sup>a</sup>
	Residual	4.624	14	.330		
	Total	9.970	15			

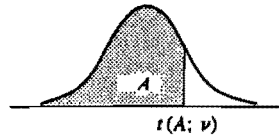
Coefficients

Model		Unstandardized Coefficients		Standardized Coefficients	t	Sig.
		B	Std. Error	Beta		
1	(Constant)	-1.517	1.076		-1.409	.181
	x	.834	.207	.732	4.023	.001

- 5.1 Find the fitted regression line.
- 5.2 Obtain a point estimate of the mean GPA for students with entrance test score of 6.
- 5.3 State the null and alternative hypotheses for the Goodness-of-Fit test of the said model and explain clearly the conclusion of the test.
- 5.4 Test  $\beta_0 = 0$  against  $\beta_0 \neq 0$  at  $\alpha = 0.01$ . Is it worth to test?
- 5.5 Test  $\beta_1 = 1$  against  $\beta_1 < 1$  at  $\alpha = 0.05$ .
- 5.6 Construct a 95% confidence interval for  $\beta_0$  and a 99% confidence interval for  $\beta_1$ .
- 5.7 Compute the coefficient of determination and interpret the value.

TABLE A.2 Percentiles of the *t* Distribution

Entry is  $t(A; \nu)$  where  $P\{t(\nu) \leq t(A; \nu)\} = A$



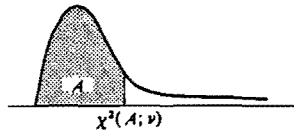
$\nu$	A						
	.60	.70	.80	.85	.90	.95	.975
1	0.325	0.727	1.376	1.963	3.078	6.314	12.706
2	0.289	0.617	1.061	1.386	1.886	2.920	4.303
3	0.277	0.584	0.978	1.250	1.638	2.353	3.182
4	0.271	0.569	0.941	1.190	1.533	2.132	2.776
5	0.267	0.559	0.920	1.156	1.476	2.015	2.571
6	0.265	0.553	0.906	1.134	1.440	1.943	2.447
7	0.263	0.549	0.896	1.119	1.415	1.895	2.365
8	0.262	0.546	0.889	1.108	1.397	1.860	2.306
9	0.261	0.543	0.883	1.100	1.383	1.833	2.262
10	0.260	0.542	0.879	1.093	1.372	1.812	2.228
11	0.260	0.540	0.876	1.088	1.363	1.796	2.201
12	0.259	0.539	0.873	1.083	1.356	1.782	2.179
13	0.259	0.537	0.870	1.079	1.350	1.771	2.160
14	0.258	0.537	0.868	1.076	1.345	1.761	2.145
15	0.258	0.536	0.866	1.074	1.341	1.753	2.131
16	0.258	0.535	0.865	1.071	1.337	1.746	2.120
17	0.257	0.534	0.863	1.069	1.333	1.740	2.110
18	0.257	0.534	0.862	1.067	1.330	1.734	2.101
19	0.257	0.533	0.861	1.066	1.328	1.729	2.093
20	0.257	0.533	0.860	1.064	1.325	1.725	2.086
21	0.257	0.532	0.859	1.063	1.323	1.721	2.080
22	0.256	0.532	0.858	1.061	1.321	1.717	2.074
23	0.256	0.532	0.858	1.060	1.319	1.714	2.069
24	0.256	0.531	0.857	1.059	1.318	1.711	2.064
25	0.256	0.531	0.856	1.058	1.316	1.708	2.060
26	0.256	0.531	0.856	1.058	1.315	1.706	2.056
27	0.256	0.531	0.855	1.057	1.314	1.703	2.052
28	0.256	0.530	0.855	1.056	1.313	1.701	2.048
29	0.256	0.530	0.854	1.055	1.311	1.699	2.045
30	0.256	0.530	0.854	1.055	1.310	1.697	2.042
40	0.255	0.529	0.851	1.050	1.303	1.684	2.021
60	0.254	0.527	0.848	1.045	1.296	1.671	2.000
120	0.254	0.526	0.845	1.041	1.289	1.658	1.980
$\infty$	0.253	0.524	0.842	1.036	1.282	1.645	1.960

TABLE A.2 (concluded) Percentiles of the *t* Distribution

$\nu$	A						
	.98	.985	.99	.9925	.995	.9975	.9995
1	15.895	21.205	31.821	42.434	63.657	127.322	636.590
2	4.849	5.643	6.965	8.073	9.925	14.089	31.598
3	3.482	3.896	4.541	5.047	5.841	7.453	12.924
4	2.999	3.298	3.747	4.088	4.604	5.598	8.610
5	2.757	3.003	3.365	3.634	4.032	4.773	6.869
6	2.612	2.829	3.143	3.372	3.707	4.317	5.959
7	2.517	2.715	2.998	3.203	3.499	4.029	5.408
8	2.449	2.634	2.896	3.085	3.355	3.833	5.041
9	2.398	2.574	2.821	2.998	3.250	3.690	4.781
10	2.359	2.527	2.764	2.932	3.169	3.581	4.587
11	2.328	2.491	2.718	2.879	3.106	3.497	4.437
12	2.303	2.461	2.681	2.836	3.055	3.428	4.318
13	2.282	2.436	2.650	2.801	3.012	3.372	4.221
14	2.264	2.415	2.624	2.771	2.977	3.326	4.140
15	2.249	2.397	2.602	2.746	2.947	3.286	4.073
16	2.235	2.382	2.583	2.724	2.921	3.252	4.015
17	2.224	2.368	2.567	2.706	2.898	3.222	3.965
18	2.214	2.356	2.552	2.689	2.878	3.197	3.922
19	2.205	2.346	2.539	2.674	2.861	3.174	3.883
20	2.197	2.336	2.528	2.661	2.845	3.153	3.849
21	2.189	2.328	2.518	2.649	2.831	3.135	3.819
22	2.183	2.320	2.508	2.639	2.819	3.119	3.792
23	2.177	2.313	2.500	2.629	2.807	3.104	3.768
24	2.172	2.307	2.492	2.620	2.797	3.091	3.745
25	2.167	2.301	2.485	2.612	2.787	3.078	3.725
26	2.162	2.296	2.479	2.605	2.779	3.067	3.707
27	2.158	2.291	2.473	2.598	2.771	3.057	3.690
28	2.154	2.286	2.467	2.592	2.763	3.047	3.674
29	2.150	2.282	2.462	2.586	2.756	3.038	3.659
30	2.147	2.278	2.457	2.581	2.750	3.030	3.646
40	2.123	2.250	2.423	2.542	2.704	2.971	3.551
60	2.099	2.223	2.390	2.504	2.660	2.915	3.460
120	2.076	2.196	2.358	2.468	2.617	2.860	3.373
$\infty$	2.054	2.170	2.326	2.432	2.576	2.807	3.291

TABLE A.3 Percentiles of the  $\chi^2$  Distribution

Entry is  $\chi^2(A; \nu)$  where  $P\{\chi^2(\nu) \leq \chi^2(A; \nu)\} = A$

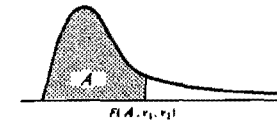


$\nu$	A									
	.005	.010	.025	.050	.100	.900	.950	.975	.990	.995
1	0.004393	0.01157	0.02982	0.07393	0.2158	2.71	3.84	5.02	6.63	7.88
2	0.0100	0.0201	0.0506	0.103	0.211	4.61	5.99	7.38	9.21	10.60
3	0.072	0.115	0.216	0.352	0.584	6.25	7.81	9.35	11.34	12.84
4	0.207	0.297	0.484	0.711	1.064	7.78	9.49	11.14	13.28	14.86
5	0.412	0.554	0.831	1.145	1.61	9.24	11.07	12.83	14.99	16.75
6	0.676	0.872	1.24	1.64	2.20	10.64	12.59	14.45	16.81	18.55
7	0.989	1.24	1.69	2.17	2.83	12.02	14.07	16.01	18.48	20.28
8	1.34	1.65	2.18	2.73	3.49	13.36	15.51	17.53	20.09	21.96
9	1.73	2.09	2.70	3.33	4.17	14.68	16.92	19.02	21.67	23.59
10	2.16	2.56	3.25	3.94	4.87	15.99	18.31	20.48	23.21	25.19
11	2.60	3.05	3.82	4.57	5.58	17.28	19.68	21.92	24.73	26.76
12	3.07	3.57	4.40	5.23	6.30	18.55	21.03	23.34	26.22	28.30
13	3.57	4.11	5.01	5.89	7.04	19.81	22.36	24.74	27.69	29.82
14	4.07	4.66	5.63	6.57	7.79	21.06	23.68	26.12	29.14	31.32
15	4.60	5.23	6.26	7.26	8.55	22.31	25.00	27.49	30.58	32.80
16	5.14	5.81	6.91	7.96	9.31	23.54	26.30	28.85	32.00	34.27
17	5.70	6.41	7.56	8.67	10.09	24.77	27.59	30.19	33.41	35.72
18	6.26	7.01	8.23	9.39	10.86	25.99	28.87	31.53	34.81	37.16
19	6.84	7.63	8.91	10.12	11.65	27.20	30.14	32.85	36.19	38.58
20	7.43	8.26	9.59	10.85	12.44	28.41	31.41	34.17	37.57	40.00
21	8.03	8.90	10.28	11.59	13.24	29.62	32.67	35.48	38.93	41.40
22	8.64	9.54	10.98	12.34	14.04	30.81	33.92	36.78	40.29	42.80
23	9.26	10.20	11.69	13.09	14.85	32.01	35.17	38.08	41.64	44.18
24	9.89	10.86	12.40	13.85	15.66	33.20	36.42	39.36	42.98	45.56
25	10.52	11.52	13.12	14.61	16.47	34.38	37.65	40.65	44.31	46.93
26	11.16	12.20	13.84	15.38	17.29	35.56	38.89	41.92	45.64	48.29
27	11.81	12.88	14.57	16.15	18.11	36.74	40.11	43.19	46.96	49.64
28	12.46	13.56	15.31	16.93	18.94	37.92	41.34	44.46	48.28	50.99
29	13.12	14.26	16.05	17.71	19.77	39.09	42.56	45.72	49.59	52.34
30	13.79	14.95	16.79	18.49	20.60	40.26	43.77	46.98	50.89	53.67
40	20.71	22.16	24.43	26.51	29.05	51.81	55.76	59.34	63.69	66.77
50	27.99	29.71	32.36	34.76	37.69	63.17	67.50	71.42	76.15	79.49
60	35.53	37.48	40.48	43.19	46.46	74.40	79.08	83.30	88.38	91.95
70	43.28	45.44	48.76	51.74	55.33	85.53	90.53	95.02	100.4	104.2
80	51.17	53.54	57.15	60.39	64.28	96.58	101.9	106.6	112.3	116.3
90	59.20	61.75	65.65	69.13	73.29	107.6	113.1	118.1	124.1	128.3
100	67.33	70.06	74.22	77.93	82.36	118.5	124.3	129.6	135.8	140.2

Source: Reprinted, with permission, from C. M. Thompson, "Table of Percentage Points of the Chi-Square Distribution," *Biometrika* 32 (1941), pp. 188-89.

TABLE A.4 Percentiles of the F Distribution

Entry is  $F(A; \nu_1, \nu_2)$  where  $P\{F(\nu_1, \nu_2) \leq F(A; \nu_1, \nu_2)\} = A$



$$F(A; \nu_1, \nu_2) = \frac{1}{F(1-A; \nu_2, \nu_1)}$$

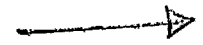








TABLE A.4 (continued) Percentiles of the F Distribution

Den. df	A	Numerator df								
		1	2	3	4	5	6	7	8	9
30	.50	0.466	0.709	0.807	0.858	0.890	0.912	0.927	0.939	0.948
	.90	2.88	2.49	2.28	2.14	2.05	1.98	1.93	1.88	1.85
	.95	4.17	3.32	2.92	2.69	2.53	2.42	2.33	2.27	2.21
	.975	5.57	4.18	3.59	3.25	3.03	2.87	2.75	2.65	2.57
	.99	7.56	5.39	4.51	4.02	3.70	3.47	3.30	3.17	3.07
60	.50	0.461	0.701	0.798	0.849	0.880	0.901	0.917	0.928	0.937
	.90	2.79	2.39	2.18	2.04	1.95	1.87	1.82	1.77	1.74
	.95	4.00	3.15	2.76	2.53	2.37	2.25	2.17	2.10	2.04
	.975	5.29	3.93	3.34	3.01	2.79	2.63	2.51	2.41	2.33
	.99	7.08	4.98	4.13	3.65	3.34	3.12	2.95	2.82	2.72
120	.50	0.458	0.697	0.793	0.844	0.875	0.896	0.912	0.923	0.932
	.90	2.75	2.35	2.13	1.99	1.90	1.82	1.77	1.72	1.68
	.95	3.92	3.07	2.68	2.45	2.29	2.18	2.09	2.02	1.96
	.975	5.15	3.80	3.23	2.89	2.67	2.52	2.39	2.30	2.22
	.99	6.85	4.79	3.95	3.48	3.17	2.96	2.79	2.66	2.56
∞	.50	0.455	0.693	0.789	0.839	0.870	0.891	0.907	0.918	0.927
	.90	2.71	2.30	2.08	1.94	1.85	1.77	1.72	1.67	1.63
	.95	3.84	3.00	2.60	2.37	2.21	2.10	2.01	1.94	1.88
	.975	5.02	3.69	3.12	2.79	2.57	2.41	2.29	2.19	2.11
	.99	6.63	4.61	3.78	3.32	3.02	2.80	2.64	2.51	2.41

TABLE A.4 (concluded) Percentiles of the F Distribution

Den. df	A	Numerator df								
		10	12	15	20	24	30	60	120	∞
30	.50	0.955	0.966	0.978	0.989	0.994	1.00	1.01	1.02	1.02
	.90	1.82	1.77	1.72	1.67	1.64	1.61	1.54	1.50	1.46
	.95	2.16	2.09	2.01	1.93	1.89	1.84	1.74	1.68	1.62
	.975	2.51	2.41	2.31	2.20	2.14	2.07	1.94	1.87	1.79
	.99	2.98	2.84	2.70	2.55	2.47	2.39	2.21	2.11	2.01
60	.50	0.945	0.956	0.967	0.978	0.983	0.989	1.00	1.01	1.01
	.90	1.71	1.66	1.60	1.54	1.51	1.48	1.40	1.35	1.29
	.95	1.99	1.92	1.84	1.75	1.70	1.65	1.53	1.47	1.39
	.975	2.27	2.17	2.06	1.94	1.88	1.82	1.67	1.58	1.48
	.99	2.63	2.50	2.35	2.20	2.12	2.03	1.84	1.73	1.60
120	.50	0.939	0.950	0.961	0.972	0.978	0.983	0.994	1.00	1.01
	.90	1.65	1.60	1.55	1.48	1.45	1.41	1.32	1.26	1.19
	.95	1.91	1.83	1.75	1.66	1.61	1.55	1.43	1.35	1.25
	.975	2.16	2.05	1.95	1.82	1.76	1.69	1.53	1.43	1.31
	.99	2.47	2.34	2.19	2.03	1.95	1.86	1.66	1.53	1.38
∞	.50	0.934	0.945	0.956	0.967	0.972	0.978	0.989	0.994	1.00
	.90	1.60	1.55	1.49	1.42	1.38	1.34	1.24	1.17	1.00
	.95	1.83	1.75	1.67	1.57	1.52	1.46	1.32	1.22	1.00
	.975	2.05	1.94	1.83	1.71	1.64	1.57	1.39	1.27	1.00
	.99	2.32	2.18	2.04	1.88	1.79	1.70	1.47	1.32	1.00

Source: Reprinted from Table 5 of Pearson and Hartley, *Biometrika Tables for Statisticians*, Volume 2, 1972, published by the Cambridge University Press, on behalf of The Biometrika Society, by permission of the authors and publishers.