

UNIVERSITY OF SWAZILAND



MAIN EXAMINATION PAPER 2015

TITLE OF PAPER : STATISTICAL INFERENCE II

COURSE CODE : ST 303

TIME ALLOWED : TWO (2) HOURS

REQUIREMENTS : CALCULATOR AND STATISTICAL TABLES

INSTRUCTIONS : ANSWER ANY THREE QUESTIONS

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Question 1

Let X_1, \dots, X_n be *iid* with common pdf

$$f(x)_\theta = \theta/(1+x)^{\theta+1}$$

for $x > 0$, where $\theta > 0$ is an unknown parameter.

Find the sufficient statistic $T(X)$, the maximum likelihood estimator, and the Cramer-Rao lower bound.

(6 + 7 + 7 Marks)

Question 2

Find the minimal sufficient statistic for θ based on an independent sample of size n from each of the following distributions:

- a) The gamma distribution with density
- b)

$$f(x; \alpha, \beta) = \frac{\beta^\alpha x^{\alpha-1} e^{-\beta x}}{\Gamma(\alpha)}, \quad x > 0$$

with $\theta = (\alpha, \beta)$.

- c) The uniform distribution on $(\theta - 1, \theta + 1)$.
- d) The Cauchy distribution with density

$$f(x; a, b) = \frac{b}{\pi\{(x-a)^2 + b^2\}}, \quad x \in \mathbb{R},$$

with $\theta = (a, b)$.

(7 + 6 + 7 Marks)

Question 3

Let X_1, \dots, X_n be *iid* with common pdf

$$f(x; \omega) = \omega e^{-\omega x}, \quad x > 0, \omega > 0$$

It is required to estimate $\theta = 1/\omega$. Find the Bayes estimator of θ , under squared error loss function, and assuming a prior density on ω of the form

$$\pi(\omega) \propto \omega^{\alpha-1} e^{-\gamma\omega}$$

With known $\alpha > 0$ and $\gamma > 0$. Find the bias, if any, of the Bayes estimator. Giving your reasoning, find the minimum variance unbiased estimator of θ , and its variance.

(20 Marks)

Question 4

- a) Let X_1, \dots, X_n be independent random variables with a common density function

$$f(x; \theta) = \theta e^{-\theta x}, x \geq 0$$

where $\theta \in (0, \infty)$ is an unknown parameter. Consider the null hypothesis $H_0: \theta \leq 1$ against the alternative hypothesis $H_1: \theta > 1$. Show how to obtain a uniformly most powerful test of size α .

(10 Marks)

- b) Suppose Y_1, Y_2, \dots, Y_n is an iid sample of size n from a population with density function

$$f(y; \theta_1, \theta_2) = \frac{1}{\theta_1} e^{-(y-\theta_2)/\theta_1}, y \geq \theta_2$$

Where $\theta_1 > 0$ and $-\infty < \theta_2 < \infty$. Find the likelihood ratio test for testing $H_0: \theta_1 = \theta_{1,0}$ versus $H_1: \theta_1 > \theta_{1,0}$.

(10 Marks)

Question 5

- (a) Explain what is meant by: the size of a statistical test, and a confidence set.

(4 Marks)

- (b) Observations X_1, X_2, \dots, X_n constitute a random sample from a distribution with unknown parameter θ , and it is required to test the hypothesis $H_0: \theta = \theta_0$ against $H_1: \theta \neq \theta_0$. A test statistic $h(X_1, \dots, X_n; \theta)$ is available which can be used to find a test for any θ_0 . Show how a 95% confidence interval for θ can be constructed based on this test statistic.

(3 Marks)

- (c) Waiting times Y_1, Y_2, \dots, Y_n constitute a random sample from the gamma distribution with probability density function of :

$$f(y) = \frac{y^k e^{-y/\lambda}}{k! \lambda^{k+1}}, y > 0$$

where k is a known non-negative integer and $\lambda > 0$ is an unknown parameter. You are given that the moment generating function of this distribution is $(1 - \lambda t)^{-k-1}$ and that the above distribution is χ^2_{2k+2} when $\lambda = 2$.

- i). Show that $\frac{2W}{\lambda}$ is a pivotal quantity for λ .

(8 Marks)

- ii). Use your answer to part (i) to find a 95% confidence interval for λ when $n = 5, \sum_{i=1}^n Y_i$ and $k = 1$.

(5 Marks)

Normal Distribution

Table C-1. Cumulative Probabilities of the Standard Normal Distribution.

Entry is area A under the standard normal curve from $-\infty$ to $z(A)$

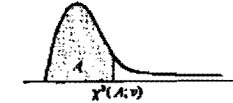


z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
-.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
-.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
-.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
-.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
-.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
-.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
-.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
-.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
-.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8483	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8663	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9013
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9533	.9543
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9980	.9981	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998

Chi-Square Distribution

Table C-2. Percentiles of the χ^2 Distribution

Entry is $\chi^2(A; \nu)$ where $P(\chi^2(\nu) \leq \chi^2(A; \nu)) = A$



ν	A									
	.005	.010	.025	.050	.100	.900	.950	.975	.990	.995
1	0.00103	0.00167	0.00382	0.00793	0.0158	2.71	3.84	5.02	6.63	7.88
2	0.0100	0.0201	0.0506	0.103	0.211	4.61	5.99	7.38	9.21	10.60
3	0.072	0.115	0.216	0.352	0.584	6.25	7.81	9.35	11.34	12.84
4	0.207	0.297	0.484	0.711	1.064	7.78	9.49	11.14	13.28	14.86
5	0.412	0.554	0.831	1.143	1.61	9.24	11.07	12.83	15.09	16.75
6	0.676	0.872	1.24	1.64	2.20	10.64	12.59	14.45	16.81	18.55
7	0.989	1.24	1.69	2.17	2.83	12.02	14.07	16.01	18.48	20.28
8	1.34	1.65	2.18	2.73	3.49	13.36	15.51	17.53	20.09	21.96
9	1.73	2.09	2.70	3.33	4.17	14.68	16.92	19.02	21.67	23.59
10	2.16	2.56	3.25	3.94	4.87	15.99	18.31	20.48	23.21	25.19
11	2.60	3.05	3.82	4.57	5.58	17.28	19.68	21.92	24.75	26.76
12	3.07	3.57	4.40	5.25	6.30	18.58	21.03	23.34	26.22	28.30
13	3.57	4.11	5.01	5.89	7.04	19.81	22.36	24.74	27.69	29.82
14	4.07	4.66	5.63	6.57	7.79	21.06	23.68	26.12	29.14	31.32
15	4.60	5.23	6.26	7.26	8.55	22.31	25.00	27.49	30.58	32.80
16	5.14	5.81	6.91	7.96	9.31	23.54	26.30	28.85	32.00	34.27
17	5.70	6.41	7.56	8.67	10.09	24.77	27.59	30.19	33.41	35.72
18	6.26	7.01	8.23	9.39	10.86	25.99	28.87	31.53	34.81	37.16
19	6.84	7.63	8.91	10.12	11.65	27.20	30.14	32.85	36.19	38.58
20	7.43	8.26	9.59	10.85	12.44	28.41	31.41	34.17	37.57	40.00
21	8.03	8.90	10.28	11.59	13.24	29.62	32.67	35.48	38.93	41.40
22	8.64	9.54	10.98	12.34	14.04	30.81	33.92	36.78	40.29	42.80
23	9.26	10.20	11.69	13.09	14.85	32.01	35.17	38.08	41.64	44.18
24	9.89	10.86	12.40	13.85	15.66	33.20	36.42	39.36	42.98	45.56
25	10.52	11.52	13.12	14.61	16.47	34.38	37.65	40.65	44.31	46.93
26	11.16	12.20	13.84	15.38	17.29	35.56	38.89	41.92	45.64	48.29
27	11.81	12.88	14.57	16.15	18.11	36.74	40.11	43.19	46.96	49.64
28	12.46	13.56	15.31	16.93	18.94	37.92	41.34	44.46	48.28	50.99
29	13.12	14.26	16.05	17.71	19.77	39.09	42.56	45.72	49.59	52.34
30	13.79	14.95	16.79	18.49	20.60	40.26	43.77	46.98	50.89	53.67
40	20.71	22.16	24.43	26.51	29.05	51.81	55.76	59.34	63.69	66.77
50	27.99	29.71	32.36	34.76	37.69	63.17	67.50	71.42	76.15	79.49
60	35.53	37.48	40.48	43.19	46.46	74.40	79.08	83.30	88.38	91.95
70	43.28	45.44	48.76	51.74	55.33	85.53	90.53	95.02	100.4	104.2
80	51.17	53.54	57.15	60.39	64.28	96.58	101.9	106.6	112.3	116.3
90	59.20	61.75	65.65	69.13	73.29	107.6	113.1	118.1	124.1	128.3
100	67.33	70.06	74.22	77.93	82.36	118.5	124.3	129.6	135.8	140.2

Student's Distribution (t Distribution)

Table C-4 Percentiles of the t Distribution

Entry is $t(A; \nu)$ where $P\{t(\nu) \leq t(A; \nu)\} = A$



ν	A						
	.60	.70	.80	.85	.90	.95	.975
1	0.325	0.727	1.376	1.963	3.078	6.314	12.706
2	0.289	0.617	1.061	1.386	1.886	2.920	4.303
3	0.277	0.584	0.978	1.250	1.638	2.353	3.182
4	0.271	0.569	0.941	1.190	1.533	2.132	2.776
5	0.267	0.559	0.920	1.156	1.476	2.015	2.571
6	0.265	0.553	0.906	1.134	1.440	1.943	2.447
7	0.263	0.549	0.896	1.119	1.415	1.895	2.365
8	0.262	0.546	0.889	1.108	1.397	1.860	2.306
9	0.261	0.543	0.883	1.100	1.383	1.833	2.262
10	0.260	0.542	0.879	1.093	1.372	1.812	2.228
11	0.260	0.540	0.876	1.088	1.363	1.796	2.201
12	0.259	0.539	0.873	1.083	1.356	1.782	2.179
13	0.259	0.537	0.870	1.079	1.350	1.771	2.160
14	0.258	0.537	0.868	1.076	1.345	1.761	2.145
15	0.258	0.536	0.866	1.074	1.341	1.753	2.131
16	0.258	0.535	0.865	1.071	1.337	1.746	2.120
17	0.257	0.534	0.863	1.069	1.333	1.740	2.110
18	0.257	0.534	0.862	1.067	1.330	1.734	2.101
19	0.257	0.533	0.861	1.066	1.328	1.729	2.093
20	0.257	0.533	0.860	1.064	1.325	1.725	2.086
21	0.257	0.532	0.859	1.063	1.323	1.721	2.080
22	0.256	0.532	0.858	1.061	1.321	1.717	2.074
23	0.256	0.532	0.858	1.060	1.319	1.714	2.069
24	0.256	0.531	0.857	1.059	1.318	1.711	2.064
25	0.256	0.531	0.856	1.058	1.316	1.708	2.060
26	0.256	0.531	0.856	1.058	1.315	1.706	2.056
27	0.256	0.531	0.855	1.057	1.314	1.703	2.052
28	0.256	0.530	0.855	1.056	1.313	1.701	2.048
29	0.256	0.530	0.854	1.055	1.311	1.699	2.045
30	0.256	0.530	0.854	1.055	1.310	1.697	2.042
40	0.255	0.529	0.851	1.050	1.303	1.684	2.021
60	0.254	0.527	0.848	1.045	1.296	1.671	2.000
120	0.254	0.526	0.845	1.041	1.289	1.658	1.980
∞	0.253	0.524	0.842	1.036	1.282	1.645	1.960

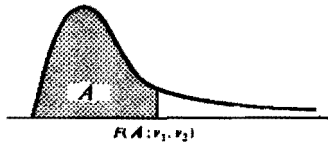
Table C-4 (Continued) Percentiles of the t Distribution

ν	A						
	.98	.985	.99	.9925	.995	.9975	.9995
1	15.895	21.205	31.821	42.434	63.657	127.322	636.590
2	4.849	5.643	6.965	8.073	9.925	14.089	31.598
3	3.482	3.896	4.541	5.047	5.841	7.453	12.924
4	2.999	3.298	3.747	4.088	4.604	5.598	8.610
5	2.757	3.003	3.365	3.634	4.032	4.773	6.869
6	2.612	2.829	3.143	3.372	3.707	4.317	5.959
7	2.517	2.715	2.998	3.203	3.499	4.029	5.408
8	2.449	2.634	2.896	3.085	3.355	3.833	5.041
9	2.398	2.574	2.821	2.998	3.250	3.690	4.781
10	2.359	2.527	2.764	2.932	3.169	3.581	4.587
11	2.328	2.491	2.718	2.879	3.106	3.497	4.437
12	2.303	2.461	2.681	2.836	3.055	3.428	4.318
13	2.282	2.436	2.650	2.801	3.012	3.372	4.221
14	2.264	2.415	2.624	2.771	2.977	3.326	4.140
15	2.249	2.397	2.602	2.746	2.947	3.286	4.073
16	2.235	2.382	2.583	2.724	2.921	3.252	4.015
17	2.224	2.368	2.567	2.706	2.898	3.222	3.965
18	2.214	2.356	2.552	2.689	2.878	3.197	3.922
19	2.205	2.346	2.539	2.674	2.861	3.174	3.883
20	2.197	2.336	2.528	2.661	2.845	3.153	3.849
21	2.189	2.328	2.518	2.649	2.831	3.135	3.819
22	2.183	2.320	2.508	2.639	2.819	3.119	3.792
23	2.177	2.313	2.500	2.629	2.807	3.104	3.768
24	2.172	2.307	2.492	2.620	2.797	3.091	3.745
25	2.167	2.301	2.485	2.612	2.787	3.078	3.725
26	2.162	2.296	2.479	2.605	2.779	3.067	3.707
27	2.158	2.291	2.473	2.598	2.771	3.057	3.690
28	2.154	2.286	2.467	2.592	2.763	3.047	3.674
29	2.150	2.282	2.462	2.586	2.756	3.038	3.659
30	2.147	2.278	2.457	2.581	2.750	3.030	3.646
40	2.123	2.250	2.423	2.542	2.704	2.971	3.551
60	2.099	2.223	2.390	2.504	2.660	2.915	3.460
120	2.076	2.196	2.358	2.468	2.617	2.860	3.373
∞	2.054	2.170	2.326	2.432	2.576	2.807	3.291

F Distribution

Table C-5 Percentiles of the F Distribution

Entry is $F(A; \nu_1, \nu_2)$ where $P\{F(\nu_1, \nu_2) \leq F(A; \nu_1, \nu_2)\} = A$



$$F(A; \nu_1, \nu_2) = \frac{1}{F(1-A; \nu_2, \nu_1)}$$

Table C-5 (Continued) Percentiles of the F Distribution

Den. df	A	Numerator df								
		1	2	3	4	5	6	7	8	9
1	.50	1.00	1.50	1.71	1.82	1.89	1.94	1.98	2.00	2.03
	.90	39.9	49.5	53.6	55.8	57.2	58.2	58.9	59.4	59.9
	.95	161	200	216	225	230	234	237	239	241
	.975	648	800	864	900	922	937	948	957	963
	.99	4,032	5,000	5,403	5,625	5,764	5,859	5,928	5,981	6,022
2	.50	0.667	1.00	1.13	1.21	1.25	1.28	1.30	1.32	1.33
	.90	8.33	9.00	9.16	9.24	9.29	9.33	9.35	9.37	9.38
	.95	18.3	19.0	19.2	19.2	19.3	19.3	19.4	19.4	19.4
	.975	38.5	39.0	39.2	39.2	39.3	39.3	39.4	39.4	39.4
	.99	98.5	99.0	99.2	99.2	99.3	99.3	99.4	99.4	99.4
3	.50	0.585	0.881	1.00	1.06	1.10	1.13	1.15	1.16	1.17
	.90	5.34	5.46	5.39	5.34	5.31	5.28	5.27	5.25	5.24
	.95	10.1	9.55	9.28	9.12	9.01	8.94	8.89	8.85	8.81
	.975	17.4	16.0	15.4	15.1	14.9	14.7	14.6	14.5	14.5
	.99	34.1	30.8	29.5	28.7	28.2	27.9	27.7	27.5	27.3
4	.50	0.549	0.828	0.941	1.00	1.04	1.06	1.08	1.09	1.10
	.90	4.54	4.32	4.19	4.11	4.05	4.01	3.98	3.95	3.94
	.95	7.71	6.94	6.59	6.39	6.26	6.16	6.09	6.04	6.00
	.975	12.2	10.6	9.98	9.60	9.36	9.20	9.07	8.98	8.90
	.99	21.2	18.0	16.7	16.0	15.5	15.2	15.0	14.8	14.7
5	.50	0.528	0.799	0.907	0.965	1.00	1.02	1.04	1.05	1.06
	.90	4.06	3.78	3.62	3.52	3.45	3.40	3.37	3.34	3.32
	.95	6.61	5.79	5.41	5.19	5.05	4.95	4.88	4.82	4.77
	.975	10.0	8.43	7.76	7.39	7.13	6.98	6.85	6.76	6.68
	.99	16.3	13.3	12.1	11.4	11.0	10.7	10.5	10.3	10.2
6	.50	0.515	0.780	0.886	0.942	0.977	1.00	1.02	1.03	1.04
	.90	3.78	3.46	3.29	3.18	3.11	3.05	3.01	2.98	2.96
	.95	5.99	5.14	4.76	4.53	4.39	4.28	4.21	4.15	4.10
	.975	8.81	7.26	6.60	6.23	5.99	5.82	5.70	5.60	5.52
	.99	13.7	10.9	9.78	9.15	8.75	8.47	8.26	8.10	7.98
7	.50	0.506	0.767	0.871	0.926	0.960	0.983	1.00	1.01	1.02
	.90	3.39	3.26	3.07	2.96	2.88	2.83	2.78	2.75	2.72
	.95	5.39	4.74	4.35	4.12	3.97	3.87	3.79	3.73	3.68
	.975	8.07	6.54	5.89	5.52	5.29	5.12	4.99	4.90	4.82
	.99	12.2	9.55	8.45	7.85	7.46	7.19	6.99	6.84	6.72
8	.50	0.500	0.759	0.862	0.916	0.950	0.973	0.995	1.00	1.01
	.90	3.30	3.17	2.98	2.87	2.79	2.74	2.69	2.66	2.63
	.95	5.29	4.64	4.25	4.02	3.87	3.77	3.69	3.63	3.58
	.975	7.97	6.44	5.79	5.42	5.19	5.02	4.89	4.80	4.72
	.99	12.1	9.43	8.33	7.73	7.34	7.07	6.87	6.72	6.60
9	.50	0.496	0.755	0.858	0.912	0.946	0.969	0.991	1.00	1.01
	.90	3.25	3.12	2.93	2.82	2.74	2.69	2.64	2.61	2.58
	.95	5.25	4.60	4.21	3.98	3.83	3.73	3.65	3.59	3.54
	.975	7.93	6.40	5.75	5.38	5.15	4.98	4.85	4.76	4.68
	.99	12.0	9.33	8.23	7.63	7.24	6.97	6.77	6.62	6.50

Table C-5 (Continued) Percentiles of the F Distribution

Den. df	A	Numerator df									
		10	12	15	20	24	30	60	120	∞	
1	.50	2.04	2.07	2.09	2.12	2.13	2.15	2.17	2.18	2.20	
	.90	60.2	60.7	61.2	61.7	62.0	62.3	62.8	63.1	63.3	
	.95	242	244	246	248	249	250	252	253	254	
	.975	969	977	985	993	997	1,001	1,010	1,014	1,018	
	.99	6,056	6,106	6,157	6,209	6,235	6,261	6,313	6,339	6,366	
	.995	24,224	24,426	24,630	24,836	24,940	25,044	25,253	25,359	25,464	
	.999	603,620	610,670	615,760	620,910	621,900	626,100	631,340	633,970	636,620	
	2	.50	1.34	1.36	1.38	1.39	1.40	1.41	1.43	1.43	1.44
	.90	9.39	9.41	9.42	9.44	9.45	9.46	9.47	9.48	9.49	
.95	19.4	19.4	19.4	19.4	19.5	19.5	19.5	19.5	19.5		
.975	39.4	39.4	39.4	39.4	39.5	39.5	39.5	39.5	39.5		
.99	99.4	99.4	99.4	99.4	99.5	99.5	99.5	99.5	99.5		
.995	199	199	199	199	199	199	199	199	200		
.999	999.4	999.4	999.4	999.4	999.5	999.5	999.5	999.5	999.5		
3	.50	1.18	1.20	1.21	1.23	1.23	1.24	1.25	1.26	1.27	
	.90	5.23	5.22	5.20	5.18	5.18	5.17	5.15	5.14	5.13	
	.95	8.79	8.74	8.70	8.66	8.64	8.62	8.57	8.55	8.53	
	.975	14.4	14.3	14.3	14.2	14.1	14.1	14.0	13.9	13.9	
	.99	27.2	27.1	26.9	26.7	26.6	26.5	26.3	26.2	26.1	
	.995	43.7	43.4	43.1	42.8	42.6	42.5	42.1	42.0	41.8	
	.999	129.2	128.3	127.4	126.4	125.9	125.4	124.5	124.0	123.5	
	4	.50	1.11	1.13	1.14	1.15	1.16	1.16	1.18	1.18	1.19
		.90	3.92	3.90	3.87	3.84	3.83	3.82	3.79	3.78	3.76
.95		5.96	5.91	5.86	5.80	5.77	5.75	5.69	5.66	5.63	
.975		8.84	8.75	8.66	8.56	8.51	8.46	8.36	8.31	8.26	
.99		14.5	14.4	14.2	14.0	13.9	13.8	13.7	13.6	13.5	
.995		21.0	20.7	20.4	20.2	20.0	19.9	19.6	19.5	19.3	
.999		48.1	47.4	46.8	46.1	45.8	45.4	44.7	44.4	44.1	
5		.50	1.07	1.09	1.10	1.11	1.12	1.12	1.14	1.14	1.15
		.90	3.30	3.27	3.24	3.21	3.19	3.17	3.14	3.12	3.11
	.95	4.74	4.68	4.62	4.55	4.53	4.50	4.43	4.40	4.37	
	.975	6.62	6.52	6.43	6.33	6.28	6.23	6.12	6.07	6.02	
	.99	10.1	9.89	9.72	9.55	9.47	9.38	9.20	9.11	9.02	
	.995	13.6	13.4	13.1	12.9	12.8	12.7	12.4	12.3	12.1	
	.999	26.9	26.4	25.9	25.4	25.1	24.9	24.3	24.1	23.8	
	6	.50	1.05	1.06	1.07	1.08	1.09	1.10	1.11	1.12	1.12
		.90	2.94	2.90	2.87	2.84	2.82	2.80	2.76	2.74	2.72
.95		4.06	4.00	3.94	3.87	3.84	3.81	3.74	3.70	3.67	
.975		5.46	5.37	5.27	5.17	5.12	5.07	4.96	4.90	4.85	
.99		7.87	7.72	7.54	7.40	7.31	7.23	7.06	6.97	6.88	
.995		10.2	10.0	9.81	9.59	9.47	9.36	9.12	9.00	8.88	
.999		18.4	18.0	17.6	17.1	16.9	16.7	16.2	16.0	15.7	
7		.50	1.03	1.04	1.05	1.07	1.07	1.08	1.09	1.10	1.10
		.90	2.70	2.67	2.63	2.59	2.58	2.56	2.51	2.49	2.47
	.95	3.64	3.57	3.51	3.44	3.41	3.38	3.30	3.27	3.23	
	.975	4.76	4.67	4.57	4.47	4.42	4.36	4.25	4.20	4.14	
	.99	6.62	6.47	6.31	6.16	6.07	5.99	5.82	5.74	5.65	
	.995	8.38	8.18	7.97	7.75	7.65	7.53	7.31	7.19	7.08	
	.999	14.1	13.7	13.3	12.9	12.7	12.5	12.1	11.9	11.7	

Table C-5 (Continued) Percentiles of the F Distribution

Den. df	A	Numerator df									
		1	2	3	4	5	6	7	8	9	
8	.50	0.499	0.737	0.860	0.915	0.948	0.971	0.988	1.00	1.01	
	.90	3.46	3.11	2.92	2.81	2.73	2.67	2.62	2.59	2.56	
	.95	5.32	4.46	4.07	3.84	3.69	3.58	3.50	3.44	3.39	
	.975	7.57	6.06	5.42	5.05	4.82	4.65	4.53	4.43	4.36	
	.99	11.3	8.65	7.59	7.01	6.63	6.37	6.18	6.03	5.91	
	.995	14.7	11.0	9.60	8.81	8.30	7.95	7.69	7.50	7.34	
	.999	25.4	18.5	15.8	14.4	13.5	12.9	12.4	12.0	11.8	
	9	.50	0.494	0.749	0.852	0.906	0.939	0.962	0.978	0.990	1.00
		.90	3.36	3.01	2.81	2.69	2.61	2.55	2.51	2.47	2.44
.95		5.12	4.26	3.86	3.63	3.48	3.37	3.29	3.23	3.18	
.975		7.21	5.71	5.08	4.72	4.48	4.32	4.20	4.10	4.03	
.99		10.6	8.02	6.99	6.42	6.06	5.80	5.61	5.47	5.35	
.995		13.6	10.1	8.72	7.96	7.47	7.13	6.88	6.69	6.54	
.999		22.9	16.4	13.9	12.6	11.7	11.1	10.7	10.4	10.1	
10		.50	0.490	0.743	0.845	0.899	0.932	0.954	0.971	0.983	0.992
		.90	3.29	2.92	2.73	2.61	2.52	2.46	2.41	2.38	2.35
	.95	4.96	4.10	3.71	3.48	3.33	3.22	3.14	3.07	3.02	
	.975	6.94	5.46	4.83	4.47	4.24	4.07	3.95	3.85	3.78	
	.99	10.0	7.56	6.35	5.99	5.64	5.39	5.20	5.06	4.94	
	.995	12.8	9.43	8.08	7.34	6.87	6.54	6.30	6.12	5.97	
	.999	21.0	14.9	12.6	11.3	10.5	9.93	9.52	9.20	8.96	
	12	.50	0.484	0.735	0.835	0.888	0.921	0.943	0.959	0.972	0.981
		.90	3.18	2.81	2.61	2.48	2.39	2.33	2.28	2.24	2.21
.95		4.75	3.89	3.49	3.26	3.11	3.00	2.91	2.85	2.80	
.975		6.55	5.10	4.47	4.12	3.89	3.73	3.61	3.51	3.44	
.99		9.33	6.93	5.95	5.41	5.06	4.82	4.64	4.50	4.39	
.995		11.8	8.51	7.23	6.52	6.07	5.76	5.52	5.35	5.20	
.999		18.6	13.0	10.8	9.63	8.89	8.38	8.00	7.71	7.48	
15		.50	0.478	0.726	0.826	0.878	0.911	0.933	0.949	0.960	0.970
		.90	3.07	2.70	2.49	2.36	2.27	2.21	2.16	2.12	2.09
	.95	4.54	3.68	3.29	3.06	2.90	2.79	2.71	2.64	2.59	
	.975	6.20	4.77	4.15	3.80	3.58	3.41	3.29	3.20	3.12	
	.99	8.68	6.36	5.42	4.89	4.56	4.32	4.14	4.00	3.89	
	.995	10.8	7.70	6.48	5.80	5.37	5.07	4.85	4.67	4.54	
	.999	16.6	11.3	9.34	8.25	7.57	7.09	6.74	6.47	6.26	
	20	.50	0.472	0.718	0.816	0.868	0.900	0.922	0.938	0.950	0.959
		.90	2.97	2.59	2.38	2.25	2.16	2.09	2.04	2.00	1.96
.95		4.35	3.49	3.10	2.87	2.71	2.60	2.51	2.45	2.39	
.975		5.87	4.46	3.86	3.51	3.29	3.13	3.01	2.91	2.84	
.99		8.10	5.85	4.94	4.43	4.10	3.87	3.70	3.56	3.46	
.995		9.94	6.99	5.82	5.17	4.76	4.47	4.26	4.09	3.96	
.999		14.8	9.95	8.10	7.10	6.46	6.02	5.69	5.44	5.24	
24		.50	0.469	0.714	0.812	0.863	0.895	0.917	0.932	0.944	0.953
		.90	2.93	2.54	2.33	2.19	2.10	2.04	1.98	1.94	1.91
	.95	4.26	3.40	3.01	2.78	2.62	2.51	2.42	2.36	2.30	
	.975	5.72	4.32	3.72	3.38	3.15	2.99	2.87	2.78	2.70	
	.99	7.82	5.61	4.72	4.22	3.90	3.67	3.50	3.36	3.26	
	.995	9.55	6.66	5.52	4.89	4.49	4.20	3.99	3.83	3.69	
	.999	14.0	9.34	7.53	6.59	5.98	5.55	5.23	4.99	4.80	

Table C-5 (Continued) Percentiles of the F Distribution

Den. df	Numerator df								
	10	12	15	20	24	30	60	120	∞
8	.50	1.02	1.03	1.04	1.05	1.06	1.07	1.08	1.09
	.90	2.54	2.90	2.46	2.42	2.40	2.38	2.34	2.32
	.95	3.35	3.28	3.22	3.15	3.12	3.08	3.01	2.97
	.975	4.30	4.20	4.10	4.00	3.95	3.89	3.78	3.73
	.99	5.81	5.67	5.52	5.36	5.28	5.20	5.03	4.95
	.995	7.21	7.01	6.81	6.61	6.50	6.40	6.18	6.06
	.999	11.5	11.2	10.8	10.5	10.3	10.1	9.73	9.53
9	.50	1.01	1.02	1.03	1.04	1.05	1.05	1.07	1.08
	.90	2.42	2.38	2.34	2.30	2.28	2.25	2.21	2.18
	.95	3.14	3.07	3.01	2.94	2.90	2.86	2.79	2.75
	.975	3.96	3.87	3.77	3.67	3.61	3.56	3.45	3.39
	.99	5.26	5.11	4.96	4.81	4.73	4.65	4.48	4.40
	.995	6.42	6.23	6.03	5.83	5.73	5.62	5.41	5.30
	.999	9.89	9.57	9.24	8.90	8.72	8.55	8.19	8.00
10	.50	1.00	1.01	1.02	1.03	1.04	1.05	1.06	1.07
	.90	2.32	2.28	2.24	2.20	2.18	2.16	2.11	2.08
	.95	2.98	2.91	2.84	2.77	2.74	2.70	2.62	2.58
	.975	3.72	3.62	3.52	3.42	3.37	3.31	3.20	3.14
	.99	4.85	4.71	4.56	4.41	4.33	4.25	4.08	4.00
	.995	5.85	5.66	5.47	5.27	5.17	5.07	4.86	4.75
	.999	8.75	8.45	8.13	7.80	7.64	7.47	7.12	6.94
12	.50	0.989	1.00	1.01	1.02	1.03	1.03	1.05	1.06
	.90	2.19	2.15	2.10	2.06	2.04	2.01	1.96	1.93
	.95	2.75	2.69	2.62	2.54	2.51	2.47	2.38	2.34
	.975	3.37	3.28	3.18	3.07	3.02	2.96	2.85	2.79
	.99	4.30	4.16	4.01	3.86	3.78	3.70	3.54	3.45
	.995	5.09	4.91	4.72	4.53	4.43	4.33	4.12	4.01
	.999	7.29	7.00	6.71	6.40	6.25	6.09	5.76	5.59
15	.50	0.977	0.989	1.00	1.01	1.02	1.02	1.03	1.04
	.90	2.06	2.02	1.97	1.92	1.90	1.87	1.82	1.79
	.95	2.54	2.48	2.40	2.33	2.29	2.25	2.16	2.11
	.975	3.06	2.96	2.86	2.76	2.70	2.64	2.52	2.46
	.99	3.80	3.67	3.52	3.37	3.29	3.21	3.05	2.96
	.995	4.42	4.25	4.07	3.88	3.79	3.69	3.48	3.37
	.999	6.08	5.81	5.54	5.25	5.10	4.93	4.64	4.48
20	.50	0.966	0.977	0.989	1.00	1.01	1.01	1.02	1.03
	.90	1.94	1.89	1.84	1.79	1.77	1.74	1.68	1.64
	.95	2.35	2.28	2.20	2.12	2.08	2.04	1.95	1.90
	.975	2.77	2.68	2.57	2.46	2.41	2.35	2.22	2.16
	.99	3.37	3.23	3.09	2.94	2.86	2.78	2.61	2.52
	.995	3.85	3.68	3.50	3.32	3.22	3.12	2.92	2.81
	.999	5.08	4.82	4.56	4.29	4.15	4.00	3.70	3.54
24	.50	0.961	0.972	0.983	0.994	1.00	1.01	1.02	1.03
	.90	1.88	1.83	1.78	1.73	1.70	1.67	1.61	1.57
	.95	2.25	2.18	2.11	2.03	1.98	1.94	1.84	1.79
	.975	2.64	2.54	2.44	2.33	2.27	2.21	2.08	2.01
	.99	3.17	3.03	2.89	2.74	2.66	2.58	2.40	2.31
	.995	3.59	3.42	3.25	3.06	2.97	2.87	2.66	2.55
	.999	4.64	4.39	4.14	3.87	3.74	3.59	3.29	3.14

Table C-5 (Continued) Percentiles of the F Distribution

Den. df	A	Numerator df								
		1	2	3	4	5	6	7	8	9
30	.50	0.466	0.709	0.807	0.858	0.890	0.912	0.927	0.939	0.948
	.90	2.88	2.49	2.28	2.14	2.05	1.98	1.93	1.88	1.85
	.95	4.17	3.32	2.92	2.69	2.53	2.42	2.33	2.27	2.21
	.975	5.57	4.18	3.59	3.25	3.03	2.87	2.75	2.65	2.57
	.99	7.56	5.39	4.51	4.02	3.70	3.47	3.30	3.17	3.07
	.995	9.18	6.35	5.24	4.62	4.23	3.95	3.74	3.58	3.45
	.999	13.3	8.77	7.05	6.12	5.53	5.12	4.82	4.58	4.39
60	.50	0.461	0.701	0.798	0.849	0.880	0.901	0.917	0.928	0.937
	.90	2.79	2.39	2.18	2.04	1.95	1.87	1.82	1.77	1.74
	.95	4.00	3.15	2.76	2.53	2.37	2.25	2.17	2.10	2.04
	.975	5.29	3.93	3.34	3.01	2.79	2.63	2.51	2.41	2.33
	.99	7.08	4.98	4.13	3.65	3.34	3.12	2.95	2.82	2.72
	.995	8.49	5.80	4.73	4.14	3.76	3.49	3.29	3.13	3.01
	.999	12.0	7.77	6.17	5.31	4.76	4.37	4.09	3.86	3.69
120	.50	0.458	0.697	0.793	0.844	0.875	0.896	0.912	0.923	0.932
	.90	2.75	2.35	2.13	1.99	1.90	1.82	1.77	1.72	1.68
	.95	3.92	3.07	2.68	2.45	2.29	2.18	2.09	2.02	1.96
	.975	5.15	3.80	3.23	2.89	2.67	2.52	2.39	2.30	2.22
	.99	6.85	4.79	3.95	3.48	3.17	2.96	2.79	2.66	2.56
	.995	8.18	5.54	4.50	3.92	3.55	3.28	3.09	2.93	2.81
	.999	11.4	7.32	5.78	4.95	4.42	4.04	3.77	3.55	3.38
∞	.50	0.455	0.693	0.789	0.839	0.870	0.891	0.907	0.918	0.927
	.90	2.71	2.30	2.08	1.94	1.85	1.77	1.72	1.67	1.63
	.95	3.84	3.00	2.60	2.37	2.21	2.10	2.01	1.94	1.88
	.975	5.02	3.69	3.12	2.79	2.57	2.41	2.29	2.19	2.11
	.99	6.63	4.61	3.78	3.32	3.02	2.80	2.64	2.51	2.41
	.995	7.88	5.30	4.28	3.72	3.35	3.09	2.90	2.74	2.62
	.999	10.8	6.91	5.42	4.62	4.10	3.74	3.47	3.27	3.10