

DEPARTMENT OF STATISTICS AND DEMOGRAPHY

MAIN EXAMINATION, 2014/15

**COURSE TITLE:** INTRODUCTION TO REGRESSION ANALYSIS

**COURSE CODE:** ST 304

**TIME ALLOWED:** TWO (2) HOURS

**INSTRUCTION:**

1. THIS PAPER CONSISTS OF FIVE (5) QUESTIONS;
2. ANSWER QUESTION ONE (1) & ANY TWO QUESTIONS

**SPECIAL REQUIREMENTS:** SCIENTIFIC CALCULATORS AND STATISTICAL TABLES

**DO NOT OPEN THIS PAGE UNTIL PERMISSION HAS BEEN GRANTED BY THE CHIEF INVIGILATOR**

**Question 1**

- (a) In a study to determine how the skill in doing a complex assembly job is influenced by the amount of training, 15 new recruits were given varying amounts of training ranging between 3 and 12 hours. After the training, their times to perform the job were recorded. Denoting  $(x)$  = duration of training (in hrs) and  $(y)$  = time to do the job (in minutes), the following summary statistics were obtained:

$$\bar{x} = 7.2; \quad SS_{xx} = 33.6; \quad SS_{xy} = -57.2; \quad \bar{y} = 45.6; \quad SS_{yy} = 160.2$$

- (i) Determine the equation for the best fitting line
- (ii) Do the data substantiate the claim that the job time decreases with more hours of training?
- (iii) Estimate the mean job time for 9 hours of training, and construct a 95% confidence interval.
- (iv) Find the predicted  $y$  for  $x=35$  hours, and comment on the result. **(5 + 12 + 8 + 5 marks)**

- (b) Management proposed the following regression model to predict sales at a fast food outlet.

$$y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + \varepsilon$$

Where:

$x_1$  = number of competitors within one mile

$x_2$  = population within one mile (1000s)

$x_3 = \begin{cases} 1, & \text{if drive-up window is present} \\ 0, & \text{otherwise} \end{cases}$

$y$  = sales (E1000s)

The following estimated regression equation was developed after 20 outlets were surveyed:

$$\hat{y} = 10.1 - 4.2x_1 + 6.8x_2 + 15.3x_3$$

- (i) What is the expected amount of sales attributed to the drive-up window?
- (ii) Predict sales for a store with two competitors, a population of 8000 within one mile, and no drive-up window.
- (iii) Predict sales for a store with one competitor, a population of 3000 within one mile, and a drive-up window. **(3 + 3 + 4 marks)**

**Question 2**

Given that  $X'X = \begin{bmatrix} n & \sum x_1 & \sum x_2 \\ \sum x_1 & \sum x_1^2 & \sum x_1 x_2 \\ \sum x_2 & \sum x_2 x_1 & \sum x_2^2 \end{bmatrix} = \begin{bmatrix} 8 & 25 & 16 \\ 25 & 87 & 55 \\ 16 & 55 & 36 \end{bmatrix}$  and

$$\sum y = 637,000, \quad \sum x_1 y = 2,031,100, \quad \sum x_2 y = 1,297,700$$

- (a) Find the least squares estimates for the multiple regression coefficients given by:

$$b = (X'X)^{-1} X'Y \quad \textbf{(10 marks)}$$

If  $\sum Y_i^2 = Y'Y = 50,907,080,000$ , Find SSE, SSR and SST and complete the ANOVA table for this regression model and test for the significance of the multiple linear regression model **(10 marks)**

**Question 3**

The regression results below were obtained for multiple linear regression model fitted on data for twelve students:

- (a) Find the values presented in asterisk **(8 marks)**  
 (b) Test for  $\beta_2=0$  given that  $t_{n-1,0.025} = 2.201$  **(8 marks)**  
 (c) Compute the F-value and make conclusions about the regression model at 5 percent level of significance. **(4 marks)**

| Regression Coefficient | Estimated regression coefficient | Standard Error | t-Statistic |
|------------------------|----------------------------------|----------------|-------------|
| $\beta_0$              | 27.5467                          | *****          | 2.204069    |
| $\beta_1$              | *****                            | 0.185827       | 4.959878    |
| $\beta_2$              | 0.28425                          | 0.753597       | *****       |

**ANOVA Results**

| Source of variation | Degrees of freedom | Sums of squares | Mean Square |
|---------------------|--------------------|-----------------|-------------|
| Regression          | *****              | 544.5959        | *****       |
| Error               | *****              | *****           | 20.40601    |
| Total               | *****              | 728.25          |             |

**Question 4**

Data concerning the weekly fuel consumption (Y), average hourly temperature ( $X_1$ ) and chill index ( $X_2$ ) at a certain factory has been observed for the last eight years.

| Week (i) | Average Hourly Temperature | Chill Index | Fuel Consumption |
|----------|----------------------------|-------------|------------------|
| 1        | 28                         | 18          | 12.4             |
| 2        | 28                         | 14          | 11.7             |
| 3        | 32.5                       | 24          | 12.4             |
| 4        | 39                         | 22          | 10.8             |
| 5        | 45.9                       | 8           | 9.4              |
| 6        | 57.8                       | 16          | 9.5              |
| 7        | 58.1                       | 1           | 8                |
| 8        | 62.5                       | 0           | 7.5              |

- (a) Fit the regression model on the fuel consumption data. **(10 marks)**  
 (b) Test for the significance of the coefficients at 5 % level of significance. **(10 marks)**

**Question 5**

- (a) What is multicollinearity? Explain, with illustrations, the estimation problem under perfect multicollinearity. **(8 marks)**
- (b) What are practical consequences of multicollinearity? **(5 marks)**
- (c) Describe various methods of detecting multicollinearity, also highlighting the weaknesses of these methods. **(7 marks)**

**END OF EXAM!!**

II. Percentage Points of the  $t$  Distribution\*

| $\nu \backslash \alpha$ | .40  | .25   | .10   | .05   | .025   | .01    | .005   | .0025  | .001   | .0005  |
|-------------------------|------|-------|-------|-------|--------|--------|--------|--------|--------|--------|
| 1                       | .325 | 1.000 | 3.078 | 6.314 | 12.706 | 31.821 | 63.657 | 127.32 | 318.31 | 636.62 |
| 2                       | .289 | .816  | 1.886 | 2.920 | 4.303  | 6.965  | 9.925  | 14.089 | 23.326 | 31.598 |
| 3                       | .277 | .765  | 1.638 | 2.353 | 3.182  | 4.541  | 5.841  | 7.453  | 10.213 | 12.924 |
| 4                       | .271 | .741  | 1.533 | 2.132 | 2.776  | 3.747  | 4.604  | 5.598  | 7.173  | 8.610  |
| 5                       | .267 | .727  | 1.476 | 2.015 | 2.571  | 3.365  | 4.032  | 4.773  | 5.893  | 6.869  |
| 6                       | .265 | .727  | 1.440 | 1.943 | 2.447  | 3.143  | 3.707  | 4.317  | 5.208  | 5.959  |
| 7                       | .263 | .711  | 1.415 | 1.895 | 2.365  | 2.998  | 3.499  | 4.019  | 4.785  | 5.408  |
| 8                       | .262 | .706  | 1.397 | 1.860 | 2.306  | 2.896  | 3.355  | 3.833  | 4.501  | 5.041  |
| 9                       | .261 | .703  | 1.383 | 1.833 | 2.262  | 2.821  | 3.250  | 3.690  | 4.297  | 4.781  |
| 10                      | .260 | .700  | 1.372 | 1.812 | 2.228  | 2.764  | 3.169  | 3.581  | 4.144  | 4.587  |
| 11                      | .260 | .697  | 1.363 | 1.796 | 2.201  | 2.718  | 3.106  | 3.497  | 4.025  | 4.437  |
| 12                      | .259 | .695  | 1.356 | 1.782 | 2.179  | 2.681  | 3.055  | 3.428  | 3.930  | 4.318  |
| 13                      | .259 | .694  | 1.350 | 1.771 | 2.160  | 2.650  | 3.012  | 3.372  | 3.852  | 4.221  |
| 14                      | .258 | .692  | 1.345 | 1.761 | 2.145  | 2.624  | 2.977  | 3.326  | 3.787  | 4.140  |
| 15                      | .258 | .691  | 1.341 | 1.753 | 2.131  | 2.602  | 2.947  | 3.286  | 3.733  | 4.073  |
| 16                      | .258 | .690  | 1.337 | 1.746 | 2.120  | 2.583  | 2.921  | 3.252  | 3.686  | 4.015  |
| 17                      | .257 | .689  | 1.333 | 1.740 | 2.110  | 2.567  | 2.898  | 3.222  | 3.646  | 3.965  |
| 18                      | .257 | .688  | 1.330 | 1.734 | 2.101  | 2.552  | 2.878  | 3.197  | 3.610  | 3.922  |
| 19                      | .257 | .688  | 1.328 | 1.729 | 2.093  | 2.539  | 2.861  | 3.174  | 3.579  | 3.883  |
| 20                      | .257 | .687  | 1.325 | 1.725 | 2.086  | 2.528  | 2.845  | 3.153  | 3.552  | 3.850  |
| 21                      | .257 | .686  | 1.323 | 1.721 | 2.080  | 2.518  | 2.831  | 3.135  | 3.527  | 3.819  |
| 22                      | .256 | .686  | 1.321 | 1.717 | 2.074  | 2.508  | 2.819  | 3.119  | 3.505  | 3.792  |
| 23                      | .256 | .685  | 1.319 | 1.714 | 2.069  | 2.500  | 2.807  | 3.104  | 3.485  | 3.767  |
| 24                      | .256 | .685  | 1.318 | 1.711 | 2.064  | 2.492  | 2.797  | 3.091  | 3.467  | 3.745  |
| 25                      | .256 | .684  | 1.316 | 1.708 | 2.060  | 2.485  | 2.787  | 3.078  | 3.450  | 3.725  |
| 26                      | .256 | .684  | 1.315 | 1.706 | 2.056  | 2.479  | 2.779  | 3.067  | 3.435  | 3.707  |
| 27                      | .256 | .684  | 1.314 | 1.703 | 2.052  | 2.473  | 2.771  | 3.057  | 3.421  | 3.690  |
| 28                      | .256 | .683  | 1.313 | 1.701 | 2.048  | 2.467  | 2.763  | 3.047  | 3.408  | 3.674  |
| 29                      | .256 | .683  | 1.311 | 1.699 | 2.045  | 2.462  | 2.756  | 3.038  | 3.396  | 3.659  |
| 30                      | .256 | .683  | 1.310 | 1.697 | 2.042  | 2.457  | 2.750  | 3.030  | 3.385  | 3.646  |
| 40                      | .255 | .681  | 1.303 | 1.684 | 2.021  | 2.423  | 2.704  | 2.971  | 3.307  | 3.551  |
| 60                      | .254 | .679  | 1.296 | 1.671 | 2.000  | 2.390  | 2.660  | 2.915  | 3.232  | 3.460  |
| 120                     | .254 | .677  | 1.289 | 1.658 | 1.980  | 2.358  | 2.617  | 2.860  | 3.160  | 3.373  |
| $\infty$                | .253 | .674  | 1.282 | 1.645 | 1.960  | 2.326  | 2.576  | 2.807  | 3.090  | 3.291  |

 $\nu$  = degrees of freedom.\* Adapted with permission from *Biometrika Tables for Statisticians*, Vol. 1, 3rd edition, by E. S. Pearson and H. O. Hartley, Cambridge University Press, Cambridge, 1966.

IV. Percentage Points of the F Distribution (continued)

$$F_{0.05, \nu_1, \nu_2}$$

| $\nu_2$   | $\nu_1$ | Degrees of Freedom for the Numerator ( $\nu_1$ ) |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |          |
|---|---------|--|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|----------|
|   |         | 1  | 2     | 3     | 4     | 5     | 6     | 7     | 8     | 9     | 10    | 12    | 15    | 20    | 24    | 30    | 40    | 60    | 120   | $\infty$ |
| 644<br>Degrees of Freedom for the Denominator ( $\nu_2$ ) | 2       | 161.4  | 199.5 | 215.7 | 224.6 | 230.2 | 234.0 | 236.8 | 238.9 | 240.5 | 241.9 | 243.9 | 245.9 | 248.0 | 249.1 | 250.1 | 251.1 | 252.2 | 253.3 | 254.3    |
|   | 3       | 18.51  | 19.00 | 19.16 | 19.25 | 19.30 | 19.33 | 19.35 | 19.37 | 19.38 | 19.40 | 19.41 | 19.43 | 19.45 | 19.45 | 19.46 | 19.47 | 19.48 | 19.49 | 19.50    |
|   | 4       | 10.13  | 9.55  | 9.28  | 9.12  | 9.01  | 8.94  | 8.89  | 8.85  | 8.81  | 8.79  | 8.74  | 8.70  | 8.66  | 8.64  | 8.62  | 8.59  | 8.57  | 8.55  | 8.53     |
|   | 5       | 7.71   | 6.94  | 6.59  | 6.39  | 6.26  | 6.16  | 6.09  | 6.04  | 6.00  | 5.96  | 5.91  | 5.86  | 5.80  | 5.77  | 5.75  | 5.72  | 5.69  | 5.66  | 5.63     |
|   | 6       | 6.61   | 5.79  | 5.41  | 5.19  | 5.05  | 4.95  | 4.88  | 4.82  | 4.77  | 4.74  | 4.68  | 4.62  | 4.56  | 4.53  | 4.50  | 4.46  | 4.43  | 4.40  | 4.36     |
|   | 7       | 5.99   | 5.14  | 4.76  | 4.53  | 4.39  | 4.28  | 4.21  | 4.15  | 4.10  | 4.06  | 4.00  | 3.94  | 3.87  | 3.84  | 3.81  | 3.77  | 3.74  | 3.70  | 3.67     |
|   | 8       | 5.59   | 4.74  | 4.35  | 4.12  | 3.97  | 3.87  | 3.79  | 3.73  | 3.68  | 3.64  | 3.57  | 3.51  | 3.44  | 3.41  | 3.38  | 3.34  | 3.30  | 3.27  | 3.23     |
|   | 9       | 5.32   | 4.46  | 4.07  | 3.84  | 3.69  | 3.58  | 3.50  | 3.44  | 3.39  | 3.35  | 3.28  | 3.22  | 3.15  | 3.12  | 3.08  | 3.04  | 3.01  | 2.97  | 2.93     |
|   | 10      | 5.12   | 4.26  | 3.86  | 3.63  | 3.48  | 3.37  | 3.29  | 3.23  | 3.18  | 3.14  | 3.07  | 3.01  | 2.94  | 2.90  | 2.86  | 2.83  | 2.79  | 2.75  | 2.71     |
|   | 11      | 4.96   | 4.10  | 3.71  | 3.48  | 3.33  | 3.22  | 3.14  | 3.07  | 3.02  | 2.98  | 2.91  | 2.85  | 2.77  | 2.74  | 2.70  | 2.66  | 2.62  | 2.58  | 2.54     |
|   | 12      | 4.84   | 3.98  | 3.59  | 3.36  | 3.20  | 3.09  | 3.01  | 2.95  | 2.90  | 2.85  | 2.79  | 2.72  | 2.65  | 2.61  | 2.57  | 2.53  | 2.49  | 2.45  | 2.40     |
|   | 13      | 4.75   | 3.89  | 3.49  | 3.26  | 3.11  | 3.00  | 2.91  | 2.85  | 2.80  | 2.75  | 2.69  | 2.62  | 2.54  | 2.51  | 2.47  | 2.43  | 2.38  | 2.34  | 2.30     |
|   | 14      | 4.67   | 3.81  | 3.41  | 3.18  | 3.03  | 2.92  | 2.83  | 2.77  | 2.71  | 2.67  | 2.60  | 2.53  | 2.46  | 2.42  | 2.38  | 2.34  | 2.30  | 2.25  | 2.21     |
|   | 15      | 4.60   | 3.74  | 3.34  | 3.11  | 2.96  | 2.85  | 2.76  | 2.70  | 2.65  | 2.60  | 2.53  | 2.46  | 2.39  | 2.35  | 2.31  | 2.27  | 2.22  | 2.18  | 2.13     |
|   | 16      | 4.54   | 3.68  | 3.29  | 3.06  | 2.90  | 2.79  | 2.71  | 2.64  | 2.59  | 2.54  | 2.48  | 2.40  | 2.33  | 2.29  | 2.25  | 2.20  | 2.16  | 2.11  | 2.07     |
|   | 17      | 4.49   | 3.63  | 3.24  | 3.01  | 2.85  | 2.74  | 2.66  | 2.59  | 2.54  | 2.49  | 2.42  | 2.35  | 2.28  | 2.24  | 2.19  | 2.15  | 2.11  | 2.06  | 2.01     |
|   | 18      | 4.45   | 3.59  | 3.20  | 2.96  | 2.81  | 2.70  | 2.61  | 2.55  | 2.49  | 2.45  | 2.38  | 2.31  | 2.23  | 2.19  | 2.15  | 2.10  | 2.06  | 2.01  | 1.96     |
|   | 19      | 4.41   | 3.55  | 3.16  | 2.93  | 2.77  | 2.66  | 2.58  | 2.51  | 2.46  | 2.41  | 2.34  | 2.27  | 2.19  | 2.15  | 2.11  | 2.06  | 2.02  | 1.97  | 1.92     |
|   | 20      | 4.38   | 3.52  | 3.13  | 2.90  | 2.74  | 2.63  | 2.54  | 2.48  | 2.42  | 2.38  | 2.31  | 2.23  | 2.16  | 2.11  | 2.07  | 2.03  | 1.98  | 1.93  | 1.88     |
|   | 21      | 4.35   | 3.49  | 3.10  | 2.87  | 2.71  | 2.60  | 2.51  | 2.45  | 2.39  | 2.35  | 2.28  | 2.20  | 2.12  | 2.08  | 2.04  | 1.99  | 1.95  | 1.90  | 1.84     |
|   | 22      | 4.32   | 3.47  | 3.07  | 2.84  | 2.68  | 2.57  | 2.49  | 2.42  | 2.37  | 2.32  | 2.25  | 2.18  | 2.10  | 2.05  | 2.01  | 1.96  | 1.92  | 1.87  | 1.81     |
|   | 23      | 4.30   | 3.44  | 3.05  | 2.82  | 2.66  | 2.55  | 2.46  | 2.40  | 2.34  | 2.30  | 2.23  | 2.15  | 2.07  | 2.03  | 1.98  | 1.94  | 1.89  | 1.84  | 1.78     |
|   | 24      | 4.28   | 3.42  | 3.03  | 2.80  | 2.64  | 2.53  | 2.44  | 2.37  | 2.32  | 2.27  | 2.20  | 2.13  | 2.05  | 2.01  | 1.96  | 1.91  | 1.86  | 1.81  | 1.76     |
|   | 25      | 4.26   | 3.40  | 3.01  | 2.78  | 2.62  | 2.51  | 2.42  | 2.36  | 2.30  | 2.25  | 2.18  | 2.11  | 2.03  | 1.98  | 1.94  | 1.89  | 1.84  | 1.79  | 1.73     |
| 26  | 4.24    | 3.39   | 2.99  | 2.76  | 2.60  | 2.49  | 2.40  | 2.34  | 2.28  | 2.24  | 2.16  | 2.09  | 2.01  | 1.96  | 1.92  | 1.87  | 1.82  | 1.77  | 1.71  |          |
| 27  | 4.23    | 3.37   | 2.98  | 2.74  | 2.59  | 2.47  | 2.39  | 2.32  | 2.27  | 2.22  | 2.15  | 2.07  | 1.99  | 1.95  | 1.90  | 1.85  | 1.80  | 1.75  | 1.69  |          |
| 28  | 4.21    | 3.35   | 2.96  | 2.73  | 2.57  | 2.46  | 2.37  | 2.31  | 2.25  | 2.20  | 2.13  | 2.06  | 1.97  | 1.93  | 1.88  | 1.84  | 1.79  | 1.73  | 1.67  |          |
| 29  | 4.20    | 3.34   | 2.95  | 2.71  | 2.56  | 2.45  | 2.36  | 2.29  | 2.24  | 2.19  | 2.12  | 2.04  | 1.96  | 1.91  | 1.87  | 1.82  | 1.77  | 1.71  | 1.65  |          |
| 30  | 4.18    | 3.33   | 2.93  | 2.70  | 2.55  | 2.43  | 2.35  | 2.28  | 2.22  | 2.18  | 2.10  | 2.03  | 1.94  | 1.90  | 1.85  | 1.81  | 1.75  | 1.70  | 1.64  |          |
| 40  | 4.17    | 3.32   | 2.92  | 2.69  | 2.53  | 2.42  | 2.33  | 2.27  | 2.21  | 2.16  | 2.09  | 2.01  | 1.93  | 1.89  | 1.84  | 1.79  | 1.74  | 1.68  | 1.62  |          |
| 60  | 4.08    | 3.23   | 2.84  | 2.61  | 2.45  | 2.34  | 2.25  | 2.18  | 2.12  | 2.08  | 2.00  | 1.92  | 1.84  | 1.79  | 1.74  | 1.69  | 1.64  | 1.58  | 1.51  |          |
| 120   | 4.00    | 3.15   | 2.76  | 2.53  | 2.37  | 2.25  | 2.17  | 2.10  | 2.04  | 1.99  | 1.92  | 1.84  | 1.75  | 1.70  | 1.65  | 1.59  | 1.53  | 1.47  | 1.39  |          |
| $\infty$  | 3.92    | 3.07   | 2.68  | 2.45  | 2.29  | 2.17  | 2.09  | 2.02  | 1.96  | 1.91  | 1.83  | 1.75  | 1.66  | 1.61  | 1.55  | 1.50  | 1.43  | 1.35  | 1.25  |          |
| $\infty$  | 3.84    | 3.00   | 2.60  | 2.37  | 2.21  | 2.10  | 2.01  | 1.94  | 1.88  | 1.83  | 1.75  | 1.67  | 1.57  | 1.52  | 1.46  | 1.39  | 1.32  | 1.22  | 1.11  |          |

IV. Percentage Points of the F Distribution (continued)

$$F_{0.025, \nu_1, \nu_2}$$

| $\nu_2$  | $\nu_1$ | Degrees of Freedom for the Numerator ( $\nu_1$ ) |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |          |
|----------|---------|--|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|----------|
|          |         | 1  | 2     | 3     | 4     | 5     | 6     | 7     | 8     | 9     | 10    | 12    | 15    | 20    | 24    | 30    | 40    | 60    | 120   | $\infty$ |
| 1        | 1       | 647.8  | 799.5 | 864.2 | 899.6 | 921.8 | 937.1 | 948.2 | 956.7 | 963.3 | 968.6 | 976.7 | 984.9 | 993.1 | 997.2 | 1001  | 1006  | 1010  | 1014  | 1018     |
| 2        | 1       | 38.51  | 39.00 | 39.17 | 39.25 | 39.30 | 39.33 | 39.36 | 39.37 | 39.39 | 39.40 | 39.41 | 39.43 | 39.45 | 39.46 | 39.46 | 39.47 | 39.48 | 39.49 | 39.50    |
| 3        | 1       | 17.44  | 16.04 | 15.44 | 15.10 | 14.88 | 14.73 | 14.62 | 14.54 | 14.47 | 14.42 | 14.34 | 14.25 | 14.17 | 14.12 | 14.08 | 14.04 | 13.99 | 13.95 | 13.90    |
| 4        | 1       | 12.22  | 10.65 | 9.98  | 9.60  | 9.36  | 9.20  | 9.07  | 8.98  | 8.90  | 8.84  | 8.75  | 8.66  | 8.56  | 8.51  | 8.46  | 8.41  | 8.36  | 8.31  | 8.26     |
| 5        | 1       | 10.01  | 8.43  | 7.76  | 7.39  | 7.15  | 6.98  | 6.85  | 6.76  | 6.68  | 6.62  | 6.52  | 6.43  | 6.33  | 6.28  | 6.23  | 6.18  | 6.12  | 6.07  | 6.02     |
| 6        | 1       | 8.81   | 7.26  | 6.60  | 6.23  | 5.99  | 5.82  | 5.70  | 5.60  | 5.52  | 5.46  | 5.37  | 5.27  | 5.17  | 5.12  | 5.07  | 5.01  | 4.96  | 4.90  | 4.85     |
| 7        | 1       | 8.07   | 6.54  | 5.89  | 5.52  | 5.29  | 5.12  | 4.99  | 4.90  | 4.82  | 4.76  | 4.67  | 4.57  | 4.47  | 4.42  | 4.36  | 4.31  | 4.25  | 4.20  | 4.14     |
| 8        | 1       | 7.57   | 6.06  | 5.42  | 5.05  | 4.82  | 4.65  | 4.53  | 4.43  | 4.36  | 4.30  | 4.20  | 4.10  | 4.00  | 3.95  | 3.89  | 3.84  | 3.78  | 3.73  | 3.67     |
| 9        | 1       | 7.21   | 5.71  | 5.08  | 4.72  | 4.48  | 4.32  | 4.20  | 4.10  | 4.03  | 3.96  | 3.87  | 3.77  | 3.67  | 3.61  | 3.56  | 3.51  | 3.45  | 3.39  | 3.33     |
| 10       | 1       | 6.94   | 5.46  | 4.83  | 4.47  | 4.24  | 4.07  | 3.95  | 3.85  | 3.78  | 3.72  | 3.62  | 3.52  | 3.42  | 3.37  | 3.31  | 3.26  | 3.20  | 3.14  | 3.08     |
| 11       | 1       | 6.72   | 5.26  | 4.63  | 4.28  | 4.04  | 3.88  | 3.76  | 3.66  | 3.59  | 3.53  | 3.43  | 3.33  | 3.23  | 3.17  | 3.12  | 3.06  | 3.00  | 2.94  | 2.88     |
| 12       | 1       | 6.55   | 5.10  | 4.47  | 4.12  | 3.89  | 3.73  | 3.61  | 3.51  | 3.44  | 3.37  | 3.28  | 3.18  | 3.07  | 3.02  | 2.96  | 2.91  | 2.85  | 2.79  | 2.72     |
| 13       | 1       | 6.41   | 4.97  | 4.35  | 4.00  | 3.77  | 3.60  | 3.48  | 3.39  | 3.31  | 3.25  | 3.15  | 3.05  | 2.95  | 2.89  | 2.84  | 2.78  | 2.72  | 2.66  | 2.60     |
| 14       | 1       | 6.30   | 4.86  | 4.24  | 3.89  | 3.66  | 3.50  | 3.38  | 3.29  | 3.21  | 3.15  | 3.05  | 2.95  | 2.84  | 2.79  | 2.73  | 2.67  | 2.61  | 2.55  | 2.49     |
| 15       | 1       | 6.20   | 4.77  | 4.15  | 3.80  | 3.58  | 3.41  | 3.29  | 3.20  | 3.12  | 3.06  | 2.96  | 2.86  | 2.76  | 2.70  | 2.64  | 2.59  | 2.52  | 2.46  | 2.40     |
| 16       | 1       | 6.12   | 4.69  | 4.08  | 3.73  | 3.50  | 3.34  | 3.22  | 3.12  | 3.05  | 2.99  | 2.89  | 2.79  | 2.68  | 2.63  | 2.57  | 2.51  | 2.45  | 2.38  | 2.32     |
| 17       | 1       | 6.04   | 4.62  | 4.01  | 3.66  | 3.44  | 3.28  | 3.16  | 3.06  | 2.98  | 2.92  | 2.82  | 2.72  | 2.62  | 2.56  | 2.50  | 2.44  | 2.38  | 2.32  | 2.25     |
| 18       | 1       | 5.98   | 4.56  | 3.95  | 3.61  | 3.38  | 3.22  | 3.10  | 3.01  | 2.93  | 2.87  | 2.77  | 2.67  | 2.56  | 2.50  | 2.44  | 2.38  | 2.32  | 2.26  | 2.19     |
| 19       | 1       | 5.92   | 4.51  | 3.90  | 3.56  | 3.33  | 3.17  | 3.05  | 2.96  | 2.88  | 2.82  | 2.72  | 2.62  | 2.51  | 2.45  | 2.39  | 2.33  | 2.27  | 2.20  | 2.13     |
| 20       | 1       | 5.87   | 4.46  | 3.86  | 3.51  | 3.29  | 3.13  | 3.01  | 2.91  | 2.84  | 2.77  | 2.68  | 2.57  | 2.46  | 2.41  | 2.35  | 2.29  | 2.22  | 2.16  | 2.09     |
| 21       | 1       | 5.83   | 4.42  | 3.82  | 3.48  | 3.25  | 3.09  | 2.97  | 2.87  | 2.80  | 2.73  | 2.64  | 2.53  | 2.42  | 2.37  | 2.31  | 2.25  | 2.18  | 2.11  | 2.04     |
| 22       | 1       | 5.79   | 4.38  | 3.78  | 3.44  | 3.22  | 3.05  | 2.93  | 2.84  | 2.76  | 2.70  | 2.60  | 2.50  | 2.39  | 2.33  | 2.27  | 2.21  | 2.14  | 2.08  | 2.00     |
| 23       | 1       | 5.75   | 4.35  | 3.75  | 3.41  | 3.18  | 3.02  | 2.90  | 2.81  | 2.73  | 2.67  | 2.57  | 2.47  | 2.36  | 2.30  | 2.24  | 2.18  | 2.11  | 2.04  | 1.97     |
| 24       | 1       | 5.72   | 4.32  | 3.72  | 3.38  | 3.15  | 2.99  | 2.87  | 2.78  | 2.70  | 2.64  | 2.54  | 2.44  | 2.33  | 2.27  | 2.21  | 2.15  | 2.08  | 2.01  | 1.94     |
| 25       | 1       | 5.69   | 4.29  | 3.69  | 3.35  | 3.13  | 2.97  | 2.85  | 2.75  | 2.68  | 2.61  | 2.51  | 2.41  | 2.30  | 2.24  | 2.18  | 2.12  | 2.05  | 1.98  | 1.91     |
| 26       | 1       | 5.66   | 4.27  | 3.67  | 3.33  | 3.10  | 2.94  | 2.82  | 2.73  | 2.65  | 2.59  | 2.49  | 2.39  | 2.28  | 2.22  | 2.16  | 2.09  | 2.03  | 1.95  | 1.88     |
| 27       | 1       | 5.63   | 4.24  | 3.65  | 3.31  | 3.08  | 2.92  | 2.80  | 2.71  | 2.63  | 2.57  | 2.47  | 2.36  | 2.25  | 2.19  | 2.13  | 2.07  | 2.00  | 1.93  | 1.85     |
| 28       | 1       | 5.61   | 4.22  | 3.63  | 3.29  | 3.06  | 2.90  | 2.78  | 2.69  | 2.61  | 2.55  | 2.45  | 2.34  | 2.23  | 2.17  | 2.11  | 2.05  | 1.98  | 1.91  | 1.83     |
| 29       | 1       | 5.59   | 4.20  | 3.61  | 3.27  | 3.04  | 2.88  | 2.76  | 2.67  | 2.59  | 2.53  | 2.43  | 2.32  | 2.21  | 2.15  | 2.09  | 2.03  | 1.96  | 1.89  | 1.81     |
| 30       | 1       | 5.57   | 4.18  | 3.59  | 3.25  | 3.03  | 2.87  | 2.75  | 2.65  | 2.57  | 2.51  | 2.41  | 2.31  | 2.20  | 2.14  | 2.07  | 2.01  | 1.94  | 1.87  | 1.79     |
| 40       | 1       | 5.42   | 4.05  | 3.46  | 3.13  | 2.90  | 2.74  | 2.62  | 2.53  | 2.45  | 2.39  | 2.29  | 2.18  | 2.07  | 2.01  | 1.94  | 1.88  | 1.80  | 1.72  | 1.64     |
| 60       | 1       | 5.29   | 3.93  | 3.34  | 3.01  | 2.79  | 2.63  | 2.51  | 2.41  | 2.33  | 2.27  | 2.17  | 2.06  | 1.94  | 1.88  | 1.82  | 1.74  | 1.67  | 1.58  | 1.48     |
| 120      | 1       | 5.15   | 3.80  | 3.23  | 2.89  | 2.67  | 2.52  | 2.39  | 2.30  | 2.22  | 2.16  | 2.05  | 1.94  | 1.82  | 1.76  | 1.69  | 1.61  | 1.53  | 1.43  | 1.31     |
| $\infty$ | 1       | 5.02   | 3.69  | 3.12  | 2.79  | 2.57  | 2.41  | 2.29  | 2.19  | 2.11  | 2.05  | 1.94  | 1.83  | 1.71  | 1.64  | 1.57  | 1.48  | 1.39  | 1.27  | 1.00     |

IV. Percentage Points of the F Distribution (continued)

$$F_{0.01, \nu_1, \nu_2}$$

| 646<br>$\nu_2$ | $\nu_1$  | Degrees of Freedom for the Numerator ( $\nu_1$ ) |        |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |          |
|----------------|----------|--|--------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|----------|
|                |          | 1  | 2      | 3     | 4     | 5     | 6     | 7     | 8     | 9     | 10    | 12    | 15    | 20    | 24    | 30    | 40    | 60    | 120   | $\infty$ |
| 1              | 1        | 4052   | 4999.5 | 5403  | 5625  | 5764  | 5859  | 5928  | 5982  | 6022  | 6056  | 6106  | 6157  | 6209  | 6235  | 6261  | 6287  | 6313  | 6339  | 6366     |
| 2              | 2        | 98.50  | 99.00  | 99.17 | 99.25 | 99.30 | 99.33 | 99.36 | 99.37 | 99.39 | 99.40 | 99.42 | 99.43 | 99.45 | 99.46 | 99.47 | 99.47 | 99.48 | 99.49 | 99.50    |
| 3              | 3        | 34.12  | 30.82  | 29.46 | 28.71 | 28.24 | 27.91 | 27.67 | 27.49 | 27.35 | 27.23 | 27.05 | 26.87 | 26.69 | 26.00 | 26.50 | 26.41 | 26.32 | 26.22 | 26.13    |
| 4              | 4        | 21.20  | 18.00  | 16.69 | 15.98 | 15.52 | 15.21 | 14.98 | 14.80 | 14.66 | 14.55 | 14.37 | 14.20 | 14.02 | 13.93 | 13.84 | 13.75 | 13.65 | 13.56 | 13.46    |
| 5              | 5        | 16.26  | 13.27  | 12.06 | 11.39 | 10.97 | 10.67 | 10.46 | 10.29 | 10.16 | 10.05 | 9.89  | 9.72  | 9.55  | 9.47  | 9.38  | 9.29  | 9.20  | 9.11  | 9.02     |
| 6              | 6        | 13.75  | 10.92  | 9.78  | 9.15  | 8.75  | 8.47  | 8.26  | 8.10  | 7.98  | 7.87  | 7.72  | 7.56  | 7.40  | 7.31  | 7.23  | 7.14  | 7.06  | 6.97  | 6.88     |
| 7              | 7        | 12.25  | 9.55   | 8.45  | 7.85  | 7.46  | 7.19  | 6.99  | 6.84  | 6.72  | 6.62  | 6.47  | 6.31  | 6.16  | 6.07  | 5.99  | 5.91  | 5.82  | 5.74  | 5.65     |
| 8              | 8        | 11.26  | 8.65   | 7.59  | 7.01  | 6.63  | 6.37  | 6.18  | 6.03  | 5.91  | 5.81  | 5.67  | 5.52  | 5.36  | 5.28  | 5.20  | 5.12  | 5.03  | 4.95  | 4.86     |
| 9              | 9        | 10.56  | 8.02   | 6.99  | 6.42  | 6.06  | 5.80  | 5.61  | 5.47  | 5.35  | 5.26  | 5.11  | 4.96  | 4.81  | 4.73  | 4.65  | 4.57  | 4.48  | 4.40  | 4.31     |
| 10             | 10       | 10.04  | 7.56   | 6.55  | 5.99  | 5.64  | 5.39  | 5.20  | 5.06  | 4.94  | 4.85  | 4.71  | 4.56  | 4.41  | 4.33  | 4.25  | 4.17  | 4.08  | 4.00  | 3.91     |
| 11             | 11       | 9.65   | 7.21   | 6.22  | 5.67  | 5.32  | 5.07  | 4.89  | 4.74  | 4.63  | 4.54  | 4.40  | 4.25  | 4.10  | 4.02  | 3.94  | 3.86  | 3.78  | 3.69  | 3.60     |
| 12             | 12       | 9.33   | 6.93   | 5.95  | 5.41  | 5.06  | 4.82  | 4.64  | 4.50  | 4.39  | 4.30  | 4.16  | 4.01  | 3.86  | 3.78  | 3.70  | 3.62  | 3.54  | 3.45  | 3.36     |
| 13             | 13       | 9.07   | 6.70   | 5.74  | 5.21  | 4.86  | 4.62  | 4.44  | 4.30  | 4.19  | 4.10  | 3.96  | 3.82  | 3.66  | 3.59  | 3.51  | 3.43  | 3.34  | 3.25  | 3.17     |
| 14             | 14       | 8.86   | 6.51   | 5.56  | 5.04  | 4.69  | 4.46  | 4.28  | 4.14  | 4.03  | 3.94  | 3.80  | 3.66  | 3.51  | 3.43  | 3.35  | 3.27  | 3.18  | 3.09  | 3.00     |
| 15             | 15       | 8.68   | 6.36   | 5.42  | 4.89  | 4.56  | 4.32  | 4.14  | 4.00  | 3.89  | 3.80  | 3.67  | 3.52  | 3.37  | 3.29  | 3.21  | 3.13  | 3.05  | 2.96  | 2.87     |
| 16             | 16       | 8.53   | 6.23   | 5.29  | 4.77  | 4.44  | 4.20  | 4.03  | 3.89  | 3.78  | 3.69  | 3.55  | 3.41  | 3.26  | 3.18  | 3.10  | 3.02  | 2.93  | 2.84  | 2.75     |
| 17             | 17       | 8.40   | 6.11   | 5.18  | 4.67  | 4.34  | 4.10  | 3.93  | 3.79  | 3.68  | 3.59  | 3.46  | 3.31  | 3.16  | 3.08  | 3.00  | 2.92  | 2.83  | 2.75  | 2.65     |
| 18             | 18       | 8.29   | 6.01   | 5.09  | 4.58  | 4.25  | 4.01  | 3.84  | 3.71  | 3.60  | 3.51  | 3.37  | 3.23  | 3.08  | 3.00  | 2.92  | 2.84  | 2.75  | 2.66  | 2.57     |
| 19             | 19       | 8.18   | 5.93   | 5.01  | 4.50  | 4.17  | 3.94  | 3.77  | 3.63  | 3.52  | 3.43  | 3.30  | 3.15  | 3.00  | 2.92  | 2.84  | 2.76  | 2.67  | 2.58  | 2.49     |
| 20             | 20       | 8.10   | 5.85   | 4.94  | 4.43  | 4.10  | 3.87  | 3.70  | 3.56  | 3.46  | 3.37  | 3.23  | 3.09  | 2.94  | 2.86  | 2.78  | 2.69  | 2.61  | 2.52  | 2.42     |
| 21             | 21       | 8.02   | 5.78   | 4.87  | 4.37  | 4.04  | 3.81  | 3.64  | 3.51  | 3.40  | 3.31  | 3.17  | 3.03  | 2.88  | 2.80  | 2.72  | 2.64  | 2.55  | 2.46  | 2.36     |
| 22             | 22       | 7.95   | 5.72   | 4.82  | 4.31  | 3.99  | 3.76  | 3.59  | 3.45  | 3.35  | 3.26  | 3.12  | 2.98  | 2.83  | 2.75  | 2.67  | 2.58  | 2.50  | 2.40  | 2.31     |
| 23             | 23       | 7.88   | 5.66   | 4.76  | 4.26  | 3.94  | 3.71  | 3.54  | 3.41  | 3.30  | 3.21  | 3.07  | 2.93  | 2.78  | 2.70  | 2.62  | 2.54  | 2.45  | 2.35  | 2.26     |
| 24             | 24       | 7.82   | 5.61   | 4.72  | 4.22  | 3.90  | 3.67  | 3.50  | 3.36  | 3.26  | 3.17  | 3.03  | 2.89  | 2.74  | 2.66  | 2.58  | 2.49  | 2.40  | 2.31  | 2.21     |
| 25             | 25       | 7.77   | 5.57   | 4.68  | 4.18  | 3.85  | 3.63  | 3.46  | 3.32  | 3.22  | 3.13  | 2.99  | 2.85  | 2.70  | 2.62  | 2.54  | 2.45  | 2.36  | 2.27  | 2.17     |
| 26             | 26       | 7.72   | 5.53   | 4.64  | 4.14  | 3.82  | 3.59  | 3.42  | 3.29  | 3.18  | 3.09  | 2.96  | 2.81  | 2.66  | 2.58  | 2.50  | 2.42  | 2.33  | 2.23  | 2.13     |
| 27             | 27       | 7.68   | 5.49   | 4.60  | 4.11  | 3.78  | 3.56  | 3.39  | 3.26  | 3.15  | 3.06  | 2.93  | 2.78  | 2.63  | 2.55  | 2.47  | 2.38  | 2.29  | 2.20  | 2.10     |
| 28             | 28       | 7.64   | 5.45   | 4.57  | 4.07  | 3.75  | 3.53  | 3.36  | 3.23  | 3.12  | 3.03  | 2.90  | 2.75  | 2.60  | 2.52  | 2.44  | 2.34  | 2.26  | 2.17  | 2.06     |
| 29             | 29       | 7.60   | 5.42   | 4.54  | 4.04  | 3.73  | 3.50  | 3.33  | 3.20  | 3.09  | 3.00  | 2.87  | 2.73  | 2.57  | 2.49  | 2.41  | 2.33  | 2.23  | 2.14  | 2.03     |
| 30             | 30       | 7.56   | 5.39   | 4.51  | 4.02  | 3.70  | 3.47  | 3.30  | 3.17  | 3.07  | 2.98  | 2.84  | 2.70  | 2.55  | 2.47  | 2.39  | 2.30  | 2.21  | 2.11  | 2.01     |
| 40             | 40       | 7.31   | 5.18   | 4.31  | 3.83  | 3.51  | 3.29  | 3.12  | 2.99  | 2.89  | 2.80  | 2.66  | 2.52  | 2.37  | 2.29  | 2.20  | 2.11  | 2.02  | 1.92  | 1.80     |
| 60             | 60       | 7.08   | 4.98   | 4.13  | 3.65  | 3.34  | 3.12  | 2.95  | 2.82  | 2.72  | 2.63  | 2.50  | 2.35  | 2.20  | 2.12  | 2.03  | 1.94  | 1.84  | 1.73  | 1.60     |
| 120            | 120      | 6.85   | 4.79   | 3.95  | 3.48  | 3.17  | 2.96  | 2.79  | 2.66  | 2.56  | 2.47  | 2.34  | 2.19  | 2.03  | 1.95  | 1.86  | 1.76  | 1.66  | 1.53  | 1.38     |
| $\infty$       | $\infty$ | 6.63   | 4.61   | 3.78  | 3.32  | 3.02  | 2.80  | 2.64  | 2.51  | 2.41  | 2.32  | 2.18  | 2.04  | 1.88  | 1.79  | 1.70  | 1.59  | 1.47  | 1.32  | 1.16     |