UNIVERSITY OF SWAZILAND

SUPPLEMENTARY EXAMINATION PAPER 2015

TITLE OF PAPER	:	SAMPLE SURVEY THEORY
COURSE CODE	:	ST306
TIME ALLOWED	:	TWO (2) HOURS
REQUIREMENTS	:	CALCULATOR AND STATISTICAL TABLES
INSTRUCTIONS	:	ANSWER ANY THREE QUESTIONS

Question 1 [20 marks, 15+5]

(a) A sociologist wants to estimate the average per capita income in a certain small city. As no list of resident adults is available, she decides that each of the city blocks will be considered one cluster. The clusters are numbered on a city map from 1 to 415, and the experimenter decides she has enough time and money to sample $n = 25$ clusters where every household will be interviewed within the clusters (blocks) chosen. The data on the next table give the number of residents and the total income for each of the 25 blocks sampled.

Cluster i	Number of Residents, M_i	Total Income per Cluster, y_i	Cluster i	Number of Residents, M_i	Total Income per Cluster, y_i
1	8	SZL192,000	14	10	SZL 98,000
2	12	SZL242,000	15	9	SZL106,000
3	4	SZL 84,000	16	3	SZL100,000
4	5	SZL130,000	17	6	SZL 64,000
5	6	SZL104,000	18	5	SZL 44,000
6	6	SZL 80,000	19	5	SZL 90,000
7	7	SZL150,000	20	4	SZL 74,000
8	5	SZL130,000	21	6	SZL102,000
9	8	SZL 90,000	22	8	SZL 60,000
10	3	SZL100,000	23	7	SZL 78,000
11	2	SZL170,000	24	3	SZL 94,000
12	6	SZL 86,000	25	8	SZL 82,000
13	5	SZL108,000			
				$\sum_{i=1}^{25} M_i = 151$	$\sum_{i=1}^{25} y_i = \text{SZL}2,658,000$

Given that $M = 2500$ residents, use these data to estimate the (unbiased) average per capita income in the city and its associated standard error.

(b) A survey of the 9^{th} graders in Ndunayithini is intended to determine the proportion intending to go to a four-year college. A preliminary estimate of $p = 0.55$ was obtained from a small informal survey. How large must the survey be to provide an estimator with error at most 0.05 with probability at least 99%?

Question 2 [20 marks, 10+5+5]

(a) Foresters want to estimate the average age of trees in a stand. Determining age is cumbersome because one needs to count the tree rings on a core taken from the tree. In general, though, the older the tree, the larger the diameter, and diameter is easy to measure. The foresters measure the diameter of all 1132 trees and find that the population mean equals 10.3. They then randomly select 20 trees for age measurement.

Tree No.	Diameter, x	Age, y	Tree No.	Diameter, x	Age, y
1	12.0	125	11	5.7	61
2	11.4	119	12	8.0	80
3	7.9	83	13	10.3	114
4	9.0	85	14	12.0	147
5	10.5	99	15	9.2	122
6	7.9	117	16	8.5	106
7	7.3	69	17	7.0	82
8	10.2	133	18	10.7	88
9	11.7	154	19	9.3	97
10	11.3	168	20	8.2	99

Estimate the population mean age of trees in the stand and give an approximate standard error for your estimate.

(b) An accounting firm is interested in estimating the error rate in a compliance audit it is conducting. The population contains 828 claims, and the firm audits an SRS of 85 of those claims. In each of the 85 sampled claims, 215 fields are checked for errors. One claim has errors in 4 of the 215 fields, 1 claim has three errors, 4 claims have two errors, 22 claims have one error, and the remaining 57 claims have no errors. (Data courtesy of Fritz Scheuren.)

   (i) Treating the claims as psu's and the observations for each field as ssu's, estimate the error rate for all 828 claims. Give a standard error for your estimate.

   (ii) Estimate (with SE) the total number of errors in the 828 claims.

Question 3 [20 marks, 6+6+8]

A village contains 175 children. Dr. Jones takes a SRS of 17 of them and counts the cavities in each ones mouth, finding the frequency table:

Number of Cavities	0	1	2	3	4	5
Number of Children	5	4	2	3	2	1

Dr. Smith examines all 175 childrens mouths and records that 55 have no cavities. Estimate the total number of cavities in the villages children using

(a) only Dr. Jones data,

(b) both Dr. Jones and Dr. Smiths data.

(c) Give approximately unbiased estimates for the variances of your (approximately) unbiased estimators in both (a) and (b).

Question 4 [20 marks, 1+2+5+2+4+2+4]

(a) Consider a population with the following values..

$$y_1 = 98 \quad y_3 = 154 \quad y_5 = 190$$
$$y_2 = 102 \quad y_4 = 133 \quad y_6 = 175$$

We are interested μ, the population mean. Two sampling plans are proposed.

- Plan 1: Eight possible samples may be chosen.

Sample Number	Sample, S	$P(S=s)$
1	{1,3,5}	$\frac{1}{8}$
2	{1,3,6}	$\frac{1}{8}$
3	{1,4,5}	$\frac{1}{8}$
4	{1,4,6}	$\frac{1}{8}$
5	{2,3,5}	$\frac{1}{8}$
6	{2,3,6}	$\frac{1}{8}$
7	{2,4,5}	$\frac{1}{8}$
8	{2,4,6}	$\frac{1}{8}$

- Plan 2: Three possible samples may be chosen.

Sample Number	Sample, S	$P(S=s)$
1	{1,4,5}	$\frac{1}{4}$
2	{2,3,6}	$\frac{1}{2}$
3	{1,3,5}	$\frac{1}{4}$

(i) What is the value of μ?

(ii) Let $E(\hat{\mu})$ be the mean of the sample values. For each sampling plan, find:

i. $E(\hat{\mu})$;

ii. Var($\hat{\mu}$);

iii. Bias($\hat{\mu}$);

iv. MSE($\hat{\mu}$).

(iii) Which sampling plan do you think is better? Why?

(b) Consider a population of farms on a 25×25 grid of varying sizes and shapes. If we randomly select a single square on this grid, then letting x_i = the area of farm i and $A = 625$ total units, the probability that farm i is selected is: $p_i = \frac{x_i}{A} = \frac{x_i}{625}$.

y_i =Workers	$p_i = \frac{x_i}{A} = \frac{\text{Size of Farm}}{\text{Total Area}}$
2	5/625
8	28/625
4	12/625
8	14/625
3	13/625

The table above shows a replacement sample of 5 farms selected with probability-proportional-to-size (PPS). Compute the estimated number of farms (and associated standard errors) using the Hansen-Hurwitz estimator.

Question 5 [20 marks, 6+8+6]

(a) Suppose we want to estimate the average number of hours of TV watched in the previous week for all adults in some county. Suppose also that the populace of this county can be grouped naturally into 3 strata (town A, town B, rural) as summarized in the table

Statistic	Town A	Town B	Rural
N_h	155	62	93
n_h	20	8	12
$\bar{y}_h$	33.90	25.12	19.00
s_h	5.95	15.24	9.36
$\hat{\tau}_h$	5254.5	1557.4	1767.0
c_h	2	2	3

(i) Compute a 95% confidence interval for the total number of hours of TV watched in the previous week for all adults in this county.

(ii) Estimate the total sample size needed to estimate the mean hours of TV watched in this particular county to within 1 hour with 99% probability using optimal allocation (unequal and equal costs).

(b) A local radio station carries out regular polls of its listeners on items of current interest. In one such poll listeners were asked to telephone the station and just answer "yes" or "no" to the following questions.

Do you think dogs should be allowed in public places only if on the lead?

The poll was carried out between 8 am and 9 am one morning. At 8:30 am the announcer said the percentage of "yes" vote was 63%. When the poll closed at 9 am he announced that the percentage was 52%. List two problems associated with this method of polling and suggest why each problem might cause misleading conclusion to be drawn.

Useful formulas

$$s^2 = \frac{\sum_{i=1}^{n}(y_i - \bar{y})^2}{n-1}$$

$$\sum_{i=1}^{n}(y_i - \bar{y})^2 = \sum_{i=1}^{n} y_i^2 - \frac{\sum_{i=1}^{n} y_i}{n}$$

$$\hat{\mu}_{srs} = \bar{y}$$

$$\hat{\mathsf{V}}(\hat{\mu}_{srs}) = \left(\frac{N-n}{N}\right)\frac{s^2}{n}$$

$$\hat{\tau}_{srs} = N\hat{\mu}_{srs}$$

$$\hat{\mathsf{V}}(\hat{\tau}_{srs}) = N^2\hat{\mathsf{V}}(\hat{\mu}_{srs})$$

$$\hat{p}_{srs} = \sum_{i=1}^{n}\frac{y_i}{n}$$

$$\left(\frac{N-n}{N}\right)\frac{\hat{p}(1-\hat{p})}{n-1}\left(\frac{N-n}{N}\right)$$

$$\hat{\tau}_{hh} = \frac{1}{n}\sum_{i=1}^{n}\frac{y_i}{p_i}$$

$$\hat{\mathsf{V}}(\hat{\mu}_{hh}) = \frac{1}{n(n-1)}\sum_{i=1}^{n}\left(\frac{y_i}{p_i} - \hat{\tau}_{hh}\right)$$

$$\hat{\mu}_{hh} = \frac{\hat{\tau}_{hh}}{N}$$

$$\hat{\mathsf{V}}(\hat{\mu}_{hh}) = \frac{1}{N^2}\hat{\mathsf{V}}(\hat{\tau}_{hh})$$

$$\hat{\tau}_{ht} = \sum_{i=1}^{\nu}\frac{y_i}{\pi_i}$$

$$\hat{\mathsf{V}}(\hat{\tau}_{ht}) = \sum_{i=1}^{\nu}\left(\frac{1}{\pi_i^2} - \frac{1}{\pi_i}\right)y_i^2 +$$

$$2\sum_{i=1}^{\nu}\sum_{j>i}^{\nu}\left(\frac{1}{\pi_i\pi_j} - \frac{1}{\pi_{ij}}\right)y_i y_j$$

$$\hat{\mu}_{ht} = \frac{\hat{\tau}_{ht}}{N}$$

$$\hat{\mathsf{V}}(\hat{\mu}_{ht}) = \frac{1}{N^2}\hat{\mathsf{V}}(\hat{\tau}_{ht})$$

$$\hat{r} = \frac{\sum_{i=1}^{n} y_i}{\sum_{i=1}^{n} x_i}$$

$$\hat{\mathsf{V}}(\hat{r}) = \left(\frac{N-n}{Nn\mu_x^2}\right)\frac{\sum_{i=1}^{n}(y_i - rx_i)^2}{n-1}$$

$$\hat{\mu}_r = r\mu_x$$

$$\hat{\mathsf{V}}(\hat{\mu}_r) = \left(\frac{N-n}{Nn}\right)\frac{\sum_{i=1}^{n}(y_i - rx_i)^2}{n-1}$$

$$\hat{\tau}_r = Nr\mu_x = r\tau_x$$

$$\hat{\mathsf{V}}(\hat{\tau}_r) = \frac{N(N-n)}{n}\frac{\sum_{i=1}^{n}(y_i - rx_i)^2}{n-1}$$

$$\hat{\mu}_L = a + b\mu_x$$

$$\hat{\mathsf{V}}(\hat{\mu}_L) = \frac{N-n}{Nn(n-1)}\sum_{i=1}^{n}(y_i - a - bx_i)^2$$

$$\hat{\tau}_L = N\mu_L$$

$$\hat{\mathsf{V}}(\hat{\tau}_L) = \frac{N(N-n)}{n(n-1)}\sum_{i=1}^{n}(y_i - a - bx_i)^2$$

$$\hat{\mu}_{str} = \sum_{h=1}^{L}\frac{N_h}{N}\bar{y}_h$$

$$\hat{\mathsf{V}}(\hat{\mu}_{str}) = \frac{1}{N^2}\sum_{h=1}^{L}N_h^2\left(\frac{N_h - n_h}{N_h}\right)\frac{s_h^2}{n_h}$$

$$\hat{\tau}_{str} = N\hat{\mu}_{str}$$

$$\hat{\mathsf{V}}(\hat{\tau}_{str}) = N^2\hat{\mathsf{V}}(\hat{\mu}_{str})$$

$$\hat{p}_{str} = \sum_{h=1}^{L}\frac{N_h}{N}\hat{p}_h$$

$$\hat{\mathsf{V}}(\hat{p}_{str}) = \frac{1}{N^2}\sum_{h=1}^{L}N_h^2\left(\frac{N_h - n_h}{N_h}\right)\left(\frac{\hat{p}_h(1-\hat{p}_h)}{n_h - 1}\right)$$

$$\hat{\mu}_{pstr} = \sum_{h=1}^{L}w_h\bar{y}_h$$

$$\hat{\mathsf{V}}(\hat{\mu}_{pstr}) = \frac{1}{n}\left(\frac{N-n}{N}\right)\sum_{h=1}^{L}w_h s_h^2 + \frac{1}{n^2}\sum_{h=1}^{L}(1-w_h)s_h^2$$

$$\hat{\tau}_{cl} = \frac{M}{nL}\sum_{i=1}^{n}\sum_{j=1}^{L} y_{ij} = \frac{N}{n}\sum_{i=1}^{n}\sum_{j=1}^{L} y_{ij} = \frac{N}{n}\sum_{i=1}^{n} y_i = N\bar{y}$$

$$\hat{\mu}_{cl} = \frac{1}{nL}\sum_{i=1}^{n}\sum_{j=1}^{L} y_{ij} = \frac{1}{nL}\sum_{i=1}^{n} y_i = \frac{\bar{y}}{L} = \frac{\hat{\tau}_{cl}}{M}$$

where $\bar{y} = \frac{1}{n}\sum_{i=1}^{n} y_i = \frac{\hat{\tau}_{cl}}{N}$

$$\hat{V}(\hat{\tau}_{cl}) = N(N-n)\frac{s_u^2}{n} \qquad \hat{V}(\hat{\mu}_{cl}) = \frac{N(N-n)}{M^2}\frac{s_u^2}{n}$$

where $s_u^2 = \frac{\sum_{i=1}^{n}(y_i - \bar{y})^2}{n-1}$.

$$\hat{\mu}_1 = \bar{y} = \frac{\hat{\tau}_{cl}}{N} \qquad \hat{V}(\hat{\mu}_1 = \frac{N-n}{N}\frac{s_u^2}{n}$$

The formulas for systematic sampling are the same as those used for one-stage cluster sampling. Change the subscript *cl* to *sys* to denote the fact that data were collected under systematic sampling.

$$\hat{\mu}_{c(a)} = \frac{\sum_{i=1}^{n} y_i}{\sum_{i=1}^{n} M_i} = \frac{\sum_{i=1}^{n} y_i}{m} \qquad \hat{V}(\hat{\mu}_{c(a)}) = \frac{(N-n)N}{n(n-1)M^2}\sum_{i=1}^{n} M_i^2(\bar{y} - \hat{\mu}_{c(a)})^2$$

$$\hat{\mu}_{c(b)} = \frac{N}{M}\frac{\sum_{i=1}^{n} y_i}{n} = \frac{N}{nM}\sum_{i=1}^{n} y_i \qquad \hat{V}(\hat{\mu}_{c(b)}) = \frac{(N-n)N}{n(n-1)M^2}\sum_{i=1}^{n}(y_i - \bar{y})^2 = \frac{(N-n)N}{nM^2}s_u^2$$

$$\hat{p}_c = \frac{\sum_{i=1}^{n} p_i}{n} \qquad \hat{V}(\hat{p}_c) = \left(\frac{N-Nn}{nN}\right)\sum_{i=1}^{n}\frac{(p_i - \hat{p}_c)^2}{n-1} = \left(\frac{1-f}{n}\right)\sum_{i=1}^{n}\frac{(p_i - \hat{p}_c)^2}{n-1}$$

$$\hat{p}_c = \frac{\sum_{i=1}^{n} y_i}{\sum_{i=1}^{n} M_i} \qquad \hat{V}(\hat{p}_c) = \left(\frac{1-f}{n\bar{m}^2}\right)\frac{\sum_{i=1}^{n}(y_i - \hat{p}_c M_i)^2}{n-1}$$

To estimate τ, multiply $\hat{\mu}_{c(\cdot)}$ by M. To get the estimated variances, multiply $\hat{V}(\hat{\mu}_{c(\cdot)})$ by M^2. If M is not known, substitute M with Nm/n. $\bar{m} = \sum_{i=1}^{n} M_i/n$.

n for μ SRS $\quad n = \frac{N\sigma^2}{(N-1)(d^2/z^2) + \sigma^2}$

n for τ SRS $\quad n = \frac{N\sigma^2}{(N-1)(d^2/z^2N^2) + \sigma^2}$

n for p SRS $\quad n = \frac{Np(1-p)}{(N-1)(d^2/z^2) + p(1-p)}$

n for μ SYS $\quad n = \frac{N\sigma^2}{(N-1)(d^2/z^2) + \sigma^2}$

n for τ SYS $\quad n = \frac{N\sigma^2}{(N-1)(d^2/z^2N^2) + \sigma^2}$

n for μ STR $\quad n = \frac{\sum_{h=1}^{L} N_h^2(\sigma_h^2/w_h)}{N^2(d^2/z^2) + \sum_{h=1}^{L} N_h\sigma_h^2}$

n for τ STR $\quad n = \frac{\sum_{h=1}^{L} N_h^2(\sigma_h^2/w_h)}{N^2(d^2/z^2N^2) + \sum_{h=1}^{L} N_h\sigma_h^2}$

where $w_h = \frac{n_h}{n}$.

Allocations for STR μ:

$$n_h = (c - c_0)\left(\frac{N_h\sigma_h/\sqrt{c_h}}{\sum_{k=1}^{L} N_k\sigma_k\sqrt{c_k}}\right) \qquad (c - c_0) = \frac{\left(\sum_{k=1}^{L} N_k\sigma_k/\sqrt{c_k}\right)\left(\sum_{k=1}^{L} N_k\sigma_k\sqrt{c_k}\right)}{N^2(d^2/z^2) + \sum_{k=1}^{L} N_k\sigma_k^2}$$

$$n_h = n\left(\frac{N_h}{N}\right) \qquad n = \frac{\sum_{k=1}^{L} N_k\sigma_k}{N^2(d^2/z^2) + \frac{1}{N}\sum_{k=1}^{L} N_k\sigma_k^2}$$

$$n_h = n\left(\frac{N_h\sigma_h}{\sum_{k=1}^{L} N_k\sigma_k}\right) \qquad n = \frac{\left(\sum_{k=1}^{L} N_k\sigma_k\right)^2}{N^2(d^2/z^2) + \sum_{k=1}^{L} N_k\sigma_k^2}$$

Allocations for STR τ:

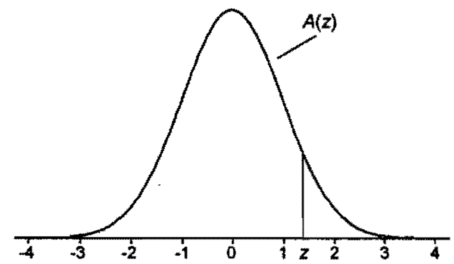
change $N^2(d^2/z^2)$ to $N^2(d^2/z^2N^2)$

Allocations for STR p:

$$n_h = n\left(\frac{N_i\sqrt{p_h(1-p_h)/c_h}}{\sum_{k=1}^{L} N_k\sqrt{p_k(1-p_k)/c_k}}\right) \qquad n = \frac{\sum_{k=1}^{L} N_k p_k(1-p_k)/w_k}{N^2(d^2/z^2) + \sum_{k=1}^{L} N_k p_k(1-p_k)}$$

Table A.1

Cumulative Standardized Normal Distribution



$A(z)$ is the integral of the standardized normal distribution from $-\infty$ to z (in other words, the area under the curve to the left of z). It gives the probability of a normal random variable not being more than z standard deviations above its mean. Values of z of particular importance:

z	$A(z)$	
1.645	0.9500	Lower limit of right 5% tail
1.960	0.9750	Lower limit of right 2.5% tail
2.326	0.9900	Lower limit of right 1% tail
2.576	0.9950	Lower limit of right 0.5% tail
3.090	0.9990	Lower limit of right 0.1% tail
3.291	0.9995	Lower limit of right 0.05% tail

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998
3.5	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998
3.6	0.9998	0.9998	0.9999							

TABLE A.2

t* Distribution: Critical Values of *t

Degrees of freedom	Two-tailed test: / One-tailed test:	Significance level					
	Two-tailed test:	10%	5%	2%	1%	0.2%	0.1%
	One-tailed test:	5%	2.5%	1%	0.5%	0.1%	0.05%
1		6.314	12.706	31.821	63.657	318.309	636.619
2		2.920	4.303	6.965	9.925	22.327	31.599
3		2.353	3.182	4.541	5.841	10.215	12.924
4		2.132	2.776	3.747	4.604	7.173	8.610
5		2.015	2.571	3.365	4.032	5.893	6.869
6		1.943	2.447	3.143	3.707	5.208	5.959
7		1.894	2.365	2.998	3.499	4.785	5.408
8		1.860	2.306	2.896	3.355	4.501	5.041
9		1.833	2.262	2.821	3.250	4.297	4.781
10		1.812	2.228	2.764	3.169	4.144	4.587
11		1.796	2.201	2.718	3.106	4.025	4.437
12		1.782	2.179	2.681	3.055	3.930	4.318
13		1.771	2.160	2.650	3.012	3.852	4.221
14		1.761	2.145	2.624	2.977	3.787	4.140
15		1.753	2.131	2.602	2.947	3.733	4.073
16		1.746	2.120	2.583	2.921	3.686	4.015
17		1.740	2.110	2.567	2.898	3.646	3.965
18		1.734	2.101	2.552	2.878	3.610	3.922
19		1.729	2.093	2.539	2.861	3.579	3.883
20		1.725	2.086	2.528	2.845	3.552	3.850
21		1.721	2.080	2.518	2.831	3.527	3.819
22		1.717	2.074	2.508	2.819	3.505	3.792
23		1.714	2.069	2.500	2.807	3.485	3.768
24		1.711	2.064	2.492	2.797	3.467	3.745
25		1.708	2.060	2.485	2.787	3.450	3.725
26		1.706	2.056	2.479	2.779	3.435	3.707
27		1.703	2.052	2.473	2.771	3.421	3.690
28		1.701	2.048	2.467	2.763	3.408	3.674
29		1.699	2.045	2.462	2.756	3.396	3.659
30		1.697	2.042	2.457	2.750	3.385	3.646
32		1.694	2.037	2.449	2.738	3.365	3.622
34		1.691	2.032	2.441	2.728	3.348	3.601
36		1.688	2.028	2.434	2.719	3.333	3.582
38		1.686	2.024	2.429	2.712	3.319	3.566
40		1.684	2.021	2.423	2.704	3.307	3.551
42		1.682	2.018	2.418	2.698	3.296	3.538
44		1.680	2.015	2.414	2.692	3.286	3.526
46		1.679	2.013	2.410	2.687	3.277	3.515
48		1.677	2.011	2.407	2.682	3.269	3.505
50		1.676	2.009	2.403	2.678	3.261	3.496
60		1.671	2.000	2.390	2.660	3.232	3.460
70		1.667	1.994	2.381	2.648	3.211	3.435
80		1.664	1.990	2.374	2.639	3.195	3.416
90		1.662	1.987	2.368	2.632	3.183	3.402
100		1.660	1.984	2.364	2.626	3.174	3.390
120		1.658	1.980	2.358	2.617	3.160	3.373
150		1.655	1.976	2.351	2.609	3.145	3.357
200		1.653	1.972	2.345	2.601	3.131	3.340
300		1.650	1.968	2.339	2.592	3.118	3.323
400		1.649	1.966	2.336	2.588	3.111	3.315
500		1.648	1.965	2.334	2.586	3.107	3.310
600		1.647	1.964	2.333	2.584	3.104	3.307
∞		1.645	1.960	2.326	2.576	3.090	3.291