UNIVERSITY OF SWAZILAND

DEPARTMENT OF STATISTICS AND DEMOGRAPHY

MAIN EXAMINATION, 2015/16

COURSE TITLE: MATHETHEMATICS FOR STATISTICS

COURSE CODE: ST 202

TIME ALLOWED: TWO (2) HOURS

INSTRUCTION: ANSWER ANY THREE QUESTIONS
ALL QUESTIONS CARRY EQUAL MARKS (25 MARKS)

SPECIAL REQUIREMENTS: SCIENTIFIC CALCULATORS AND GRAPH PAPER

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Question 1

(a) Use langrange multipliers to find the extremum of 'f' subject to the given constraints, assuming that x, y and z are non-negative.
Maximise
$f(x, y, z) = xy + yz$
$Constraints: x + 2y = 6; x = 3z$ **(15 marks)**

(b) Use partial derivatives to find the values of 'a' and 'b' such that the linear model $f(x) = ax + b$ has a minimum sum of squared errors for the given points: (-2,0), (-1,1), (0,1), (1,2), (2,3). **(10 marks)**

Question 2

(a) The marketing department of a business has determined that the demand for a product can be modelled by $p = \frac{50}{\sqrt{x}}$
The cost of producing 'x' units is given by: $C = 0.5x + 500$. What price will yield maximum profit? **(7 marks)**

(b) The cost 'C' of inventory depends on ordering and storage costs such that: $C = \left(\frac{Q}{x}\right)s + \left(\frac{x}{2}\right)r$, where Q is the number of units sold per year, 'r' is the cost of storing one unit for one year, 's' is the cost of placing an order, and 'x' is the number of units in the order. Determine the order size that will minimise the cost when Q = 10,000, s = 4.5 and r = 5.76. **(5 marks)**

(c) Analyse the graph of a function $f(x) = x^3 - 4x^2 + 5x - 4$. Determine all critical values and critical points of $f'(x)$ to find the relative/absolute maxima and minima of this function, and then indicate the direction of the slope around each test interval. Sketch the graph of this function. **(5+5+3 marks)**

Question 3

(a) For the function $f(x) = 12x(1-x)^2$

(i) Show that this is a probability density function over the interval (0,1).

(ii) Find the probability that 'x' lies in the interval $\frac{1}{2} \le x \le \frac{3}{4}$

(10 marks)

(b) A baseball fan examined the record of a favourite baseball player's performance during his last 50 games. The number of games in which the player scored zero, one, two, three and four hits are recored in the table shown below:

Number of Hits	0	1	2	3	4
Frequency	14	26	7	2	1

(i) Complete the table below, where 'x' is the number of hits: **(2 marks)**

x	0	1	2	3	4
P(x)					

(ii) Use the table in (a) to find $P(1 \le x \le 3)$ **(3 marks)**

(iii) Determine E(x), V(x) and σ. Explain your results. **(10 marks)**

Question 4

(a) Find A^{-1} for the following matrix:

$$A = \begin{bmatrix} 1 & 0 & 4 \\ 4 & 1 & -2 \\ 3 & 1 & -1 \end{bmatrix}$$

(7 marks)

(b) Solve the following linear system of equations using the method of determinants:

$$\begin{aligned} x + \quad 4z &= 4 \\ 4x + y - 2z &= 0 \\ 3x + y - z &= 2 \end{aligned}$$

(7 marks)

(c) Suppose $x = \begin{bmatrix} 2 \\ 0 \\ -4 \end{bmatrix}$ and $y = \begin{bmatrix} 0 \\ -1 \\ -3 \end{bmatrix}$

Use vector algebra to find the least squares regression line through the set of points determined by vectors x and y. **(6 marks)**

Question 5

(a) Solve the following system of equations using the Gauss-Jordan elimination method:

$$x_1 + 2x_2 + 3x_3 = 6$$
$$2x_1 - 4x_2 + 2x_3 = 16$$
$$3x_1 + x_2 - x_3 = -2$$

(15 marks)

(b) Given the system of equations: $\begin{cases} x_1 + x_2 = 3 \\ x_1 + (a^2 - 8)x_2 = a \end{cases}$

Find all values of "a" such that the system has:
(i) no solution
(ii) a unique solution
(iii)Infinitely many solutions.

(10 marks)

END OF EXAM!!!!!