UNIVERSITY OF SWAZILAND

FINAL EXAMINATION PAPER 2016

TITLE OF PAPER	:	SAMPLE SURVEY THEORY
COURSE CODE	:	ST306
TIME ALLOWED	:	TWO (2) HOURS
REQUIREMENTS	:	CALCULATOR AND STATISTICAL TABLES
INSTRUCTIONS	:	ANSWER ANY THREE QUESTIONS

Question 1 [20 marks, 9+3+8]

(a) Consider a population of farms on a 25×25 grid of varying sizes and shapes. If we randomly select a single square on this grid, then letting x_i = the area of farm i and $A = 625$ total units, the probability that farm i is selected is: $p_i = \frac{x_i}{A} = \frac{x_i}{625}$.

y_i =Workers	$p_i = \frac{x_i}{A} = \frac{\text{Size of Farm}}{\text{Total Area}}$
2	5/625
8	28/625
4	12/625
3	13/625

The table above shows a replacement sample of 4 farms selected with probability-proportional-to-size (PPS). Compute:

(i) The estimated number of workers (and associated standard errors).

(ii) The estimated number of farms.

using the Horvitz-Thompson estimator.

(b) The following coefficients of variation per unit were obtained in a farm survey in Iowa, the unit being an area 1 mile square (data of R.J. Jessen):

Item	Estimated cv (%)
Acres in farms	38
Acres in corn	39
Acres in oats	44
Number of family workers	100
Number of hired workers	110
Number of unemployed	317

A survey is planned to estimate acreage items with a cv of $2\frac{1}{2}\%$ and numbers of workers (excluding unemployed) with a cv of 5%. With simple random sampling, how many units are needed? How well would this sample be expected to estimate the number of unemployed?

Question 2 [20 marks, 5+5+7+3]

A manufacturer of band saws wants to estimate the average repair cost per month for the saws he has sold to certain industries. He cannot obtain a repair cost for each saw, but he can obtain the total amount spent for saw repairs and the total number of saws owned by each industry. Thus he decides to use cluster sampling, with each industry as a cluster. The manufacturer selects a simple random sample of size $n = 20$ from the $N = 82$ industries he services. The data on total cost of repairs per industry and the number of saws per industry are as given in the accompanying table.

Industry	Number of Saws	Total Repair Cost for Past Month (SZL)	Industry	Number of Saws	Total Repair Cost for Past Month (SZL)
1	3	50	11	8	140
2	7	110	12	6	120
3	11	230	13	3	70
4	9	140	14	2	50
5	2	50	15	1	10
6	12	260	16	4	60
7	14	240	17	12	280
8	3	45	18	6	150
9	5	60	19	5	110
10	9	230	20	8	120

(a) Estimate the average repair cost per saw for the past month, and give the standard error of this estimate.

(b) Estimate the total amount spent by the 82 industries on band saw repairs and give the standard error of this estimate.

(c) After checking his sales records, the manufacturer finds that he sold a total of 690 band saws to these industries. Using this additional information, estimate the total amount spent on saw repairs by these industries, and give the standard error.

(d) The manufacturer wants to estimate the average repair cost per saw for next month. How many clusters should he select for his sample if he wants to estimate this average cost to within SZL2.00 with 95% confidence?

Question 3 [20 marks, 2+2+4+4+4+4]

(a) A village contains 175 children. Dr. Jones takes a SRS of 17 of them and counts the cavities in each ones mouth, finding the frequency table:

Number of Cavities	0	1	2	3	4	5
Number of Children	5	4	2	3	2	1

Dr. Smith examines all 175 childrens mouths and records that 55 have no cavities. Estimate the total number of cavities in the villages children using

(i) only Dr. Jones data,

(ii) both Dr. Jones and Dr. Smiths data.

(iii) Give approximately unbiased estimate for the variance of the estimator in (ii).

(b) A simple random sample of 290 households was chosen from a city area containing 14,828 households. Each family was asked whether it owned or rented the house and also whether it had the exclusive use of an indoor toilet. Results were as follows.

	Exclusive use of toilet		
	Yes	No	Total
Owned	141	6	147
Rented	109	34	143
Total	250	40	290

(i) For families who rent, estimate the percentage in the area with exclusive use of an indoor toilet and give the standard error of your estimate;

(ii) estimate the total number of renting families in the area who do not have exclusive indoor toilet facilities and give the standard error of this estimate.

(c) A stratified random sample is better for estimating the population mean (in the sense of having a smaller variance) than a simple random sample of the same size, when the variability between strata is high compared to the variability within strata. What do you think will be the case for cluster sampling in terms of the variability between clusters as compared to the variability within clusters? Why?

Question 4 [20 marks, 8+6+6]

(a) In a study to estimate the total sugar content of a truckload of oranges, a random sample of $n = 10$ oranges was juiced and weighed. The data for the 10 oranges are given in the table below and displayed in a plot of sugar content versus weight.

Orange	Sugar Content (in pounds)	Weight of Orange (in pounds)
1	0.021	0.40
2	0.030	0.48
3	0.025	0.43
4	0.022	0.42
5	0.033	0.50
6	0.027	0.46
7	0.019	0.39
8	0.021	0.41
9	0.023	0.42
10	0.025	0.44

(i) The total weight of all the oranges, obtained by first weighing the truck loaded and then unloaded, was found to be 1800 pounds. Estimate τ_y, the total sugar content for the oranges, and place a bound on the error of estimation.

(ii) Roughly how many oranges must be sampled from the truck of oranges weighing 1800 pounds in order for the standard error of the estimator to be about 3 pounds, where

$$\sum_{i=1}^{N} \frac{(y_i - Rx_i)^2}{N-1} = 0.0066^2$$

You may assume that the mean weight on an orange is 0.396 pounds.

(b) In a district containing 4000 houses the percentage of owned houses is to be estimated with a standard error of not more than 2% and the percentage of two-car households with a standard error of not more than 1%. The true percentage of owners is thought to lie between 45 an 65% and the percentage of two-car households between 5 and 10%. How large a sample is necessary to satisfy both aims?

Question 5 [20 marks, 6+8+6]

(a) Suppose we want to estimate the average number of hours of TV watched in the previous week for all adults in some county. Suppose also that the populace of this county can be grouped naturally into 3 strata (town A, town B, rural) as summarized in the table

Statistic	Town A	Town B	Rural
N_h	155	62	93
n_h	20	8	12
$\bar{y}_h$	33.90	25.12	19.00
s_h	5.95	15.24	9.36
$\hat{\tau}_h$	5254.5	1557.4	1767.0
c_h	2	2	3

(i) Compute a 95% confidence interval for the total number of hours of TV watched in the previous week for all adults in this county.

(ii) Estimate the total sample size needed to estimate the mean hours of TV watched in this particular county to within 1 hour with 99% probability using optimal allocation (unequal and equal costs).

(b) A local radio station carries out regular polls of its listeners on items of current interest. In one such poll listeners were asked to telephone the station and just answer "yes" or "no" to the following questions.

Do you think dogs should be allowed in public places only if on the lead?

The poll was carried out between 8 am and 9 am one morning. At 8:30 am the announcer said the percentage of "yes" vote was 63%. When the poll closed at 9 am he announced that the percentage was 52%. List two problems associated with this method of polling and suggest why each problem might cause misleading conclusion to be drawn.

Useful formulas

$$s^2 = \frac{\sum_{i=1}^{n}(y_i - \bar{y})^2}{n-1}$$

$$\sum_{i=1}^{n}(y_i - \bar{y})^2 = \sum_{i=1}^{n} y_i^2 - \frac{\sum_{i=1}^{n} y_i}{n}$$

$$\hat{\mu}_{srs} = \bar{y}$$

$$\hat{\mathsf{V}}(\hat{\mu}_{srs}) = \left(\frac{N-n}{N}\right)\frac{s^2}{n}$$

$$\hat{\tau}_{srs} = N\hat{\mu}_{srs}$$

$$\hat{\mathsf{V}}(\hat{\tau}_{srs}) = N^2\hat{\mathsf{V}}(\hat{\mu}_{srs})$$

$$\hat{p}_{srs} = \sum_{i=1}^{n}\frac{y_i}{n}$$

$$\left(\frac{N-n}{N}\right)\frac{\hat{p}(1-\hat{p})}{n-1}\left(\frac{N-n}{N}\right)$$

$$\hat{\tau}_{hh} = \frac{1}{n}\sum_{i=1}^{n}\frac{y_i}{p_i}$$

$$\hat{\mathsf{V}}(\hat{\mu}_{hh}) = \frac{1}{n(n-1)}\sum_{i=1}^{n}\left(\frac{y_i}{p_i} - \hat{\tau}_{hh}\right)^2$$

$$\hat{\mu}_{hh} = \frac{\hat{\tau}_{hh}}{N}$$

$$\hat{\mathsf{V}}(\hat{\mu}_{hh}) = \frac{1}{N^2}\hat{\mathsf{V}}(\hat{\tau}_{hh})$$

$$\hat{\tau}_{ht} = \sum_{i=1}^{\nu}\frac{y_i}{\pi_i}$$

$$\hat{\mathsf{V}}(\hat{\tau}_{ht}) = \sum_{i=1}^{\nu}\left(\frac{1}{\pi_i^2} - \frac{1}{\pi_i}\right)y_i^2 +$$

$$2\sum_{i=1}^{\nu}\sum_{j>i}^{\nu}\left(\frac{1}{\pi_i\pi_j} - \frac{1}{\pi_{ij}}\right)y_i y_j$$

$$\hat{\mu}_{ht} = \frac{\hat{\tau}_{ht}}{N}$$

$$\hat{\mathsf{V}}(\hat{\mu}_{ht}) = \frac{1}{N^2}\hat{\mathsf{V}}(\hat{\tau}_{ht})$$

$$\hat{r} = \frac{\sum_{i=1}^{n} y_i}{\sum_{i=1}^{n} x_i}$$

$$\hat{\mathsf{V}}(\hat{r}) = \left(\frac{N-n}{Nn\mu_x^2}\right)\frac{\sum_{i=1}^{n}(y_i - rx_i)^2}{n-1}$$

$$\hat{\mu}_r = r\mu_x$$

$$\hat{\mathsf{V}}(\hat{\mu}_r) = \left(\frac{N-n}{Nn}\right)\frac{\sum_{i=1}^{n}(y_i - rx_i)^2}{n-1}$$

$$\hat{\tau}_r = Nr\mu_x = r\tau_x$$

$$\hat{\mathsf{V}}(\hat{\tau}_r) = \frac{N(N-n)}{n}\frac{\sum_{i=1}^{n}(y_i - rx_i)^2}{n-1}$$

$$\hat{\mu}_L = a + b\mu_x$$

$$\hat{\mathsf{V}}(\hat{\mu}_L) = \frac{N-n}{Nn(n-1)}\sum_{i=1}^{n}(y_i - a - bx_i)^2$$

$$\hat{\tau}_L = N\mu_L$$

$$\hat{\mathsf{V}}(\hat{\tau}_L) = \frac{N(N-n)}{n(n-1)}\sum_{i=1}^{n}(y_i - a - bx_i)^2$$

$$\hat{\mu}_{str} = \sum_{h=1}^{L}\frac{N_h}{N}\bar{y}_h$$

$$\hat{\mathsf{V}}(\hat{\mu}_{str}) = \frac{1}{N^2}\sum_{h=1}^{L}N_h^2\left(\frac{N_h - n_h}{N_h}\right)\frac{s_h^2}{n_h}$$

$$\hat{\tau}_{str} = N\hat{\mu}_{str}$$

$$\hat{\mathsf{V}}(\hat{\tau}_{str}) = N^2\hat{\mathsf{V}}(\hat{\mu}_{str})$$

$$\hat{p}_{str} = \sum_{h=1}^{L}\frac{N_h}{N}\hat{p}_h$$

$$\hat{\mathsf{V}}(\hat{p}_{str}) = \frac{1}{N^2}\sum_{h=1}^{L}N_h^2\left(\frac{N_h - n_h}{N_h}\right)\left(\frac{\hat{p}_h(1-\hat{p}_h)}{n_h - 1}\right)$$

$$\hat{\mu}_{pstr} = \sum_{h=1}^{L} w_h\bar{y}_h$$

$$\hat{\mathsf{V}}(\hat{\mu}_{pstr}) = \frac{1}{n}\left(\frac{N-n}{N}\right)\sum_{h=1}^{L} w_h s_h^2 + \frac{1}{n^2}\sum_{h=1}^{L}(1-w_h)s_h^2$$

$$\hat{\tau}_{cl} = \frac{M}{nL}\sum_{i=1}^{n}\sum_{j=1}^{L} y_{ij} = \frac{N}{n}\sum_{i=1}^{n}\sum_{j=1}^{L} y_{ij} = \frac{N}{n}\sum_{i=1}^{n} y_i = N\bar{y}$$

$$\hat{\mu}_{cl} = \frac{1}{nL}\sum_{i=1}^{n}\sum_{j=1}^{L} y_{ij} = \frac{1}{nL}\sum_{i=1}^{n} y_i = \frac{\bar{y}}{L} = \frac{\hat{\tau}_{cl}}{M}$$

where $\bar{y} = \frac{1}{n}\sum_{i=1}^{n} y_i = \frac{\hat{\tau}_{cl}}{N}$

$$\hat{\mathsf{V}}(\hat{\tau}_{cl}) = N(N-n)\frac{s_u^2}{n} \qquad \hat{\mathsf{V}}(\hat{\mu}_{cl}) = \frac{N(N-n)}{M^2}\frac{s_u^2}{n}$$

where $s_u^2 = \frac{\sum_{i=1}^{n}(y_i - \bar{y})^2}{n-1}$.

$$\hat{\mu}_1 = \bar{y} = \frac{\hat{\tau}_{cl}}{N} \qquad \hat{\mathsf{V}}(\hat{\mu}_1 = \frac{N-n}{N}\frac{s_u^2}{n}$$

The formulas for systematic sampling are the same as those used for one-stage cluster sampling. Change the subscript *cl* to *sys* to denote the fact that data were collected under systematic sampling.

$$\hat{\mu}_{c(a)} = \frac{\sum_{i=1}^{n} y_i}{\sum_{i=1}^{n} M_i} = \frac{\sum_{i=1}^{n} y_i}{m} \qquad \hat{\mathsf{V}}(\hat{\mu}_{c(a)}) = \frac{(N-n)N}{n(n-1)M^2}\sum_{i=1}^{n} M_i^2(\bar{y} - \hat{\mu}_{c(a)})^2$$

$$\hat{\mu}_{c(b)} = \frac{N}{M}\frac{\sum_{i=1}^{n} y_i}{n} = \frac{N}{nM}\sum_{i=1}^{n} y_i \qquad \hat{\mathsf{V}}(\hat{\mu}_{c(b)}) = \frac{(N-n)N}{n(n-1)M^2}\sum_{i=1}^{n}(y_i - \bar{y})^2 = \frac{(N-n)N}{nM^2}s_u^2$$

$$\hat{p}_c = \frac{\sum_{i=1}^{n} p_i}{n} \quad \hat{\mathsf{V}}(\hat{p}_c) = \left(\frac{N-Nn}{nN}\right)\sum_{i=1}^{n}\frac{(p_i - \hat{p}_c)^2}{n-1} = \left(\frac{1-f}{n}\right)\sum_{i=1}^{n}\frac{(p_i - \hat{p}_c)^2}{n-1}$$

$$\hat{p}_c = \frac{\sum_{i=1}^{n} y_i}{\sum_{i=1}^{n} M_i} \qquad \hat{\mathsf{V}}(\hat{p}_c) = \left(\frac{1-f}{n\bar{m}^2}\right)\frac{\sum_{i=1}^{n}(y_i - \hat{p}_c M_i)^2}{n-1}$$

To estimate τ, multiply $\hat{\mu}_{c(\cdot)}$ by M. To get the estimated variances, multiply $\hat{\mathsf{V}}(\hat{\mu}_{c(\cdot)})$ by M^2. If M is not known, substitute M with Nm/n. $\bar{m} = \sum_{i=1}^{n} M_i/n$.

n for μ SRS $\quad n = \dfrac{N\sigma^2}{(N-1)(d^2/z^2) + \sigma^2}$

n for τ SRS $\quad n = \dfrac{N\sigma^2}{(N-1)(d^2/z^2N^2) + \sigma^2}$

n for p SRS $\quad n = \dfrac{Np(1-p)}{(N-1)(d^2/z^2) + p(1-p)}$

n for μ SYS $\quad n = \dfrac{N\sigma^2}{(N-1)(d^2/z^2) + \sigma^2}$

n for τ SYS $\quad n = \dfrac{N\sigma^2}{(N-1)(d^2/z^2N^2) + \sigma^2}$

n for μ STR $\quad n = \dfrac{\sum_{h=1}^{L} N_h^2(\sigma_h^2/w_h)}{N^2(d^2/z^2) + \sum_{h=1}^{L} N_h\sigma_h^2}$

n for τ STR $\quad n = \dfrac{\sum_{h=1}^{L} N_h^2(\sigma_h^2/w_h)}{N^2(d^2/z^2N^2) + \sum_{h=1}^{L} N_h\sigma_h^2}$

where $w_h = \frac{n_h}{n}$.

Allocations for STR μ:

$$n_h = (c - c_0)\left(\frac{N_h\sigma_h/\sqrt{c_h}}{\sum_{k=1}^{L} N_k\sigma_k\sqrt{c_k}}\right) \qquad (c - c_0) = \frac{\left(\sum_{k=1}^{L} N_k\sigma_k/\sqrt{c_k}\right)\left(\sum_{k=1}^{L} N_k\sigma_k\sqrt{c_k}\right)}{N^2(d^2/z^2) + \sum_{k=1}^{L} N_k\sigma_k^2}$$

$$n_h = n\left(\frac{N_h}{N}\right) \qquad n = \frac{\sum_{k=1}^{L} N_k\sigma_k}{N^2(d^2/z^2) + \frac{1}{N}\sum_{k=1}^{L} N_k\sigma_k^2}$$

$$n_h = n\left(\frac{N_h\sigma_h}{\sum_{k=1}^{L} N_k\sigma_k}\right) \qquad n = \frac{\left(\sum_{k=1}^{L} N_k\sigma_k\right)^2}{N^2(d^2/z^2) + \sum_{k=1}^{L} N_k\sigma_k^2}$$

Allocations for STR τ:

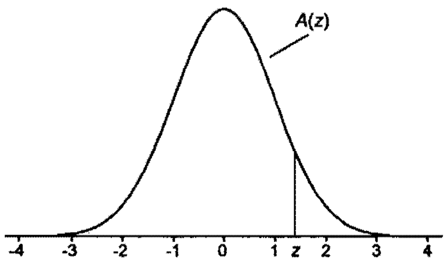
change $N^2(d^2/z^2)$ to $N^2(d^2/z^2N^2)$

Allocations for STR p:

$$n_h = n\left(\frac{N_i\sqrt{p_h(1-p_h)/c_h}}{\sum_{k=1}^{L} N_k\sqrt{p_k(1-p_k)/c_k}}\right) \qquad n = \frac{\sum_{k=1}^{L} N_k p_k(1-p_k)/w_k}{N^2(d^2/z^2) + \sum_{k=1}^{L} N_k p_k(1-p_k)}$$

TABLE A.1

Cumulative Standardized Normal Distribution



$A(z)$ is the integral of the standardized normal distribution from $-\infty$ to z (in other words, the area under the curve to the left of z). It gives the probability of a normal random variable not being more than z standard deviations above its mean. Values of z of particular importance:

z	$A(z)$	
1.645	0.9500	Lower limit of right 5% tail
1.960	0.9750	Lower limit of right 2.5% tail
2.326	0.9900	Lower limit of right 1% tail
2.576	0.9950	Lower limit of right 0.5% tail
3.090	0.9990	Lower limit of right 0.1% tail
3.291	0.9995	Lower limit of right 0.05% tail

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998
3.5	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998
3.6	0.9998	0.9998	0.9999							

Table A.2

t Distribution: Critical Values of t

Degrees of freedom	Two-tailed test: / One-tailed test:	10% / 5%	5% / 2.5%	2% / 1%	1% / 0.5%	0.2% / 0.1%	0.1% / 0.05%
	Significance level						
1		6.314	12.706	31.821	63.657	318.309	636.619
2		2.920	4.303	6.965	9.925	22.327	31.599
3		2.353	3.182	4.541	5.841	10.215	12.924
4		2.132	2.776	3.747	4.604	7.173	8.610
5		2.015	2.571	3.365	4.032	5.893	6.869
6		1.943	2.447	3.143	3.707	5.208	5.959
7		1.894	2.365	2.998	3.499	4.785	5.408
8		1.860	2.306	2.896	3.355	4.501	5.041
9		1.833	2.262	2.821	3.250	4.297	4.781
10		1.812	2.228	2.764	3.169	4.144	4.587
11		1.796	2.201	2.718	3.106	4.025	4.437
12		1.782	2.179	2.681	3.055	3.930	4.318
13		1.771	2.160	2.650	3.012	3.852	4.221
14		1.761	2.145	2.624	2.977	3.787	4.140
15		1.753	2.131	2.602	2.947	3.733	4.073
16		1.746	2.120	2.583	2.921	3.686	4.015
17		1.740	2.110	2.567	2.898	3.646	3.965
18		1.734	2.101	2.552	2.878	3.610	3.922
19		1.729	2.093	2.539	2.861	3.579	3.883
20		1.725	2.086	2.528	2.845	3.552	3.850
21		1.721	2.080	2.518	2.831	3.527	3.819
22		1.717	2.074	2.508	2.819	3.505	3.792
23		1.714	2.069	2.500	2.807	3.485	3.768
24		1.711	2.064	2.492	2.797	3.467	3.745
25		1.708	2.060	2.485	2.787	3.450	3.725
26		1.706	2.056	2.479	2.779	3.435	3.707
27		1.703	2.052	2.473	2.771	3.421	3.690
28		1.701	2.048	2.467	2.763	3.408	3.674
29		1.699	2.045	2.462	2.756	3.396	3.659
30		1.697	2.042	2.457	2.750	3.385	3.646
32		1.694	2.037	2.449	2.738	3.365	3.622
34		1.691	2.032	2.441	2.728	3.348	3.601
36		1.688	2.028	2.434	2.719	3.333	3.582
38		1.686	2.024	2.429	2.712	3.319	3.566
40		1.684	2.021	2.423	2.704	3.307	3.551
42		1.682	2.018	2.418	2.698	3.296	3.538
44		1.680	2.015	2.414	2.692	3.286	3.526
46		1.679	2.013	2.410	2.687	3.277	3.515
48		1.677	2.011	2.407	2.682	3.269	3.505
50		1.676	2.009	2.403	2.678	3.261	3.496
60		1.671	2.000	2.390	2.660	3.232	3.460
70		1.667	1.994	2.381	2.648	3.211	3.435
80		1.664	1.990	2.374	2.639	3.195	3.416
90		1.662	1.987	2.368	2.632	3.183	3.402
100		1.660	1.984	2.364	2.626	3.174	3.390
120		1.658	1.980	2.358	2.617	3.160	3.373
150		1.655	1.976	2.351	2.609	3.145	3.357
200		1.653	1.972	2.345	2.601	3.131	3.340
300		1.650	1.968	2.339	2.592	3.118	3.323
400		1.649	1.966	2.336	2.588	3.111	3.315
500		1.648	1.965	2.334	2.586	3.107	3.310
600		1.647	1.964	2.333	2.584	3.104	3.307
∞		1.645	1.960	2.326	2.576	3.090	3.291