UNIVERSITY OF SWAZILAND

SUPPLEMENTARY EXAMINATION PAPER 2016

TITLE OF PAPER	:	SAMPLE SURVEY THEORY
COURSE CODE	:	ST306
TIME ALLOWED	:	TWO (2) HOURS
REQUIREMENTS	:	CALCULATOR AND STATISTICAL TABLES
INSTRUCTIONS	:	ANSWER ANY THREE QUESTIONS

Question 1 **[20 marks, 12+8]**

(a) There are about 80,000 public schools, 23% of them in central cities, 24% in non-central urban areas, 25% in towns, and 28% in rural areas. We regard these 4 types of locations as strata, and wish to estimate the average yearly income earned by teachers. Assume that the standard deviations of yearly income in these strata are respectively $S_1 = 4200$, $S_2 = 3000$, $S_3 = 1900$, and $S_4 = 2400$. Find the optimal stratum sample-sizes for a stratified sample of total size n = 1000 schools to estimate average yearly income, and find the theoretical MSE for the unbiased estimator to be derived from that stratified sample.

(b) For a health survey of a large population, estimates are wanted for two proportions, each measuring the yearly incidence of a disease? For designing the sample, we guess that one occurs with a frequency of 50 percent and the other with a frequency of only 1 percent. To obtain the same standard error of $\frac{1}{2}$ percent, how large an srs is needed for each disease. The large difference in the needed n causes a re-evaluation of the requirements. Now the same coefficient of variation of 0.05 is declared desirable for each disease; how large a sample is needed for each disease?

Question 2 **[20 marks, 10+10]**

A campus population of size $N = 9000$ is to be surveyed by a stratified sample for the prevalence of a certain disease, based upon three strata of respective sizes $N_h = 1000, 3000, 5000$ for $h = 1, 2, 3$. The costs of sampling individuals from these strata are estimated to be respectively 40, 20, and 10 Swazi Emalangeni (SZL) per person. The campus health authorities believe that roughly 1% of stratum 1, 5% of stratum 2, and 12% of stratum 3 will test positive for the disease.

(a) What is the optimal number of individuals to sample in each stratum if the total budget for data-collection in the survey is $SZL20000$.

(b) Suppose that the same population were to be sampled by SRS. About how much would the SRS cost if you want to achieve the same MSE as in (a) in estimating the proportion of the population who have the disease ?

Question 3 **[20 marks, 6+6+8]**

A village contains 175 children. Dr. Jones takes a SRS of 17 of them and counts the cavities in each ones mouth, finding the frequency table:

Number of Cavities	0	1	2	3	4	5
Number of Children	5	4	2	3	2	1

Dr. Smith examines all 175 childrens mouths and records that 55 have no cavities. Estimate the total number of cavities in the villages children using

(a) only Dr. Jones data,

(b) both Dr. Jones and Dr. Smiths data.

(c) Give approximately unbiased estimates for the variances of your (approximately) unbiased estimators in both (a) and (b).

Question 4 [20 marks, 3+12+5]

(a) An investigator wishes to estimate the total number of trees on a 250-acre plantation. She divides the plantation into 1000 $\frac{1}{4}$-acre plots. She has aerial photographs from which she can easily estimate the total number of trees on each plot. She counts the actual number of trees on an SRS of 10 plots in order to calibrate her estimates from the photographs. Based on the aerial photographs, she estimates there are a total of 23100 trees on the whole plantation or 23.1 per plot. For the SRS of ten plots, she finds the following:

Plot	1	2	3	4	5	6	7	8	9	10
Actual Number Trees	25	15	22	24	13	18	35	30	10	29
Photo Estimate	23	14	20	25	12	18	30	27	8	31

(i) Draw a scatterplot of the data. Does it appear that a SRS estimate or ratio estimate is appropriate or should a regression estimate be used and why?

(ii) Use the appropriate method to estimate the total number of trees and its associated standard error.

(b) In a district containing 4000 houses the percentage of owned houses is to be estimated with a standard error of not more than 2% and the percentage of two-car households with a standard error of not more than 1%. The true percentage of owners is thought to lie between 45 an 65% and the percentage of two-car households between 5 and 10%. How large a sample is necessary to satisfy both aims?

Question 5 [20 marks, 15+5]

(a) A sociologist wants to estimate the average per capita income in a certain small city. As no list of resident adults is available, she decides that each of the city blocks will be considered one cluster. The clusters are numbered on a city map from 1 to 415, and the experimenter decides she has enough time and money to sample $n = 25$ clusters where every household will be interviewed within the clusters (blocks) chosen. The data on the next table give the number of residents and the total income for each of the 25 blocks sampled.

Cluster i	Number of Residents, M_i	Total Income per Cluster, y_i	Cluster i	Number of Residents, M_i	Total Income per Cluster, y_i
1	8	SZL192,000	14	10	SZL 98,000
2	12	SZL242,000	15	9	SZL106,000
3	4	SZL 84,000	16	3	SZL100,000
4	5	SZL130,000	17	6	SZL 64,000
5	6	SZL104,000	18	5	SZL 44,000
6	6	SZL 80,000	19	5	SZL 90,000
7	7	SZL150,000	20	4	SZL 74,000
8	5	SZL130,000	21	6	SZL102,000
9	8	SZL 90,000	22	8	SZL 60,000
10	3	SZL100,000	23	7	SZL 78,000
11	2	SZL170,000	24	3	SZL 94,000
12	6	SZL 86,000	25	8	SZL 82,000
13	5	SZL108,000			
				$\sum_{i=1}^{25} M_i = 151$	$\sum_{i=1}^{25} y_i =$ SZL$2,658,000$

Given that $M = 2500$ residents, use these data to estimate the (unbiased) average per capita income in the city and its associated standard error.

(b) A survey of the 9^{th} graders in Ndunayithini is intended to determine the proportion intending to go to a four-year college. A preliminary estimate of $p = 0.55$ was obtained from a small informal survey. How large must the survey be to provide an estimator with error at most 0.05 with probability at least 99%?

Useful formulas

$$s^2 = \frac{\sum_{i=1}^{n}(y_i - \bar{y})^2}{n-1} \qquad \sum_{i=1}^{n}(y_i - \bar{y})^2 = \sum_{i=1}^{n} y_i^2 - \frac{\sum_{i=1}^{n} y_i}{n}$$

$$\hat{\mu}_{srs} = \bar{y} \qquad \hat{\mathsf{V}}(\hat{\mu}_{srs}) = \left(\frac{N-n}{N}\right)\frac{s^2}{n}$$

$$\hat{\tau}_{srs} = N\hat{\mu}_{srs} \qquad \hat{\mathsf{V}}(\hat{\tau}_{srs}) = N^2\hat{\mathsf{V}}(\hat{\mu}_{srs})$$

$$\hat{p}_{srs} = \sum_{i=1}^{n}\frac{y_i}{n} \qquad \left(\frac{N-n}{N}\right)\frac{\hat{p}(1-\hat{p})}{n-1}\left(\frac{N-n}{N}\right)$$

$$\hat{\tau}_{hh} = \frac{1}{n}\sum_{i=1}^{n}\frac{y_i}{p_i} \qquad \hat{\mathsf{V}}(\hat{\mu}_{hh}) = \frac{1}{n(n-1)}\sum_{i=1}^{n}\left(\frac{y_i}{p_i} - \hat{\tau}_{hh}\right)$$

$$\hat{\mu}_{hh} = \frac{\hat{\tau}_{hh}}{N} \qquad \hat{\mathsf{V}}(\hat{\mu}_{hh}) = \frac{1}{N^2}\hat{\mathsf{V}}(\hat{\tau}_{hh})$$

$$\hat{\tau}_{ht} = \sum_{i=1}^{\nu}\frac{y_i}{\pi_i} \qquad \hat{\mathsf{V}}(\hat{\tau}_{ht}) = \sum_{i=1}^{\nu}\left(\frac{1}{\pi_i^2} - \frac{1}{\pi_i}\right)y_i^2 + 2\sum_{i=1}^{\nu}\sum_{j>i}^{\nu}\left(\frac{1}{\pi_i\pi_j} - \frac{1}{\pi_{ij}}\right)y_iy_j$$

$$\hat{\mu}_{ht} = \frac{\hat{\tau}_{ht}}{N} \qquad \hat{\mathsf{V}}(\hat{\mu}_{ht}) = \frac{1}{N^2}\hat{\mathsf{V}}(\hat{\tau}_{ht})$$

$$\hat{r} = \frac{\sum_{i=1}^{n} y_i}{\sum_{i=1}^{n} x_i} \qquad \hat{\mathsf{V}}(\hat{r}) = \left(\frac{N-n}{Nn\mu_x^2}\right)\frac{\sum_{i=1}^{n}(y_i - rx_i)^2}{n-1}$$

$$\hat{\mu}_r = r\mu_x \qquad \hat{\mathsf{V}}(\hat{\mu}_r) = \left(\frac{N-n}{Nn}\right)\frac{\sum_{i=1}^{n}(y_i - rx_i)^2}{n-1}$$

$$\hat{\tau}_r = Nr\mu_x = r\tau_x \qquad \hat{\mathsf{V}}(\hat{\tau}_r) = \frac{N(N-n)}{n}\frac{\sum_{i=1}^{n}(y_i - rx_i)^2}{n-1}$$

$$\hat{\mu}_L = a + b\mu_x \qquad \hat{\mathsf{V}}(\hat{\mu}_L) = \frac{N-n}{Nn(n-1)}\sum_{i=1}^{n}(y_i - a - bx_i)^2$$

$$\hat{\tau}_L = N\mu_L \qquad \hat{\mathsf{V}}(\hat{\tau}_L) = \frac{N(N-n)}{n(n-1)}\sum_{i=1}^{n}(y_i - a - bx_i)^2$$

$$\hat{\mu}_{str} = \sum_{h=1}^{L}\frac{N_h}{N}\bar{y}_h \qquad \hat{\mathsf{V}}(\hat{\mu}_{str}) = \frac{1}{N^2}\sum_{h=1}^{L}N_h^2\left(\frac{N_h - n_h}{N_h}\right)\frac{s_h^2}{n_h}$$

$$\hat{\tau}_{str} = N\hat{\mu}_{str} \qquad \hat{\mathsf{V}}(\hat{\tau}_{str}) = N^2\hat{\mathsf{V}}(\hat{\mu}_{str})$$

$$\hat{p}_{str} = \sum_{h=1}^{L}\frac{N_h}{N}\hat{p}_h \qquad \hat{\mathsf{V}}(\hat{p}_{str}) = \frac{1}{N^2}\sum_{h=1}^{L}N_h^2\left(\frac{N_h - n_h}{N_h}\right)\left(\frac{\hat{p}_h(1-\hat{p}_h)}{n_h - 1}\right)$$

$$\hat{\mu}_{pstr} = \sum_{h=1}^{L}w_h\bar{y}_h \qquad \hat{\mathsf{V}}(\hat{\mu}_{pstr}) = \frac{1}{n}\left(\frac{N-n}{N}\right)\sum_{h=1}^{L}w_hs_h^2 + \frac{1}{n^2}\sum_{h=1}^{L}(1-w_h)s_h^2$$

$$\hat{\tau}_{cl} = \frac{M}{nL}\sum_{i=1}^{n}\sum_{j=1}^{L} y_{ij} = \frac{N}{n}\sum_{i=1}^{n}\sum_{j=1}^{L} y_{ij} = \frac{N}{n}\sum_{i=1}^{n} y_i = N\bar{y}$$

$$\hat{\mu}_{cl} = \frac{1}{nL}\sum_{i=1}^{n}\sum_{j=1}^{L} y_{ij} = \frac{1}{nL}\sum_{i=1}^{n} y_i = \frac{\bar{y}}{L} = \frac{\hat{\tau}_{cl}}{M}$$

where $\bar{y} = \frac{1}{n}\sum_{i=1}^{n} y_i = \frac{\hat{\tau}_{cl}}{N}$

$$\hat{V}(\hat{\tau}_{cl}) = N(N-n)\frac{s_u^2}{n} \qquad \hat{V}(\hat{\mu}_{cl}) = \frac{N(N-n)}{M^2}\frac{s_u^2}{n}$$

where $s_u^2 = \frac{\sum_{i=1}^{n}(y_i-\bar{y})^2}{n-1}$.

$$\hat{\mu}_1 = \bar{y} = \frac{\hat{\tau}_{cl}}{N} \qquad \hat{V}(\hat{\mu}_1 = \frac{N-n}{N}\frac{s_u^2}{n}$$

The formulas for systematic sampling are the same as those used for one-stage cluster sampling. Change the subscript *cl* to *sys* to denote the fact that data were collected under systematic sampling.

$$\hat{\mu}_{c(a)} = \frac{\sum_{i=1}^{n} y_i}{\sum_{i=1}^{n} M_i} = \frac{\sum_{i=1}^{n} y_i}{m} \qquad \hat{V}(\hat{\mu}_{c(a)}) = \frac{(N-n)N}{n(n-1)M^2}\sum_{i=1}^{n} M_i^2(\bar{y}-\hat{\mu}_{c(a)})^2$$

$$\hat{\mu}_{c(b)} = \frac{N}{M}\frac{\sum_{i=1}^{n} y_i}{n} = \frac{N}{nM}\sum_{i=1}^{n} y_i \qquad \hat{V}(\hat{\mu}_{c(b)}) = \frac{(N-n)N}{n(n-1)M^2}\sum_{i=1}^{n}(y_i-\bar{y})^2 = \frac{(N-n)N}{nM^2}s_u^2$$

$$\hat{p}_c = \frac{\sum_{i=1}^{n} p_i}{n} \qquad \hat{V}(\hat{p}_c) = \left(\frac{N-Nn}{nN}\right)\sum_{i=1}^{n}\frac{(p_i-\hat{p}_c)^2}{n-1} = \left(\frac{1-f}{n}\right)\sum_{i=1}^{n}\frac{(p_i-\hat{p}_c)^2}{n-1}$$

$$\hat{p}_c = \frac{\sum_{i=1}^{n} y_i}{\sum_{i=1}^{n} M_i} \qquad \hat{V}(\hat{p}_c) = \left(\frac{1-f}{n\bar{m}^2}\right)\frac{\sum_{i=1}^{n}(y_i-\hat{p}_c M_i)^2}{n-1}$$

To estimate τ, multiply $\hat{\mu}_{c(\cdot)}$ by M. To get the estimated variances, multiply $\hat{V}(\hat{\mu}_{c(\cdot)})$ by M^2. If M is not known, substitute M with Nm/n. $\bar{m} = \sum_{i=1}^{n} M_i/n$.

n for μ SRS $\quad n = \frac{N\sigma^2}{(N-1)(d^2/z^2)+\sigma^2}$

n for τ SRS $\quad n = \frac{N\sigma^2}{(N-1)(d^2/z^2N^2)+\sigma^2}$

n for p SRS $\quad n = \frac{Np(1-p)}{(N-1)(d^2/z^2)+p(1-p)}$

n for μ SYS $\quad n = \frac{N\sigma^2}{(N-1)(d^2/z^2)+\sigma^2}$

n for τ SYS $\quad n = \frac{N\sigma^2}{(N-1)(d^2/z^2N^2)+\sigma^2}$

n for μ STR $\quad n = \frac{\sum_{h=1}^{L} N_h^2(\sigma_h^2/w_h)}{N^2(d^2/z^2)+\sum_{h=1}^{L} N_h\sigma_h^2}$

n for τ STR $\quad n = \frac{\sum_{h=1}^{L} N_h^2(\sigma_h^2/w_h)}{N^2(d^2/z^2N^2)+\sum_{h=1}^{L} N_h\sigma_h^2}$

where $w_h = \frac{n_h}{n}$.

Allocations for STR μ:

$$n_h = (c - c_0)\left(\frac{N_h\sigma_h/\sqrt{c_h}}{\sum_{k=1}^{L} N_k\sigma_k\sqrt{c_k}}\right) \qquad (c - c_0) = \frac{\left(\sum_{k=1}^{L} N_k\sigma_k/\sqrt{c_k}\right)\left(\sum_{k=1}^{L} N_k\sigma_k\sqrt{c_k}\right)}{N^2(d^2/z^2) + \sum_{k=1}^{L} N_k\sigma_k^2}$$

$$n_h = n\left(\frac{N_h}{N}\right) \qquad n = \frac{\sum_{k=1}^{L} N_k\sigma_k}{N^2(d^2/z^2) + \frac{1}{N}\sum_{k=1}^{L} N_k\sigma_k^2}$$

$$n_h = n\left(\frac{N_h\sigma_h}{\sum_{k=1}^{L} N_k\sigma_k}\right) \qquad n = \frac{\left(\sum_{k=1}^{L} N_k\sigma_k\right)^2}{N^2(d^2/z^2) + \sum_{k=1}^{L} N_k\sigma_k^2}$$

Allocations for STR τ:

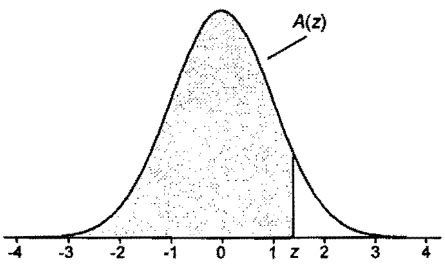
change $N^2(d^2/z^2)$ to $N^2(d^2/z^2N^2)$

Allocations for STR p:

$$n_h = n\left(\frac{N_i\sqrt{p_h(1-p_h)/c_h}}{\sum_{k=1}^{L} N_k\sqrt{p_k(1-p_k)/c_k}}\right) \qquad n = \frac{\sum_{k=1}^{L} N_k p_k(1-p_k)/w_k}{N^2(d^2/z^2) + \sum_{k=1}^{L} N_k p_k(1-p_k)}$$

TABLE A.1

Cumulative Standardized Normal Distribution



$A(z)$ is the integral of the standardized normal distribution from $-\infty$ to z (in other words, the area under the curve to the left of z). It gives the probability of a normal random variable not being more than z standard deviations above its mean. Values of z of particular importance:

z	$A(z)$	
1.645	0.9500	Lower limit of right 5% tail
1.960	0.9750	Lower limit of right 2.5% tail
2.326	0.9900	Lower limit of right 1% tail
2.576	0.9950	Lower limit of right 0.5% tail
3.090	0.9990	Lower limit of right 0.1% tail
3.291	0.9995	Lower limit of right 0.05% tail

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998
3.5	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998
3.6	0.9998	0.9998	0.9999							

TABLE A.2

t* Distribution: Critical Values of *t

		Significance level					
Degrees of freedom	*Two-tailed test:*	10%	5%	2%	1%	0.2%	0.1%
	One-tailed test:	5%	2.5%	1%	0.5%	0.1%	0.05%
1		6.314	12.706	31.821	63.657	318.309	636.619
2		2.920	4.303	6.965	9.925	22.327	31.599
3		2.353	3.182	4.541	5.841	10.215	12.924
4		2.132	2.776	3.747	4.604	7.173	8.610
5		2.015	2.571	3.365	4.032	5.893	6.869
6		1.943	2.447	3.143	3.707	5.208	5.959
7		1.894	2.365	2.998	3.499	4.785	5.408
8		1.860	2.306	2.896	3.355	4.501	5.041
9		1.833	2.262	2.821	3.250	4.297	4.781
10		1.812	2.228	2.764	3.169	4.144	4.587
11		1.796	2.201	2.718	3.106	4.025	4.437
12		1.782	2.179	2.681	3.055	3.930	4.318
13		1.771	2.160	2.650	3.012	3.852	4.221
14		1.761	2.145	2.624	2.977	3.787	4.140
15		1.753	2.131	2.602	2.947	3.733	4.073
16		1.746	2.120	2.583	2.921	3.686	4.015
17		1.740	2.110	2.567	2.898	3.646	3.965
18		1.734	2.101	2.552	2.878	3.610	3.922
19		1.729	2.093	2.539	2.861	3.579	3.883
20		1.725	2.086	2.528	2.845	3.552	3.850
21		1.721	2.080	2.518	2.831	3.527	3.819
22		1.717	2.074	2.508	2.819	3.505	3.792
23		1.714	2.069	2.500	2.807	3.485	3.768
24		1.711	2.064	2.492	2.797	3.467	3.745
25		1.708	2.060	2.485	2.787	3.450	3.725
26		1.706	2.056	2.479	2.779	3.435	3.707
27		1.703	2.052	2.473	2.771	3.421	3.690
28		1.701	2.048	2.467	2.763	3.408	3.674
29		1.699	2.045	2.462	2.756	3.396	3.659
30		1.697	2.042	2.457	2.750	3.385	3.646
32		1.694	2.037	2.449	2.738	3.365	3.622
34		1.691	2.032	2.441	2.728	3.348	3.601
36		1.688	2.028	2.434	2.719	3.333	3.582
38		1.686	2.024	2.429	2.712	3.319	3.566
40		1.684	2.021	2.423	2.704	3.307	3.551
42		1.682	2.018	2.418	2.698	3.296	3.538
44		1.680	2.015	2.414	2.692	3.286	3.526
46		1.679	2.013	2.410	2.687	3.277	3.515
48		1.677	2.011	2.407	2.682	3.269	3.505
50		1.676	2.009	2.403	2.678	3.261	3.496
60		1.671	2.000	2.390	2.660	3.232	3.460
70		1.667	1.994	2.381	2.648	3.211	3.435
80		1.664	1.990	2.374	2.639	3.195	3.416
90		1.662	1.987	2.368	2.632	3.183	3.402
100		1.660	1.984	2.364	2.626	3.174	3.390
120		1.658	1.980	2.358	2.617	3.160	3.373
150		1.655	1.976	2.351	2.609	3.145	3.357
200		1.653	1.972	2.345	2.601	3.131	3.340
300		1.650	1.968	2.339	2.592	3.118	3.323
400		1.649	1.966	2.336	2.588	3.111	3.315
500		1.648	1.965	2.334	2.586	3.107	3.310
600		1.647	1.964	2.333	2.584	3.104	3.307
∞		1.645	1.960	2.326	2.576	3.090	3.291