UNIVERSITY OF SWAZILAND

DEPARTMENT OF STATISTICS AND DEMOGRAPHY

MAIN EXAMINATION 2015/16

COURSE TITLE: OPERATIONS RESEARCH 1

COURSE CODE: ST 307

TIME ALLOWED: TWO (2) HOURS

INSTRUCTION: ANSWER ANY FOUR QUESTIONS

SPECIAL REQUIREMENTS: SCIENTIFIC CALCULATOR AND GRAPH PAPER

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Question 1

Swaziland Feed (Pty) Ltd is using two ingredients A and B in a feed mix for livestock. Each kg of ingredient A contributes 16 grams of the required vitamins and 24 grams of the required minerals to the feed mix. Each kg of ingredient B contributes 32 grams of the required vitamins and 16 grams of the required minerals to the feed mix. The feed mix must contain at least 128 grams of the required vitamins and 96 grams of the required minerals. A kilogram of ingredient A costs E8 and a kilogram of ingredient B costs E6. There are no other pressing requirements to be met. The company wants to know the quantity of each ingredient to use in the mix that will minimize the total cost of the two ingredients in the mix.

i) Formulate a Linear Programming Model for optimization problem. [15 marks]

ii) There are 5 assumptions behind Linear Programming Models namely; Proportionality, Additivity, Divisibility, Certainty and Non-negativity. Use the results in a) to briefly show that the assumptions hold. [10 marks]

Question 2

UNISWA Technology Centre (Pty) Ltd wants to raise at least 6000 million yen from the sales of two types of space technology equipments they recently invented to a space technology company in Japan. As from their stocks, they can sell a maximum of 50 units of equipment type A and a maximum of 70 units of equipment type B. They have a transport facility for a combined consignment of both types of equipment of 80 units. A unit of type A will fetch 200 million yen, while a unit of type B will fetch 60 million yen. They will like to maximise the amount raised from the sales of the space technology equipments to be shipped in a single consignment. We would like to establish a Linear Programming model to help to advise management on the number of units of each type of equipment to be shipped in the consignment.

Given that the Linear Programming Model for the above problem is as follows:

Maximise $Z = 200x_1 + 60x_2$

Subject to:

$$200x_1 + 60x_2 \geq 6000$$
$$x_1 \leq 50$$
$$x_2 \leq 70$$
$$x_1 + x_2 \leq 80$$
$$x_1 \geq 0\,,\ x_2 \geq 0$$

i) Use the graphical method to solve the problem and [20 marks]

ii) Advise the company on the number of each type of equipment to be shipped in the consignment. [5 marks]

Question 3

Matsapha Soft Drinks (Pty) Ltd produces two kinds of soft drinks **Cola** and **Fantasy**, which bring in profit of E150 and E120 per hectolitre respectively. Each hectolitre of the soft drink require the following quantities of the two raw materials X and Y in kilograms

Soft Drink	Raw Material X	Raw Material Y
Cola	32	48
Fantasy	64	16

The company has a weekly supply of 960 kg of raw material X and 720 kg of raw material Y. There are no other bottlenecks or restrictions. The company would like to maximise the profit from the sale of the two soft drinks.

Given the Primal Model and its Optimal Solution Vector as follows:

Maximise $Z = 150x_1 + 120x_2$

Subject to:

$$32x_1 + 64x_2 \leq 960$$
$$48x_1 + 16x_2 \leq 720$$

$$x_1 \geq 0, \; x_2 \geq 0$$

Optimal Solution Vector $X_0 = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 12 \\ 9 \end{bmatrix}$

i) Derive the Dual Model and find its Optimal Solution [15 marks]

ii) Demonstrate that the Duality Theorem is satisfied [8 marks]

iii) Give shadow prices for material X and material Y [2 marks]

Question 4

Lowveld Animal Feed Ltd produces three types of animal feed; **Poli, Khomo,** and **Koena.** Each tonne of poli requires 4 tonnes of ingredient A and 2 tonnes of ingredient B, each tonne of khomo requires 1 tonne of ingredient A and 6 tonnes of ingredient B, and each tonne of koena requires 3 tonnes of ingredient A and 3 tonnes of ingredient B. the company has 24 tonnes of ingredient A and 30 tonnes of ingredient B available weekly. There are no other bottlenecks in the production and marketing of the animal feeds. A tonne of poli brings E600, a tonne of khomo brings in a profit of E600 and a tonne of koena brings in a profit of E800. The company wants to maximise the weekly profit.

Model Formulation

Let x_1 be the quantity of poli produced in tonnes, let x_1 be the quantity of khomo produced in tonnes and let x_3 be the quantity of koena produced in tonnes. Then we have the following;

The total weekly profit in Emalangeni
$= 600x_1 + 400x_2 + 800x_3$

The quantity of ingredient A used in tonnes
$= 4x_1 + x_2 + 3x_3$

The quantity of ingredient B used in tonnes
$= 2x_1 + 6x_2 + 3x_3$

The Linear Programming Model is given as;

Maximise $\quad Z = 600x_1 + 400x_2 + 800x_3$

Subject to:

$$4x_1 + x_2 + 3x_3 \leq 24$$
$$2x_1 + 6x_2 + 3x_3 \leq 30$$

i) Solve the Linear Programing Model using the Simplex Method [12 marks]

ii) Give advice on the quantities of each type of feed to produce in order to maximise weekly profit. [5 marks]

iii) Give range of optimality in the objective function. [8 marks]

Question 5

Consider a Transportation Problem whose rates of resource utilization, activity flow supplies and activity flow requirements are given below.

Source i	Destination j		Activity Flow Supply, S_i
	1	2	
1	15	18	100
2	10	24	100
Activity Flow Requirement D_j	120	80	200

i) Set up a Linear Programming Model and state your assumption for this model. [5 marks]

ii) Find the dual model from i) [4 marks]

iii) Use the Northwest corner method to solve the problem. [8 marks]

iv) Use the Least Cost Method to solve the problem. [8 marks]

END OF EXAMINATION