UNIVERSITY OF SWAZILAND

SUPPLEMENTARY EXAMINATION PAPER 2017

TITLE OF PAPER	: STATISTICAL INFERENCE II
COURSE CODE	: ST 303
TIME ALLOWED	: 2 HRS
REQUIREMENTS	: CALCULATOR
INSTRUCTIONS	: ANSWER ANY THREE QUESTIONS

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Question 1

If the random variable Y has a probability density function given by;

$$f(y;\theta) = \theta a^{\theta}/y^{(\theta+1)}, \qquad y > a, \theta > 0, a > 0.$$

a) Find the method of moments estimate of θ.

(6 Marks)

b) Find the MLE, and asymptotic variance of the MLE.

(14 Marks)

Question 2

Suppose

$$f(y;\theta) = \theta e^{-\theta y}, \quad y > 0, 1 \le i \le n$$

Show that this is an exponential family form distribution, with natural parameter π = - θ.
Find the sufficient statistic and its distribution, and find the MLE for each π, θ.

(5+5+5+5 Marks)

Question 3

Let $X_1,\ldots\ldots,X_n$ be independent, identically distributed with common density function

$$f(x;\omega) = \omega \exp(-\omega x), \quad x > 0, \omega > 0$$

It is required to estimate = $1/\omega$. Find the Bayes estimator of θ, under squared error loss function, and assuming a prior density on ω of the form

$$\pi(\omega) \propto \omega^{\alpha-1} \exp(-\gamma\omega),$$

With known α > 0 and $\gamma > 0$. Find the bias, if any, of the Bayes estimator. Giving your reasoning, find the minimum variance unbiased estimator of θ, and its variance.

(8+6+6 Marks)

Question 4

a) Assume $X_1,\ldots,X_{25}$ are iid N(μ,100). Consider the following test $H_0: \mu \le 4$ versus $H_1: \mu > 4$. Suppose we reject H_0 if $\bar{X} > 7.92$. Compute the type I error when μ = 2. Compute the type II error when μ = 6.Find the significance level α.

(10 Marks)

b) Assume $X \sim Gamma(2,\beta)$, in which the density is

$$f(x) = \frac{1}{\beta^2} x e^{-x/\beta}.$$

Consider the following test $H_0: \beta \le 1$ versus $H_1: \beta > 1$.Suppose we reject H_0 if $X > 4$.

Compute the type I error , and type II error when $\beta = 2$. Find the significance level α.

(10 Marks)

Question 5

Suppose the household incomes in Swaziland have a probability distribution with *pdf*

$$f(x) = \frac{\theta v^{\theta}}{x^{\theta+1}}, \quad v \le x \le \infty$$

where θ > 0 is unknown and $v > 0$ is known. Let $x_1, x_2 \ldots\ldots, x_n$ denote the incomes for random sample of *n* households. We wish to test the null hypothesis $\theta = 1$ against the alternative hypothesis that $\theta \neq 1$.

a) Show that the generalised likelihood ratio test statistic, $\lambda(\boldsymbol{x})$, satisfies

$$ln\{\lambda(\boldsymbol{x})\} = n - \ln(\hat{\theta}) - \frac{n}{\hat{\theta}}$$

(8 Marks)

b) Show that the test fails to reject the null hypothesis if

$$k_1 < \sum_{i=1}^{n} \ln(x_i) < k_2$$

and state how the values of k_1 and k_2 may be determined.

(12 Marks)