## UNIVERSITY OF SWAZILAND

DEPARTMENT OF STATISTICS AND DEMOGRAPHY

## SUPPLEMENTARY EXAMINATION, 2016/7

| COURSE TITLE: | OPERATIONS RESEARCH II |
| :--- | :--- |
| COURSE CODE: | ST 408 |
| TIME ALLOWED: | THREE (3) HOURS |
|  |  |
| INSTRUCTION: | ANSWER A TOTAL OF FOUR OUESTIONS |
|  | SECTION A: ANSWER BOTH OUESTIONS |
|  | ALL QUESTIONS CARRY EQUAL MARKS (25 <br> MARKS |
| SPECIAL REQUIREMENTS: | SCIENTIFIC CALCULATORS AND STATISTICAL |

## TABLES

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## SECTION A

## Question 1

Emangcamane Investments is developing a new fertilizer. If the company markets the product and it is successful, the company will earn $\$ 50,000$ profit; if it is unsuccessful, the company will loose $\$ 35,000$. In the past, similar products have been successful $60 \%$ of the time. At a cost of $\$ 5,000$, the effectiveness of the new fertilizer can be tested. If the test result is favourable, there is an $80 \%$ chance that the fertilizer will be successful. If the test result is unfavourable, there is only a $30 \%$ chance that the fertilizer will be successful. There is a $60 \%$ chance of a favourable test result and $40 \%$ chance of an unfavourable test result.
(a) Determine Emangcamane's optimal strategy.
(15 marks)
(b) Find EVSI and EVPI
(10 marks)

## Question 2

In a project involving the development of a new production process, the various costs associated with various time reductions are as shown in the following table (Assuming Activity B connects nodes 1 and 2 , and a dummy activity connects 2 and 3 ).

| Activity | Event (i to j) | Normal Time <br> (Days) | Crash Time <br> (Days) | Daily Crash Cost <br> (E) |
| :--- | :--- | :--- | :--- | :--- |
| A | 1 to 3 | 5 | 3 | 100 |
| B | 1 to 2 | 6 | 3 | 150 |
| C | 2 to 4 | 5 | 4 | 200 |
| D | 2 to 5 | 8 | 5 | 125 |
| E | 3 to 4 | 9 | 6 | 175 |
| F | 3 to 5 | 7 | 5 | 225 |
| G | 3 to 6 | 10 | 6 | 200 |
| H | 4 to 7 | 11 | 180 |  |
| I | 4 to 8 | 12 | 6 | 200 |
| J | 5 to 6 | 11 | 8 | 190 |
| K | 5 to 7 | 10 | 8 | 150 |
| L | 5 to 8 | 13 | 7 | 175 |
| M | 6 to 9 | 9 | 10 | 225 |
| N | 7 to 9 | 10 | 6 | 210 |
| O | 8 to 9 | 11 | 7 | 200 |

(a) Draw the network diagram for this project and determine the critical path, assuming no activities are crashed;
(b) Crash the project one-day at a time to 35 days;
(c) Construct a time-cost trade-off curve for this problem showing the behaviour of the incremental cost of crashing the project to 35 days.
( $15+5+5$ marks)

## SECTION B

## Question 3

(a) Given the following payoff table:

| Decision | States of Nature |  |  |
| :--- | :--- | :--- | :--- |
| Alternative | $\theta 1$ | $\theta 2$ | $\theta 3$ |
| al | E1000 | E3000 | E1500 |
| a2 | 900 | 1100 | 800 |
| a3 | 700 | 600 | 600 |
| Probability | 0.3 | 0.5 | 0.2 |

(i) Compute the expected monetary value of each of the alternatives and select the best alternative;
(ii) Develop the opportunity loss table and compute the expected opportunity loss for each alternative;
(iii)Determine the expected value of perfect information for this problem situation.
(5+8+2 marks)
(b) Janet Smith has recently purchased a ski resort in Boone, North Carolina. Her financial success in this new adventure will mostly depend upon the weather conditions (i.e. snowfall) during the winter months. Probabilities associated with the three levels of snowfall have been obtained from the local weather bureau. Janet has developed the following payoff table, with associated probabilities for her new venture:

|  | State of Nature |  |  |
| :--- | :--- | :--- | :--- |
|  | Heavy Snowfall | Moderate Snowfall | Light Snowfall |
| Season return | $\$ 150,000$ | $\$ 50,000$ | $-\$ 25,000$ |
| Probability | 0.3 | 0.4 | 0.3 |

Janet is contemplating an offer from the syndicate of local investors who wish to lease the ski resort from her for $\$ 50,000$ annually. Should she accept their offer or should she operate the ski resort herself?
(10 marks)

## Question 4

(a) Given the following: annual demand $=32,000$ units per year; annual carrying cost $=\$ 0.35$ per unit per year, and cost per order $=\$ 500$ per order, compute the economic order quantity, the total annual minimum cost, and the length of an inventory cycle.
(10 marks)
(b) A local paint retailer is attempting to determine how paint should be ordered. The retailer experiences an annual demand of 100,000 litres. If the retailer orders the paint by the case, it costs E8.00 per litre, but if he orders the paint by the truck ( 10000 litres or more), the cost is E5.50 per litre. The annual holding cost is 30 percent of the purchase price and it costs E500 to place an order. Should the dealer buy a truck load?
( 15 marks)

## Question 5

(a) Trucks arrive at the Safe Trade supermarket according to a Poisson distribution, at a rate of 4 trucks per hour. A worker unloads them at a rate of 6 trucks per hour, following approximately the exponential distribution of service times. Management of the supermarket is considering hiring a second worker to unload trucks, believing that to do so will result in a total of 12 trucks per hour being unloaded. The hourly labour cost associated with each worker unloading trucks is E100. The hourly cost associated with having a truck waiting to be unloaded is E200 per hour (i.e. once the truck is actually being unloaded, the waiting cost is not incurred). Perform a queuing analysis and economic analysis for this situation.
( 10 marks)
(b) Trucks using a single-channel loading dock have a mean arrival rate of 12 trucks per day. The loading/unloading rate is 18 trucks per day.
(i) What is the probability that the loading dock will be idle?
(ii) What is the probability that there will be at least one truck waiting to be unloaded?
(iii) What is the average number of trucks in the system?
( 15 marks)

## Standard Normal Probabilities



Table entry for $z$ is the area under the standard normal curve to the left of $z$.

|  | 00 | . 01 | 02 | . 03 | . 0 | 05 | . 06 | . 07 | . 08 | . 09 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| -3.4 | . 0003 | . 0003 | . 000 | , | . 0003 | . 0003 | . 0003 | . 0003 | . 0003 | 0002 |
|  |  |  |  |  |  |  |  |  |  | .0003 |
| -3.2 | . 0007 | . 0007 | . 0006 | . 0006 | . 0006 | . 0006 | . 0006 | . 0005 | . 0005 | . 0005 |
| -3.1 | 001 | 00 | 0009 | 000 |  |  |  |  | 0007 | 0007 |
| -3.0 | . 0013 | . 0013 | . 0013 | . 0012 | . 0012 | . 0011 | . 0011 | . 0011 | . 0010 | . 0010 |
|  |  |  |  |  |  |  |  |  |  | 14 |
| -2.8 | . 0026 | . 0025 | . 0024 | . 0023 | . 0023 | . 0022 | . 0021 | . 0021 | . 0020 | 0019 |
| -2.78 | 0035 | 0 | 0 | 0 |  |  | 0029 | 0028 |  | 0026 |
| -2.6 | . 0047 | . 0045 | . 0044 | . 0043 | . 0041 | . 0040 | . 0039 | . 0038 | . 0037 | 0036 |
|  | 00 | 0 | 00 | 00 |  |  |  |  |  | 8 |
| -2.4 | . 0082 | . 0080 | . 0078 | . 0075 | . 0073 | . 0071 | . 0069 | . 0068 | . 0066 | . 0064 |
| , | 010 |  | 010 | 00 | 00 |  |  |  | , |  |
| -2.2 | . 0139 | . 0136 | . 0132 | . 0129 | . 0125 | . 0122 | . 0119 | . 0116 | . 0113 | 10 |
| 1 | 01 | 01 | 01 | 18 | 0 | 015 | T | 0150 |  | 3. |
| -2.0 | . 0228 | . 0222 | . 0217 | . 0212 | . 0207 | . 0202 | . 0197 | . 0192 | . 0188 | . 0183 |
|  | . 0287 |  | 02 |  |  | . | , | 02 | 0239 | 3 |
| -1.8 | . 0359 | . 0351 | . 0344 | . 0336 | . 0329 | . 0322 | . 0314 | . 0307 | . 0301 | 0294 |
| -17 |  | 0436 |  | 014 | 0409 |  |  | \$33 | 0375 | . 0367 |
| -1.6 | . 0548 | . 0537 | . 0526 | . 0516 | . 0505 | . 0495 | . 0485 | . 0475 | . 0465 | 5 |
| 1.5 | 066 | 2065 | 06 | .0630 | 061 |  |  | 0 | 8 | 59 |
| -1.4 | . 0808 | . 0793 | . 0778 | . 0764 | . 0749 | . 0735 | . 0721 | . 0708 | . 0694 | . 0681 |
|  | 096 |  |  | . |  |  |  | - | 0838 |  |
| -1.2 | . 1151 | . 1131 | . 1112 | . 1093 | 1075 | . 1056 | . 1038 | 1020 | 1003 | . 0985 |
| 411. |  |  |  |  |  |  | 1230 |  |  |  |
| -1.0 | . 1587 | . 1562 | . 1539 | . 1515 | . 1492 | . 1469 | . 1446 | . 1423 | . 1401 | . 1379 |
|  |  | 1814 |  |  |  |  |  |  |  |  |
| -0.8 | . 2119 | . 2090 | . 2061 | . 2033 | . 2005 | . 1977 | . 1949 | . 1922 | 1894 | 1867 |
|  |  |  |  |  |  |  |  |  |  |  |
| -0.6 | . 2743 | . 2709 | . 2676 | . 2643 | . 2611 | . 2578 | . 2546 | . 2514 | . 2483 | . 2451 |
| . | . 308 |  | , | 2 | 2 |  |  |  | , | $2776$ |
| -0.4 | . 3446 | . 3409 | . 3372 | . 3336 | . 3300 | . 3264 | . 3228 | . 3192 | . 3156 | . 3121 |
| -03 | 38821 | 378 |  |  |  |  |  | 33557 |  | \%3. |
| -0.2 | . 4207 | . 4168 | . 4129 | . 4090 | . 4052 | . 4013 | . 3974 | . 3936 | . 3897 | . 3859 |
| -0.1 | 4460 |  | 4522 | 4 | , | 4404 | 4364 | . 4325 |  | . 4247 |
| -0.0 | . 5000 | . 4960 | . 4920 | . 4880 | . 4840 | . 4801 | . 4761 | . 4721 | . 4681 | . 4641 |

## Standard Normal Probabilities



Table entry for $z$ is the area under the standard normal curve to the left of $z$.

|  | . 00 | . 01 | 02 | 03 | . 04 | 05 | 06 | 07 | 08 | 09 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0.0 | 500 | . 5040 | . 5080 | . 5120 | 51 | . 5199 | . | . 5279 | . 5319 | . 5359 |
| 0.1. |  | 543 | 5 |  | , | 5 | 56 | 5 | 5714 | 3 |
| 0.2 | . 5793 | . 5832 | . 5871 | . 5910 | . 5948 | . 5987 | . 6026 | . 6064 . | 6103 | . 6141 |
|  | 6179 | 621 |  |  |  |  | . 6406 |  | 6480 | 7 |
| 0.4 | . 6554 | . 6591 | . 6628 | . 6664 | . 6700 | . 6736 | . 6772 | . 6808 | . 6844 | . 6879 |
|  |  |  |  |  |  |  |  |  |  |  |
| 0.6 | . 7257 | . 7291 | . 7324 | . 7357 | . 7389 | . 7422 | . 7454 | . 7486 | . 7517 | 7549 |
| 0.7 |  |  | T |  |  |  | 27764 |  | 3 | 7852 |
| 0.8 | . 7881 | . 7910 | . 7939 | . 7967 | . 7995 | . 8023 | . 8051 | . 8078 | . 8106 | . 8133 |
| 0.9 | 815 | 818 |  |  |  |  |  | 8340 | 3365 | 8389 |
| 1.0 | . 8413 | . 8438 | . 8461 | . 8485 | . 8508 | . 8531 | . 8554 | . 8577 | 8599 | . 8621 |
|  | 86 | . 86 | . 8686 | 8 | ¢ | 0 | . 8770 |  | 8810 | 8830 |
| 1.2 | . 8849 | . 8869 | . 8888 | . 8907 | . 8925 | . 8944 | . 8962 | . 8980 | . 8997 | 15 |
| 1.30 | 9 | 9080 | 986 |  | 9 | 9115 | 0 |  |  | 7. |
| 1.4 | . 9192 | . 9207 | . 9222 | . 9236 | . 9251 | . 9265 | . 9279 | . 9292 | . 9306 | 9319 |
| 1.5 | 938 | 93 | 93 |  |  |  |  |  |  |  |
| 1.6 | . 9452 | . 9463 | . 9474 | . 9484 | . 9495 | . 9505 | . 9515 | . 9525 | . 9535 | . 9545 |
| 1.74 | 055 |  | 9 |  |  |  |  |  |  | , |
| 1.8 | . 9641 | . 9649 | . 9656 | . 9664 | . 9671 | . 9678 | . 9686 | . 9693 | . 9699 | . 9706 |
| , | - |  |  |  |  |  |  | 9756 | 9761 | 67 |
| 2.0 | . 9772 | . 9778 | . 9783 | . 9788 | . 9793 | . 9798 | . 980 | . 9808 | 12 | . 9817 |
| 21. | -9821 | \%.982 | , |  |  |  | 9846 | . 9850 | 9854 | . 9857 |
| 2.2 | . 9861 | . 9864 | . 9868 | . 9871 | . 9875 | . 9878 | . 9881 | . 9884 | . 9887 | . 9890 |
| 2, | \% |  |  |  |  |  | 9909 |  |  |  |
| 2.4 | . 9918 | . 9920 | . 9922 | . 9925 | . 9927 | . 99 | . 993 | . 9932 | . 9934 | . 9936 |
|  |  |  |  |  |  |  |  |  |  | 9952 |
| 2.6 | . 9953 | . 9955 | . 9956 | . 9957 | . 9959 | . 9960 | . 9961 | 9962 | . 9963 | . 9964 |
|  |  |  |  |  |  |  |  |  |  | .9974 |
| 2.8 | . 9974 | . 9975 | . 9976 | . 9977 | . 9977 | . 9978 | . 9979 | . 9979 | . 9980 | . 9981 |
| - | . 2 |  |  |  |  |  |  | 5 |  | 6 |
| 3.0 | . 9987 | . 9987 | . 9987 | . 9988 | . 9988 | . 9989 | . 9989 | . 9989 | . 9990 |  |
| 31.2 | F 9990 | 98 | . 99 |  |  |  |  |  | 9993 | 9993. |
| 3.2 | . 9993 | . 9993 | . 9994 | . 9994 | . 9994 | . 9994 | . 9994 | . 9995 | . 9995 | . 9995 |
| , |  |  | . 9 |  | 9 |  |  | . 99 | \%9996 | . 9997 |
| . 4 | . 9997 | . 9997 | . 9997 | . 9997 | . 9997 | . 9997 | . 9997 | . 9997 | . 9997 | . 9998 |

