# UNIVERSITY OF SWAZILAND
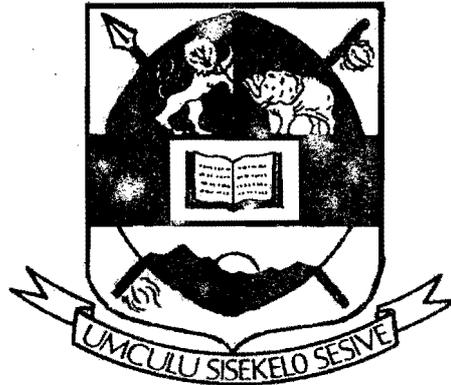


# RE SIT EXAMINATION PAPER 2016/2017

| | |
|---|---|
| TITLE OF PAPER: | DESCRIPTIVE STATISTICS |
| COURSE CODE: | STA 131/ ST 132 |
| TIME ALLOCATED: | 2 (TWO) HOURS |
| REQUIREMENTS: | STATISTICAL TABLES AND CALCULATOR |
| INSTRUCTION: | ANSWER ALL PARTS OF SECTION A AND ANY 2 (TWO) QUESTIONS OF YOUR CHOICE FROM SECTION B. SECTION A IS **COMPULSORY**. ALL QUESTIONS CARRY THE MARKS AS INDICATED WITHIN THE PARENTHESIS. |

## SECTION A : COMPULSORY QUESTION

**QUESTION ONE** $[3+2+3+1+3+3+3+2]$

The following scores represent the final examination grade for an elementary statistics course:

| 23 | 60 | 79 | 32 | 57 | 74 | 52 | 70 | 82 | 36 |
|----|----|----|----|----|----|----|----|----|----|
| 55 | 98 | 88 | 48 | 69 | 80 | 76 | 81 | 62 | 84 |
| 74 | 77 | 52 | 67 | 74 | 90 | 81 | 10 | 41 | 43 |
| 15 | 80 | 95 | 41 | 65 | 92 | 85 | 64 | 75 | 78 |
| 25 | 80 | 71 | 83 | 54 | 64 | 72 | 60 | 78 | 89 |
| 76 | 84 | 79 | 34 | 67 | 17 | 82 | 85 | 61 |    |

a. Using the following stems : 1, 2, 3, ..., 9. , construct **a stem and leaf plot** for the examination grades.
b. Furthermore
    i. Construct a **Relative Frequency Distribution Table** (RFDT) using the intervals 10-20, 20-30, etc.
    ii. Using the above (RFDT), draw an estimate **Relative Frequency Histogram** of the distribution.
    iii. Comment on the **Skewness** of the distribution.
c. Compute the following from the Relative Frequency Distribution Table :
    i. the grouped sample mean
    ii. the grouped sample median
    iii. the grouped sample variance
    iv. and the grouped sample standard deviation .

## SECTION B : Choose Any Two Questions

**QUESTION TWO** $[4+8+4+3+1]$

The insurance ratings below are based on collision claim experience and the theft frequency for 12 makes of small, two door cars. Higher numbers reflect higher claims and more frequent thefts, respectively.

| Collision | 103 | 79 | 105 | 151 | 127 | 140 | 106 | 193 | 101 | 96 | 84 | 150 |
|-----------|-----|-----|-----|-----|-----|-----|-----|-----|-----|----|----|-----|
| Theft | 103 | 131 | 81 | 68 | 98 | 97 | 97 | 452 | 82 | 81 | 59 | 176 |

a. State which is the x and y variables respectively and why.
b. Determine the least squares regression line for predicting the rate of collision claims on the basis of theft frequency rating.
c. Calculate the Correlation Coefficient *(r)* and comment on it.
d. Interpret the Coefficient of Determination *($R^2$)*

e. If a new model were to have a theft rating of 101, what would be the predicted rating for collision claims?

## QUESTION THREE                                                                [4+4+4+4+4]

a. A dealer invest E5,000.00 in a certain stock at a price of E10.00 per share, E5,000.00 at E2.50 per share , and E5,000.00 at E8.00 per share. Determine the average price per share of the stock.

b. A classroom section of 32 students of statistics received grades averaging 70 percent in a standardized test, and another section of 48 students of statistics received grades averaging 84 percent on the same test. What is the overall average of these grades?

c. In a market experiment a supermarket chain of three stores sold lettuce, on a certain day, at a different price in each of its stores. One of the store sold 75 heads at E0.98; the second sold 125 heads at E0.88; and the third store which had an advertised special, sold 300 heads at E0.69.
   i. How much was the average price per head of lettuce sold by the supermarket chain?
   ii. How many additional lettuce heads must be sold at E0.98 a head to raise the mean price of lettuce sold that day by the supermarket chain to E0.80 per head?

d. If an office manager spends E30.00 on pencils which cost E10.00 per gross (11 items per gross) and another E30.00 on pencils which cost E15.00 per gross, what is the average cost per gross?

## QUESTION FOUR                                                                [3+1+7+7+2]

Consider the following time series data.

| Week  | 1  | 2  | 3  | 4  | 5  | 6  | 7  |
|-------|----|----|----|----|----|----|----|
| Value | 24 | 13 | 20 | 12 | 19 | 23 | 15 |

a. Construct a time series plot. What type of pattern exists in the data?
b. Develop the three-week moving average forecasts for this time series.
   i. Compute MSE and a forecast for week 8.
c. Use $\alpha = 0.2$ to compute the exponential smoothing forecasts for the time series.
   i. Compute MSE and a forecast for week 8.
d. Compare the three-week moving average approach with the exponential smoothing approach using $\alpha = 0.2$. Which appears to provide more accurate forecasts based on MSE?

### END OF EXAMINATION
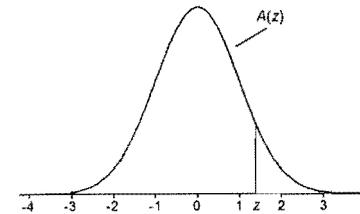
# STATISTICAL TABLES

Cumulative normal distribution

Critical values of the *t* distribution

Critical values of the *F* distribution

Critical values of the chi-squared distribution

## TABLE A.1

### Cumulative Standardized Normal Distribution



$A(z)$ is the integral of the standardized normal distribution from $-\infty$ to $z$ (in other words, the area under the curve to the left of $z$). It gives the probability of a normal random variable not being more than $z$ standard deviations above its mean. Values of $z$ of particular importance:

| $z$ | $A(z)$ | |
|---|---|---|
| 1.645 | 0.9500 | Lower limit of right 5% tail |
| 1.960 | 0.9750 | Lower limit of right 2.5% tail |
| 2.326 | 0.9900 | Lower limit of right 1% tail |
| 2.576 | 0.9950 | Lower limit of right 0.5% tail |
| 3.090 | 0.9990 | Lower limit of right 0.1% tail |
| 3.291 | 0.9995 | Lower limit of right 0.05% tail |

| $z$ | 0.00 | 0.01 | 0.02 | 0.03 | 0.04 | 0.05 | 0.06 | 0.07 | 0.08 | 0.09 |
|---|---|---|---|---|---|---|---|---|---|---|
| 0.0 | 0.5000 | 0.5040 | 0.5080 | 0.5120 | 0.5160 | 0.5199 | 0.5239 | 0.5279 | 0.5319 | 0.5359 |
| 0.1 | 0.5398 | 0.5438 | 0.5478 | 0.5517 | 0.5557 | 0.5596 | 0.5636 | 0.5675 | 0.5714 | 0.5753 |
| 0.2 | 0.5793 | 0.5832 | 0.5871 | 0.5910 | 0.5948 | 0.5987 | 0.6026 | 0.6064 | 0.6103 | 0.6141 |
| 0.3 | 0.6179 | 0.6217 | 0.6255 | 0.6293 | 0.6331 | 0.6368 | 0.6406 | 0.6443 | 0.6480 | 0.6517 |
| 0.4 | 0.6554 | 0.6591 | 0.6628 | 0.6664 | 0.6700 | 0.6736 | 0.6772 | 0.6808 | 0.6844 | 0.6879 |
| 0.5 | 0.6915 | 0.6950 | 0.6985 | 0.7019 | 0.7054 | 0.7088 | 0.7123 | 0.7157 | 0.7190 | 0.7224 |
| 0.6 | 0.7257 | 0.7291 | 0.7324 | 0.7357 | 0.7389 | 0.7422 | 0.7454 | 0.7486 | 0.7517 | 0.7549 |
| 0.7 | 0.7580 | 0.7611 | 0.7642 | 0.7673 | 0.7704 | 0.7734 | 0.7764 | 0.7794 | 0.7823 | 0.7852 |
| 0.8 | 0.7881 | 0.7910 | 0.7939 | 0.7967 | 0.7995 | 0.8023 | 0.8051 | 0.8078 | 0.8106 | 0.8133 |
| 0.9 | 0.8159 | 0.8186 | 0.8212 | 0.8238 | 0.8264 | 0.8289 | 0.8315 | 0.8340 | 0.8365 | 0.8389 |
| 1.0 | 0.8413 | 0.8438 | 0.8461 | 0.8485 | 0.8508 | 0.8531 | 0.8554 | 0.8577 | 0.8599 | 0.8621 |
| 1.1 | 0.8643 | 0.8665 | 0.8686 | 0.8708 | 0.8729 | 0.8749 | 0.8770 | 0.8790 | 0.8810 | 0.8830 |
| 1.2 | 0.8849 | 0.8869 | 0.8888 | 0.8907 | 0.8925 | 0.8944 | 0.8962 | 0.8980 | 0.8997 | 0.9015 |
| 1.3 | 0.9032 | 0.9049 | 0.9066 | 0.9082 | 0.9099 | 0.9115 | 0.9131 | 0.9147 | 0.9162 | 0.9177 |
| 1.4 | 0.9192 | 0.9207 | 0.9222 | 0.9236 | 0.9251 | 0.9265 | 0.9279 | 0.9292 | 0.9306 | 0.9319 |
| 1.5 | 0.9332 | 0.9345 | 0.9357 | 0.9370 | 0.9382 | 0.9394 | 0.9406 | 0.9418 | 0.9429 | 0.9441 |
| 1.6 | 0.9452 | 0.9463 | 0.9474 | 0.9484 | 0.9495 | 0.9505 | 0.9515 | 0.9525 | 0.9535 | 0.9545 |
| 1.7 | 0.9554 | 0.9564 | 0.9573 | 0.9582 | 0.9591 | 0.9599 | 0.9608 | 0.9616 | 0.9625 | 0.9633 |
| 1.8 | 0.9641 | 0.9649 | 0.9656 | 0.9664 | 0.9671 | 0.9678 | 0.9686 | 0.9693 | 0.9699 | 0.9706 |
| 1.9 | 0.9713 | 0.9719 | 0.9726 | 0.9732 | 0.9738 | 0.9744 | 0.9750 | 0.9756 | 0.9761 | 0.9767 |
| 2.0 | 0.9772 | 0.9778 | 0.9783 | 0.9788 | 0.9793 | 0.9798 | 0.9803 | 0.9808 | 0.9812 | 0.9817 |
| 2.1 | 0.9821 | 0.9826 | 0.9830 | 0.9834 | 0.9838 | 0.9842 | 0.9846 | 0.9850 | 0.9854 | 0.9857 |
| 2.2 | 0.9861 | 0.9864 | 0.9868 | 0.9871 | 0.9875 | 0.9878 | 0.9881 | 0.9884 | 0.9887 | 0.9890 |
| 2.3 | 0.9893 | 0.9896 | 0.9898 | 0.9901 | 0.9904 | 0.9906 | 0.9909 | 0.9911 | 0.9913 | 0.9916 |
| 2.4 | 0.9918 | 0.9920 | 0.9922 | 0.9925 | 0.9927 | 0.9929 | 0.9931 | 0.9932 | 0.9934 | 0.9936 |
| 2.5 | 0.9938 | 0.9940 | 0.9941 | 0.9943 | 0.9945 | 0.9946 | 0.9948 | 0.9949 | 0.9951 | 0.9952 |
| 2.6 | 0.9953 | 0.9955 | 0.9956 | 0.9957 | 0.9959 | 0.9960 | 0.9961 | 0.9962 | 0.9963 | 0.9964 |
| 2.7 | 0.9965 | 0.9966 | 0.9967 | 0.9968 | 0.9969 | 0.9970 | 0.9971 | 0.9972 | 0.9973 | 0.9974 |
| 2.8 | 0.9974 | 0.9975 | 0.9976 | 0.9977 | 0.9977 | 0.9978 | 0.9979 | 0.9979 | 0.9980 | 0.9981 |
| 2.9 | 0.9981 | 0.9982 | 0.9982 | 0.9983 | 0.9984 | 0.9984 | 0.9985 | 0.9985 | 0.9986 | 0.9986 |
| 3.0 | 0.9987 | 0.9987 | 0.9987 | 0.9988 | 0.9988 | 0.9989 | 0.9989 | 0.9989 | 0.9990 | 0.9990 |
| 3.1 | 0.9990 | 0.9991 | 0.9991 | 0.9991 | 0.9992 | 0.9992 | 0.9992 | 0.9992 | 0.9993 | 0.9993 |
| 3.2 | 0.9993 | 0.9993 | 0.9994 | 0.9994 | 0.9994 | 0.9994 | 0.9994 | 0.9995 | 0.9995 | 0.9995 |
| 3.3 | 0.9995 | 0.9995 | 0.9995 | 0.9996 | 0.9996 | 0.9996 | 0.9996 | 0.9996 | 0.9996 | 0.9997 |
| 3.4 | 0.9997 | 0.9997 | 0.9997 | 0.9997 | 0.9997 | 0.9997 | 0.9997 | 0.9997 | 0.9997 | 0.9998 |
| 3.5 | 0.9998 | 0.9998 | 0.9998 | 0.9998 | 0.9998 | 0.9998 | 0.9998 | 0.9998 | 0.9998 | 0.9998 |
| 3.6 | 0.9998 | 0.9998 | 0.9999 | | | | | | | |

## TABLE A.2

### t Distribution: Critical Values of t

| Degrees of freedom | Two-tailed test: One-tailed test: | 10% / 5% | 5% / 2.5% | 2% / 1% | 1% / 0.5% | 0.2% / 0.1% | 0.1% / 0.05% |
|---|---|---|---|---|---|---|---|
| 1 | | 6.314 | 12.706 | 31.821 | 63.657 | 318.309 | 636.619 |
| 2 | | 2.920 | 4.303 | 6.965 | 9.925 | 22.327 | 31.599 |
| 3 | | 2.353 | 3.182 | 4.541 | 5.841 | 10.215 | 12.924 |
| 4 | | 2.132 | 2.776 | 3.747 | 4.604 | 7.173 | 8.610 |
| 5 | | 2.015 | 2.571 | 3.365 | 4.032 | 5.893 | 6.869 |
| 6 | | 1.943 | 2.447 | 3.143 | 3.707 | 5.208 | 5.959 |
| 7 | | 1.894 | 2.365 | 2.998 | 3.499 | 4.785 | 5.408 |
| 8 | | 1.860 | 2.306 | 2.896 | 3.355 | 4.501 | 5.041 |
| 9 | | 1.833 | 2.262 | 2.821 | 3.250 | 4.297 | 4.781 |
| 10 | | 1.812 | 2.228 | 2.764 | 3.169 | 4.144 | 4.587 |
| 11 | | 1.796 | 2.201 | 2.718 | 3.106 | 4.025 | 4.437 |
| 12 | | 1.782 | 2.179 | 2.681 | 3.055 | 3.930 | 4.318 |
| 13 | | 1.771 | 2.160 | 2.650 | 3.012 | 3.852 | 4.221 |
| 14 | | 1.761 | 2.145 | 2.624 | 2.977 | 3.787 | 4.140 |
| 15 | | 1.753 | 2.131 | 2.602 | 2.947 | 3.733 | 4.073 |
| 16 | | 1.746 | 2.120 | 2.583 | 2.921 | 3.686 | 4.015 |
| 17 | | 1.740 | 2.110 | 2.567 | 2.898 | 3.646 | 3.965 |
| 18 | | 1.734 | 2.101 | 2.552 | 2.878 | 3.610 | 3.922 |
| 19 | | 1.729 | 2.093 | 2.539 | 2.861 | 3.579 | 3.883 |
| 20 | | 1.725 | 2.086 | 2.528 | 2.845 | 3.552 | 3.850 |
| 21 | | 1.721 | 2.080 | 2.518 | 2.831 | 3.527 | 3.819 |
| 22 | | 1.717 | 2.074 | 2.508 | 2.819 | 3.505 | 3.792 |
| 23 | | 1.714 | 2.069 | 2.500 | 2.807 | 3.485 | 3.768 |
| 24 | | 1.711 | 2.064 | 2.492 | 2.797 | 3.467 | 3.745 |
| 25 | | 1.708 | 2.060 | 2.485 | 2.787 | 3.450 | 3.725 |
| 26 | | 1.706 | 2.056 | 2.479 | 2.779 | 3.435 | 3.707 |
| 27 | | 1.703 | 2.052 | 2.473 | 2.771 | 3.421 | 3.690 |
| 28 | | 1.701 | 2.048 | 2.467 | 2.763 | 3.408 | 3.674 |
| 29 | | 1.699 | 2.045 | 2.462 | 2.756 | 3.396 | 3.659 |
| 30 | | 1.697 | 2.042 | 2.457 | 2.750 | 3.385 | 3.646 |
| 32 | | 1.694 | 2.037 | 2.449 | 2.738 | 3.365 | 3.622 |
| 34 | | 1.691 | 2.032 | 2.441 | 2.728 | 3.348 | 3.601 |
| 36 | | 1.688 | 2.028 | 2.434 | 2.719 | 3.333 | 3.582 |
| 38 | | 1.686 | 2.024 | 2.429 | 2.712 | 3.319 | 3.566 |
| 40 | | 1.684 | 2.021 | 2.423 | 2.704 | 3.307 | 3.551 |
| 42 | | 1.682 | 2.018 | 2.418 | 2.698 | 3.296 | 3.538 |
| 44 | | 1.680 | 2.015 | 2.414 | 2.692 | 3.286 | 3.526 |
| 46 | | 1.679 | 2.013 | 2.410 | 2.687 | 3.277 | 3.515 |
| 48 | | 1.677 | 2.011 | 2.407 | 2.682 | 3.269 | 3.505 |
| 50 | | 1.676 | 2.009 | 2.403 | 2.678 | 3.261 | 3.496 |
| 60 | | 1.671 | 2.000 | 2.390 | 2.660 | 3.232 | 3.460 |
| 70 | | 1.667 | 1.994 | 2.381 | 2.648 | 3.211 | 3.435 |
| 80 | | 1.664 | 1.990 | 2.374 | 2.639 | 3.195 | 3.416 |
| 90 | | 1.662 | 1.987 | 2.368 | 2.632 | 3.183 | 3.402 |
| 100 | | 1.660 | 1.984 | 2.364 | 2.626 | 3.174 | 3.390 |
| 120 | | 1.658 | 1.980 | 2.358 | 2.617 | 3.160 | 3.373 |
| 150 | | 1.655 | 1.976 | 2.351 | 2.609 | 3.145 | 3.357 |
| 200 | | 1.653 | 1.972 | 2.345 | 2.601 | 3.131 | 3.340 |
| 300 | | 1.650 | 1.968 | 2.339 | 2.592 | 3.118 | 3.323 |
| 400 | | 1.649 | 1.966 | 2.336 | 2.588 | 3.111 | 3.315 |
| 500 | | 1.648 | 1.965 | 2.334 | 2.586 | 3.107 | 3.310 |
| 600 | | 1.647 | 1.964 | 2.333 | 2.584 | 3.104 | 3.307 |
| ∞ | | 1.645 | 1.960 | 2.326 | 2.576 | 3.090 | 3.291 |

## TABLE A.3

### F Distribution: Critical Values of F (5% significance level)

| $\nu_2$ \ $\nu_1$ | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 12 | 14 | 16 | 18 | 20 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 161.45 | 199.50 | 215.71 | 224.58 | 230.16 | 233.99 | 236.77 | 238.88 | 240.54 | 241.88 | 243.91 | 245.36 | 246.46 | 247.32 | 248.01 |
| 2 | 18.51 | 19.00 | 19.16 | 19.25 | 19.30 | 19.33 | 19.35 | 19.37 | 19.38 | 19.40 | 19.41 | 19.42 | 19.43 | 19.44 | 19.45 |
| 3 | 10.13 | 9.55 | 9.28 | 9.12 | 9.01 | 8.94 | 8.89 | 8.85 | 8.81 | 8.79 | 8.74 | 8.71 | 8.69 | 8.67 | 8.66 |
| 4 | 7.71 | 6.94 | 6.59 | 6.39 | 6.26 | 6.16 | 6.09 | 6.04 | 6.00 | 5.96 | 5.91 | 5.87 | 5.84 | 5.82 | 5.80 |
| 5 | 6.61 | 5.79 | 5.41 | 5.19 | 5.05 | 4.95 | 4.88 | 4.82 | 4.77 | 4.74 | 4.68 | 4.64 | 4.60 | 4.58 | 4.56 |
| 6 | 5.99 | 5.14 | 4.76 | 4.53 | 4.39 | 4.28 | 4.21 | 4.15 | 4.10 | 4.06 | 4.00 | 3.96 | 3.92 | 3.90 | 3.87 |
| 7 | 5.59 | 4.74 | 4.35 | 4.12 | 3.97 | 3.87 | 3.79 | 3.73 | 3.68 | 3.64 | 3.57 | 3.53 | 3.49 | 3.47 | 3.44 |
| 8 | 5.32 | 4.46 | 4.07 | 3.84 | 3.69 | 3.58 | 3.50 | 3.44 | 3.39 | 3.35 | 3.28 | 3.24 | 3.20 | 3.17 | 3.15 |
| 9 | 5.12 | 4.26 | 3.86 | 3.63 | 3.48 | 3.37 | 3.29 | 3.23 | 3.18 | 3.14 | 3.07 | 3.03 | 2.99 | 2.96 | 2.94 |
| 10 | 4.96 | 4.10 | 3.71 | 3.48 | 3.33 | 3.22 | 3.14 | 3.07 | 3.02 | 2.98 | 2.91 | 2.86 | 2.83 | 2.80 | 2.77 |
| 11 | 4.84 | 3.98 | 3.59 | 3.36 | 3.20 | 3.09 | 3.01 | 2.95 | 2.90 | 2.85 | 2.79 | 2.74 | 2.70 | 2.67 | 2.65 |
| 12 | 4.75 | 3.89 | 3.49 | 3.26 | 3.11 | 3.00 | 2.91 | 2.85 | 2.80 | 2.75 | 2.69 | 2.64 | 2.60 | 2.57 | 2.54 |
| 13 | 4.67 | 3.81 | 3.41 | 3.18 | 3.03 | 2.92 | 2.83 | 2.77 | 2.71 | 2.67 | 2.60 | 2.55 | 2.51 | 2.48 | 2.46 |
| 14 | 4.60 | 3.74 | 3.34 | 3.11 | 2.96 | 2.85 | 2.76 | 2.70 | 2.65 | 2.60 | 2.53 | 2.48 | 2.44 | 2.41 | 2.39 |
| 15 | 4.54 | 3.68 | 3.29 | 3.06 | 2.90 | 2.79 | 2.71 | 2.64 | 2.59 | 2.54 | 2.48 | 2.42 | 2.38 | 2.35 | 2.33 |
| 16 | 4.49 | 3.63 | 3.24 | 3.01 | 2.85 | 2.74 | 2.66 | 2.59 | 2.54 | 2.49 | 2.42 | 2.37 | 2.33 | 2.30 | 2.28 |
| 17 | 4.45 | 3.59 | 3.20 | 2.96 | 2.81 | 2.70 | 2.61 | 2.55 | 2.49 | 2.45 | 2.38 | 2.33 | 2.29 | 2.26 | 2.23 |
| 18 | 4.41 | 3.55 | 3.16 | 2.93 | 2.77 | 2.66 | 2.58 | 2.51 | 2.46 | 2.41 | 2.34 | 2.29 | 2.25 | 2.22 | 2.19 |
| 19 | 4.38 | 3.52 | 3.13 | 2.90 | 2.74 | 2.63 | 2.54 | 2.48 | 2.42 | 2.38 | 2.31 | 2.26 | 2.21 | 2.18 | 2.16 |
| 20 | 4.35 | 3.49 | 3.10 | 2.87 | 2.71 | 2.60 | 2.51 | 2.45 | 2.39 | 2.35 | 2.28 | 2.22 | 2.18 | 2.15 | 2.12 |
| 21 | 4.32 | 3.47 | 3.07 | 2.84 | 2.68 | 2.57 | 2.49 | 2.42 | 2.37 | 2.32 | 2.25 | 2.20 | 2.16 | 2.12 | 2.10 |
| 22 | 4.30 | 3.44 | 3.05 | 2.82 | 2.66 | 2.55 | 2.46 | 2.40 | 2.34 | 2.30 | 2.23 | 2.17 | 2.13 | 2.10 | 2.07 |
| 23 | 4.28 | 3.42 | 3.03 | 2.80 | 2.64 | 2.53 | 2.44 | 2.37 | 2.32 | 2.27 | 2.20 | 2.15 | 2.11 | 2.08 | 2.05 |
| 24 | 4.26 | 3.40 | 3.01 | 2.78 | 2.62 | 2.51 | 2.42 | 2.36 | 2.30 | 2.25 | 2.18 | 2.13 | 2.09 | 2.05 | 2.03 |
| 25 | 4.24 | 3.39 | 2.99 | 2.76 | 2.60 | 2.49 | 2.40 | 2.34 | 2.28 | 2.24 | 2.16 | 2.11 | 2.07 | 2.04 | 2.01 |
| 26 | 4.22 | 3.37 | 2.98 | 2.74 | 2.59 | 2.47 | 2.39 | 2.32 | 2.27 | 2.22 | 2.15 | 2.09 | 2.05 | 2.02 | 1.99 |
| 27 | 4.21 | 3.35 | 2.96 | 2.73 | 2.57 | 2.46 | 2.37 | 2.31 | 2.25 | 2.20 | 2.13 | 2.08 | 2.04 | 2.00 | 1.97 |
| 28 | 4.20 | 3.34 | 2.95 | 2.71 | 2.56 | 2.45 | 2.36 | 2.29 | 2.24 | 2.19 | 2.12 | 2.06 | 2.02 | 1.99 | 1.96 |
| 29 | 4.18 | 3.33 | 2.93 | 2.70 | 2.55 | 2.43 | 2.35 | 2.28 | 2.22 | 2.18 | 2.10 | 2.05 | 2.01 | 1.97 | 1.94 |
| 30 | 4.17 | 3.32 | 2.92 | 2.69 | 2.53 | 2.42 | 2.33 | 2.27 | 2.21 | 2.16 | 2.09 | 2.04 | 1.99 | 1.96 | 1.93 |
| 35 | 4.12 | 3.27 | 2.87 | 2.64 | 2.49 | 2.37 | 2.29 | 2.22 | 2.16 | 2.11 | 2.04 | 1.99 | 1.94 | 1.91 | 1.88 |
| 40 | 4.08 | 3.23 | 2.84 | 2.61 | 2.45 | 2.34 | 2.25 | 2.18 | 2.12 | 2.08 | 2.00 | 1.95 | 1.90 | 1.87 | 1.84 |
| 50 | 4.03 | 3.18 | 2.79 | 2.56 | 2.40 | 2.29 | 2.20 | 2.13 | 2.07 | 2.03 | 1.95 | 1.89 | 1.85 | 1.81 | 1.78 |
| 60 | 4.00 | 3.15 | 2.76 | 2.53 | 2.37 | 2.25 | 2.17 | 2.10 | 2.04 | 1.99 | 1.92 | 1.86 | 1.82 | 1.78 | 1.75 |
| 70 | 3.98 | 3.13 | 2.74 | 2.50 | 2.35 | 2.23 | 2.14 | 2.07 | 2.02 | 1.97 | 1.89 | 1.84 | 1.79 | 1.75 | 1.72 |
| 80 | 3.96 | 3.11 | 2.72 | 2.49 | 2.33 | 2.21 | 2.13 | 2.06 | 2.00 | 1.95 | 1.88 | 1.82 | 1.77 | 1.73 | 1.70 |
| 90 | 3.95 | 3.10 | 2.71 | 2.47 | 2.32 | 2.20 | 2.11 | 2.04 | 1.99 | 1.94 | 1.86 | 1.80 | 1.76 | 1.72 | 1.69 |
| 100 | 3.94 | 3.09 | 2.70 | 2.46 | 2.31 | 2.19 | 2.10 | 2.03 | 1.97 | 1.93 | 1.85 | 1.79 | 1.75 | 1.71 | 1.68 |
| 120 | 3.92 | 3.07 | 2.68 | 2.45 | 2.29 | 2.18 | 2.09 | 2.02 | 1.96 | 1.91 | 1.83 | 1.78 | 1.73 | 1.69 | 1.66 |
| 150 | 3.90 | 3.06 | 2.66 | 2.43 | 2.27 | 2.16 | 2.07 | 2.00 | 1.94 | 1.89 | 1.82 | 1.76 | 1.71 | 1.67 | 1.64 |
| 200 | 3.89 | 3.04 | 2.65 | 2.42 | 2.26 | 2.14 | 2.06 | 1.98 | 1.93 | 1.88 | 1.80 | 1.74 | 1.69 | 1.66 | 1.62 |
| 250 | 3.88 | 3.03 | 2.64 | 2.41 | 2.25 | 2.13 | 2.05 | 1.98 | 1.92 | 1.87 | 1.79 | 1.73 | 1.68 | 1.65 | 1.61 |
| 300 | 3.87 | 3.03 | 2.63 | 2.40 | 2.24 | 2.13 | 2.04 | 1.97 | 1.91 | 1.86 | 1.78 | 1.72 | 1.68 | 1.64 | 1.61 |
| 400 | 3.86 | 3.02 | 2.63 | 2.39 | 2.24 | 2.12 | 2.03 | 1.96 | 1.90 | 1.85 | 1.78 | 1.72 | 1.67 | 1.63 | 1.60 |
| 500 | 3.86 | 3.01 | 2.62 | 2.39 | 2.23 | 2.12 | 2.03 | 1.96 | 1.90 | 1.85 | 1.77 | 1.71 | 1.66 | 1.62 | 1.59 |
| 600 | 3.86 | 3.01 | 2.62 | 2.39 | 2.23 | 2.11 | 2.02 | 1.95 | 1.90 | 1.85 | 1.77 | 1.71 | 1.66 | 1.62 | 1.59 |
| 750 | 3.85 | 3.01 | 2.62 | 2.38 | 2.23 | 2.11 | 2.02 | 1.95 | 1.89 | 1.84 | 1.77 | 1.70 | 1.66 | 1.62 | 1.58 |
| 1000 | 3.85 | 3.00 | 2.61 | 2.38 | 2.22 | 2.11 | 2.02 | 1.95 | 1.89 | 1.84 | 1.76 | 1.70 | 1.65 | 1.61 | 1.58 |

### Table A.3 (continued)

**F Distribution: Critical Values of F (5% significance level)**

| $\nu_2$ \ $\nu_1$ | 25 | 30 | 35 | 40 | 50 | 60 | 75 | 100 | 150 | 200 |
|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 249.26 | 250.10 | 250.69 | 251.14 | 251.77 | 252.20 | 252.62 | 253.04 | 253.46 | 253.68 |
| 2 | 19.46 | 19.46 | 19.47 | 19.47 | 19.48 | 19.48 | 19.48 | 19.49 | 19.49 | 19.49 |
| 3 | 8.63 | 8.62 | 8.60 | 8.59 | 8.58 | 8.57 | 8.56 | 8.55 | 8.54 | 8.54 |
| 4 | 5.77 | 5.75 | 5.73 | 5.72 | 5.70 | 5.69 | 5.68 | 5.66 | 5.65 | 5.65 |
| 5 | 4.52 | 4.50 | 4.48 | 4.46 | 4.44 | 4.43 | 4.42 | 4.41 | 4.39 | 4.39 |
| 6 | 3.83 | 3.81 | 3.79 | 3.77 | 3.75 | 3.74 | 3.73 | 3.71 | 3.70 | 3.69 |
| 7 | 3.40 | 3.38 | 3.36 | 3.34 | 3.32 | 3.30 | 3.29 | 3.27 | 3.26 | 3.25 |
| 8 | 3.11 | 3.08 | 3.06 | 3.04 | 3.02 | 3.01 | 2.99 | 2.97 | 2.96 | 2.95 |
| 9 | 2.89 | 2.86 | 2.84 | 2.83 | 2.80 | 2.79 | 2.77 | 2.76 | 2.74 | 2.73 |
| 10 | 2.73 | 2.70 | 2.68 | 2.66 | 2.64 | 2.62 | 2.60 | 2.59 | 2.57 | 2.56 |
| 11 | 2.60 | 2.57 | 2.55 | 2.53 | 2.51 | 2.49 | 2.47 | 2.46 | 2.44 | 2.43 |
| 12 | 2.50 | 2.47 | 2.44 | 2.43 | 2.40 | 2.38 | 2.37 | 2.35 | 2.33 | 2.32 |
| 13 | 2.41 | 2.38 | 2.36 | 2.34 | 2.31 | 2.30 | 2.28 | 2.26 | 2.24 | 2.23 |
| 14 | 2.34 | 2.31 | 2.28 | 2.27 | 2.24 | 2.22 | 2.21 | 2.19 | 2.17 | 2.16 |
| 15 | 2.28 | 2.25 | 2.22 | 2.20 | 2.18 | 2.16 | 2.14 | 2.12 | 2.10 | 2.10 |
| 16 | 2.23 | 2.19 | 2.17 | 2.15 | 2.12 | 2.11 | 2.09 | 2.07 | 2.05 | 2.04 |
| 17 | 2.18 | 2.15 | 2.12 | 2.10 | 2.08 | 2.06 | 2.04 | 2.02 | 2.00 | 1.99 |
| 18 | 2.14 | 2.11 | 2.08 | 2.06 | 2.04 | 2.02 | 2.00 | 1.98 | 1.96 | 1.95 |
| 19 | 2.11 | 2.07 | 2.05 | 2.03 | 2.00 | 1.98 | 1.96 | 1.94 | 1.92 | 1.91 |
| 20 | 2.07 | 2.04 | 2.01 | 1.99 | 1.97 | 1.95 | 1.93 | 1.91 | 1.89 | 1.88 |
| 21 | 2.05 | 2.01 | 1.98 | 1.96 | 1.94 | 1.92 | 1.90 | 1.88 | 1.86 | 1.84 |
| 22 | 2.02 | 1.98 | 1.96 | 1.94 | 1.91 | 1.89 | 1.87 | 1.85 | 1.83 | 1.82 |
| 23 | 2.00 | 1.96 | 1.93 | 1.91 | 1.88 | 1.86 | 1.84 | 1.82 | 1.80 | 1.79 |
| 24 | 1.97 | 1.94 | 1.91 | 1.89 | 1.86 | 1.84 | 1.82 | 1.80 | 1.78 | 1.77 |
| 25 | 1.96 | 1.92 | 1.89 | 1.87 | 1.84 | 1.82 | 1.80 | 1.78 | 1.76 | 1.75 |
| 26 | 1.94 | 1.90 | 1.87 | 1.85 | 1.82 | 1.80 | 1.78 | 1.76 | 1.74 | 1.73 |
| 27 | 1.92 | 1.88 | 1.86 | 1.84 | 1.81 | 1.79 | 1.76 | 1.74 | 1.72 | 1.71 |
| 28 | 1.91 | 1.87 | 1.84 | 1.82 | 1.79 | 1.77 | 1.75 | 1.73 | 1.70 | 1.69 |
| 29 | 1.89 | 1.85 | 1.83 | 1.81 | 1.77 | 1.75 | 1.73 | 1.71 | 1.69 | 1.67 |
| 30 | 1.88 | 1.84 | 1.81 | 1.79 | 1.76 | 1.74 | 1.72 | 1.70 | 1.67 | 1.66 |
| 35 | 1.82 | 1.79 | 1.76 | 1.74 | 1.70 | 1.68 | 1.66 | 1.63 | 1.61 | 1.60 |
| 40 | 1.78 | 1.74 | 1.72 | 1.69 | 1.66 | 1.64 | 1.61 | 1.59 | 1.56 | 1.55 |
| 50 | 1.73 | 1.69 | 1.66 | 1.63 | 1.60 | 1.58 | 1.55 | 1.52 | 1.50 | 1.48 |
| 60 | 1.69 | 1.65 | 1.62 | 1.59 | 1.56 | 1.53 | 1.51 | 1.48 | 1.45 | 1.44 |
| 70 | 1.66 | 1.62 | 1.59 | 1.57 | 1.53 | 1.50 | 1.48 | 1.45 | 1.42 | 1.40 |
| 80 | 1.64 | 1.60 | 1.57 | 1.54 | 1.51 | 1.48 | 1.45 | 1.43 | 1.39 | 1.38 |
| 90 | 1.63 | 1.59 | 1.55 | 1.53 | 1.49 | 1.46 | 1.44 | 1.41 | 1.38 | 1.36 |
| 100 | 1.62 | 1.57 | 1.54 | 1.52 | 1.48 | 1.45 | 1.42 | 1.39 | 1.36 | 1.34 |
| 120 | 1.60 | 1.55 | 1.52 | 1.50 | 1.46 | 1.43 | 1.40 | 1.37 | 1.33 | 1.32 |
| 150 | 1.58 | 1.54 | 1.50 | 1.48 | 1.44 | 1.41 | 1.38 | 1.34 | 1.31 | 1.29 |
| 200 | 1.56 | 1.52 | 1.48 | 1.46 | 1.41 | 1.39 | 1.35 | 1.32 | 1.28 | 1.26 |
| 250 | 1.55 | 1.50 | 1.47 | 1.44 | 1.40 | 1.37 | 1.34 | 1.31 | 1.27 | 1.25 |
| 300 | 1.54 | 1.50 | 1.46 | 1.43 | 1.39 | 1.36 | 1.33 | 1.30 | 1.26 | 1.23 |
| 400 | 1.53 | 1.49 | 1.45 | 1.42 | 1.38 | 1.35 | 1.32 | 1.28 | 1.24 | 1.22 |
| 500 | 1.53 | 1.48 | 1.45 | 1.42 | 1.38 | 1.35 | 1.31 | 1.28 | 1.23 | 1.21 |
| 600 | 1.52 | 1.48 | 1.44 | 1.41 | 1.37 | 1.34 | 1.31 | 1.27 | 1.23 | 1.20 |
| 750 | 1.52 | 1.47 | 1.44 | 1.41 | 1.37 | 1.34 | 1.30 | 1.26 | 1.22 | 1.20 |
| 1000 | 1.52 | 1.47 | 1.43 | 1.41 | 1.36 | 1.33 | 1.30 | 1.26 | 1.22 | 1.19 |

### Table A.3 (continued)

**F Distribution: Critical Values of F (1% significance level)**

| $\nu_2$ \ $\nu_1$ | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 12 | 14 | 16 | 18 | 20 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 4052.18 | 4999.50 | 5403.35 | 5624.58 | 5763.65 | 5858.99 | 5928.36 | 5981.07 | 6022.47 | 6055.85 | 6106.32 | 6142.67 | 6170.10 | 6191.53 | 6208.73 |
| 2 | 98.50 | 99.00 | 99.17 | 99.25 | 99.30 | 99.33 | 99.36 | 99.37 | 99.39 | 99.40 | 99.42 | 99.43 | 99.44 | 99.44 | 99.45 |
| 3 | 34.12 | 30.82 | 29.46 | 28.71 | 28.24 | 27.91 | 27.67 | 27.49 | 27.35 | 27.23 | 27.05 | 26.92 | 26.83 | 26.75 | 26.69 |
| 4 | 21.20 | 18.00 | 16.69 | 15.98 | 15.52 | 15.21 | 14.98 | 14.80 | 14.66 | 14.55 | 14.37 | 14.25 | 14.15 | 14.08 | 14.02 |
| 5 | 16.26 | 13.27 | 12.06 | 11.39 | 10.97 | 10.67 | 10.46 | 10.29 | 10.16 | 10.05 | 9.89 | 9.77 | 9.68 | 9.61 | 9.55 |
| 6 | 13.75 | 10.92 | 9.78 | 9.15 | 8.75 | 8.47 | 8.26 | 8.10 | 7.98 | 7.87 | 7.72 | 7.60 | 7.52 | 7.45 | 7.40 |
| 7 | 12.25 | 9.55 | 8.45 | 7.85 | 7.46 | 7.19 | 6.99 | 6.84 | 6.72 | 6.62 | 6.47 | 6.36 | 6.28 | 6.21 | 6.16 |
| 8 | 11.26 | 8.65 | 7.59 | 7.01 | 6.63 | 6.37 | 6.18 | 6.03 | 5.91 | 5.81 | 5.67 | 5.56 | 5.48 | 5.41 | 5.36 |
| 9 | 10.56 | 8.02 | 6.99 | 6.42 | 6.06 | 5.80 | 5.61 | 5.47 | 5.35 | 5.26 | 5.11 | 5.01 | 4.92 | 4.86 | 4.81 |
| 10 | 10.04 | 7.56 | 6.55 | 5.99 | 5.64 | 5.39 | 5.20 | 5.06 | 4.94 | 4.85 | 4.71 | 4.60 | 4.52 | 4.46 | 4.41 |
| 11 | 9.65 | 7.21 | 6.22 | 5.67 | 5.32 | 5.07 | 4.89 | 4.74 | 4.63 | 4.54 | 4.40 | 4.29 | 4.21 | 4.15 | 4.10 |
| 12 | 9.33 | 6.93 | 5.95 | 5.41 | 5.06 | 4.82 | 4.64 | 4.50 | 4.39 | 4.30 | 4.16 | 4.05 | 3.97 | 3.91 | 3.86 |
| 13 | 9.07 | 6.70 | 5.74 | 5.21 | 4.86 | 4.62 | 4.44 | 4.30 | 4.19 | 4.10 | 3.96 | 3.86 | 3.78 | 3.72 | 3.66 |
| 14 | 8.86 | 6.51 | 5.56 | 5.04 | 4.69 | 4.46 | 4.28 | 4.14 | 4.03 | 3.94 | 3.80 | 3.70 | 3.62 | 3.56 | 3.51 |
| 15 | 8.68 | 6.36 | 5.42 | 4.89 | 4.56 | 4.32 | 4.14 | 4.00 | 3.89 | 3.80 | 3.67 | 3.56 | 3.49 | 3.42 | 3.37 |
| 16 | 8.53 | 6.23 | 5.29 | 4.77 | 4.44 | 4.20 | 4.03 | 3.89 | 3.78 | 3.69 | 3.55 | 3.45 | 3.37 | 3.31 | 3.26 |
| 17 | 8.40 | 6.11 | 5.18 | 4.67 | 4.34 | 4.10 | 3.93 | 3.79 | 3.68 | 3.59 | 3.46 | 3.35 | 3.27 | 3.21 | 3.16 |
| 18 | 8.29 | 6.01 | 5.09 | 4.58 | 4.25 | 4.01 | 3.84 | 3.71 | 3.60 | 3.51 | 3.37 | 3.27 | 3.19 | 3.13 | 3.08 |
| 19 | 8.18 | 5.93 | 5.01 | 4.50 | 4.17 | 3.94 | 3.77 | 3.63 | 3.52 | 3.43 | 3.30 | 3.19 | 3.12 | 3.05 | 3.00 |
| 20 | 8.10 | 5.85 | 4.94 | 4.43 | 4.10 | 3.87 | 3.70 | 3.56 | 3.46 | 3.37 | 3.23 | 3.13 | 3.05 | 2.99 | 2.94 |
| 21 | 8.02 | 5.78 | 4.87 | 4.37 | 4.04 | 3.81 | 3.64 | 3.51 | 3.40 | 3.31 | 3.17 | 3.07 | 2.99 | 2.93 | 2.88 |
| 22 | 7.95 | 5.72 | 4.82 | 4.31 | 3.99 | 3.76 | 3.59 | 3.45 | 3.35 | 3.26 | 3.12 | 3.02 | 2.94 | 2.88 | 2.83 |
| 23 | 7.88 | 5.66 | 4.76 | 4.26 | 3.94 | 3.71 | 3.54 | 3.41 | 3.30 | 3.21 | 3.07 | 2.97 | 2.89 | 2.83 | 2.78 |
| 24 | 7.82 | 5.61 | 4.72 | 4.22 | 3.90 | 3.67 | 3.50 | 3.36 | 3.26 | 3.17 | 3.03 | 2.93 | 2.85 | 2.79 | 2.74 |
| 25 | 7.77 | 5.57 | 4.68 | 4.18 | 3.85 | 3.63 | 3.46 | 3.32 | 3.22 | 3.13 | 2.99 | 2.89 | 2.81 | 2.75 | 2.70 |
| 26 | 7.72 | 5.53 | 4.64 | 4.14 | 3.82 | 3.59 | 3.42 | 3.29 | 3.18 | 3.09 | 2.96 | 2.86 | 2.78 | 2.72 | 2.66 |
| 27 | 7.68 | 5.49 | 4.60 | 4.11 | 3.78 | 3.56 | 3.39 | 3.26 | 3.15 | 3.06 | 2.93 | 2.82 | 2.75 | 2.68 | 2.63 |
| 28 | 7.64 | 5.45 | 4.57 | 4.07 | 3.75 | 3.53 | 3.36 | 3.23 | 3.12 | 3.03 | 2.90 | 2.79 | 2.72 | 2.65 | 2.60 |
| 29 | 7.60 | 5.42 | 4.54 | 4.04 | 3.73 | 3.50 | 3.33 | 3.20 | 3.09 | 3.00 | 2.87 | 2.77 | 2.69 | 2.63 | 2.57 |
| 30 | 7.56 | 5.39 | 4.51 | 4.02 | 3.70 | 3.47 | 3.30 | 3.17 | 3.07 | 2.98 | 2.84 | 2.74 | 2.66 | 2.60 | 2.55 |
| 35 | 7.42 | 5.27 | 4.40 | 3.91 | 3.59 | 3.37 | 3.20 | 3.07 | 2.96 | 2.88 | 2.74 | 2.64 | 2.56 | 2.50 | 2.44 |
| 40 | 7.31 | 5.18 | 4.31 | 3.83 | 3.51 | 3.29 | 3.12 | 2.99 | 2.89 | 2.80 | 2.66 | 2.56 | 2.48 | 2.42 | 2.37 |
| 50 | 7.17 | 5.06 | 4.20 | 3.72 | 3.41 | 3.19 | 3.02 | 2.89 | 2.78 | 2.70 | 2.56 | 2.46 | 2.38 | 2.32 | 2.27 |
| 60 | 7.08 | 4.98 | 4.13 | 3.65 | 3.34 | 3.12 | 2.95 | 2.82 | 2.72 | 2.63 | 2.50 | 2.39 | 2.31 | 2.25 | 2.20 |
| 70 | 7.01 | 4.92 | 4.07 | 3.60 | 3.29 | 3.07 | 2.91 | 2.78 | 2.67 | 2.59 | 2.45 | 2.35 | 2.27 | 2.20 | 2.15 |
| 80 | 6.96 | 4.88 | 4.04 | 3.56 | 3.26 | 3.04 | 2.87 | 2.74 | 2.64 | 2.55 | 2.42 | 2.31 | 2.23 | 2.17 | 2.12 |
| 90 | 6.93 | 4.85 | 4.01 | 3.53 | 3.23 | 3.01 | 2.84 | 2.72 | 2.61 | 2.52 | 2.39 | 2.29 | 2.21 | 2.14 | 2.09 |
| 100 | 6.90 | 4.82 | 3.98 | 3.51 | 3.21 | 2.99 | 2.82 | 2.69 | 2.59 | 2.50 | 2.37 | 2.27 | 2.19 | 2.12 | 2.07 |
| 120 | 6.85 | 4.79 | 3.95 | 3.48 | 3.17 | 2.96 | 2.79 | 2.66 | 2.56 | 2.47 | 2.34 | 2.23 | 2.15 | 2.09 | 2.03 |
| 150 | 6.81 | 4.75 | 3.91 | 3.45 | 3.14 | 2.92 | 2.76 | 2.63 | 2.53 | 2.44 | 2.31 | 2.20 | 2.12 | 2.06 | 2.00 |
| 200 | 6.76 | 4.71 | 3.88 | 3.41 | 3.11 | 2.89 | 2.73 | 2.60 | 2.50 | 2.41 | 2.27 | 2.17 | 2.09 | 2.03 | 1.97 |
| 250 | 6.74 | 4.69 | 3.86 | 3.40 | 3.09 | 2.87 | 2.71 | 2.58 | 2.48 | 2.39 | 2.26 | 2.15 | 2.07 | 2.01 | 1.95 |
| 300 | 6.72 | 4.68 | 3.85 | 3.38 | 3.08 | 2.86 | 2.70 | 2.57 | 2.47 | 2.38 | 2.24 | 2.14 | 2.06 | 1.99 | 1.94 |
| 400 | 6.70 | 4.66 | 3.83 | 3.37 | 3.06 | 2.85 | 2.68 | 2.56 | 2.45 | 2.37 | 2.23 | 2.13 | 2.05 | 1.98 | 1.92 |
| 500 | 6.69 | 4.65 | 3.82 | 3.36 | 3.05 | 2.84 | 2.68 | 2.55 | 2.44 | 2.36 | 2.22 | 2.12 | 2.04 | 1.97 | 1.92 |
| 600 | 6.68 | 4.64 | 3.81 | 3.35 | 3.05 | 2.83 | 2.67 | 2.54 | 2.44 | 2.35 | 2.21 | 2.11 | 2.03 | 1.96 | 1.91 |
| 750 | 6.67 | 4.63 | 3.81 | 3.34 | 3.04 | 2.83 | 2.66 | 2.53 | 2.43 | 2.34 | 2.21 | 2.11 | 2.02 | 1.96 | 1.90 |
| 1000 | 6.66 | 4.63 | 3.80 | 3.34 | 3.04 | 2.82 | 2.66 | 2.53 | 2.43 | 2.34 | 2.20 | 2.10 | 2.02 | 1.95 | 1.90 |

### TABLE A.3 (continued)

**F Distribution: Critical Values of $F$ (1% significance level)**

| $v_2$ \ $v_1$ | 25 | 30 | 35 | 40 | 50 | 60 | 75 | 100 | 150 | 200 |
|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 6239.83 | 6260.65 | 6275.57 | 6286.78 | 6302.52 | 6313.03 | 6323.56 | 6334.11 | 6344.68 | 6349.97 |
| 2 | 99.46 | 99.47 | 99.47 | 99.47 | 99.48 | 99.48 | 99.49 | 99.49 | 99.49 | 99.49 |
| 3 | 26.58 | 26.50 | 26.45 | 26.41 | 26.35 | 26.32 | 26.28 | 26.24 | 26.20 | 26.18 |
| 4 | 13.91 | 13.84 | 13.79 | 13.75 | 13.69 | 13.65 | 13.61 | 13.58 | 13.54 | 13.52 |
| 5 | 9.45 | 9.38 | 9.33 | 9.29 | 9.24 | 9.20 | 9.17 | 9.13 | 9.09 | 9.08 |
| 6 | 7.30 | 7.23 | 7.18 | 7.14 | 7.09 | 7.06 | 7.02 | 6.99 | 6.95 | 6.93 |
| 7 | 6.06 | 5.99 | 5.94 | 5.91 | 5.86 | 5.82 | 5.79 | 5.75 | 5.72 | 5.70 |
| 8 | 5.26 | 5.20 | 5.15 | 5.12 | 5.07 | 5.03 | 5.00 | 4.96 | 4.93 | 4.91 |
| 9 | 4.71 | 4.65 | 4.60 | 4.57 | 4.52 | 4.48 | 4.45 | 4.41 | 4.38 | 4.36 |
| 10 | 4.31 | 4.25 | 4.20 | 4.17 | 4.12 | 4.08 | 4.05 | 4.01 | 3.98 | 3.96 |
| 11 | 4.01 | 3.94 | 3.89 | 3.86 | 3.81 | 3.78 | 3.74 | 3.71 | 3.67 | 3.66 |
| 12 | 3.76 | 3.70 | 3.65 | 3.62 | 3.57 | 3.54 | 3.50 | 3.47 | 3.43 | 3.41 |
| 13 | 3.57 | 3.51 | 3.46 | 3.43 | 3.38 | 3.34 | 3.31 | 3.27 | 3.24 | 3.22 |
| 14 | 3.41 | 3.35 | 3.30 | 3.27 | 3.22 | 3.18 | 3.15 | 3.11 | 3.08 | 3.06 |
| 15 | 3.28 | 3.21 | 3.17 | 3.13 | 3.08 | 3.05 | 3.01 | 2.98 | 2.94 | 2.92 |
| 16 | 3.16 | 3.10 | 3.05 | 3.02 | 2.97 | 2.93 | 2.90 | 2.86 | 2.83 | 2.81 |
| 17 | 3.07 | 3.00 | 2.96 | 2.92 | 2.87 | 2.83 | 2.80 | 2.76 | 2.73 | 2.71 |
| 18 | 2.98 | 2.92 | 2.87 | 2.84 | 2.78 | 2.75 | 2.71 | 2.68 | 2.64 | 2.62 |
| 19 | 2.91 | 2.84 | 2.80 | 2.76 | 2.71 | 2.67 | 2.64 | 2.60 | 2.57 | 2.55 |
| 20 | 2.84 | 2.78 | 2.73 | 2.69 | 2.64 | 2.61 | 2.57 | 2.54 | 2.50 | 2.48 |
| 21 | 2.79 | 2.72 | 2.67 | 2.64 | 2.58 | 2.55 | 2.51 | 2.48 | 2.44 | 2.42 |
| 22 | 2.73 | 2.67 | 2.62 | 2.58 | 2.53 | 2.50 | 2.46 | 2.42 | 2.38 | 2.36 |
| 23 | 2.69 | 2.62 | 2.57 | 2.54 | 2.48 | 2.45 | 2.41 | 2.37 | 2.34 | 2.32 |
| 24 | 2.64 | 2.58 | 2.53 | 2.49 | 2.44 | 2.40 | 2.37 | 2.33 | 2.29 | 2.27 |
| 25 | 2.60 | 2.54 | 2.49 | 2.45 | 2.40 | 2.36 | 2.33 | 2.29 | 2.25 | 2.23 |
| 26 | 2.57 | 2.50 | 2.45 | 2.42 | 2.36 | 2.33 | 2.29 | 2.25 | 2.21 | 2.19 |
| 27 | 2.54 | 2.47 | 2.42 | 2.38 | 2.33 | 2.29 | 2.26 | 2.22 | 2.18 | 2.16 |
| 28 | 2.51 | 2.44 | 2.39 | 2.35 | 2.30 | 2.26 | 2.23 | 2.19 | 2.15 | 2.13 |
| 29 | 2.48 | 2.41 | 2.36 | 2.33 | 2.27 | 2.23 | 2.20 | 2.16 | 2.12 | 2.10 |
| 30 | 2.45 | 2.39 | 2.34 | 2.30 | 2.25 | 2.21 | 2.17 | 2.13 | 2.09 | 2.07 |
| 35 | 2.35 | 2.28 | 2.23 | 2.19 | 2.14 | 2.10 | 2.06 | 2.02 | 1.98 | 1.96 |
| 40 | 2.27 | 2.20 | 2.15 | 2.11 | 2.06 | 2.02 | 1.98 | 1.94 | 1.90 | 1.87 |
| 50 | 2.17 | 2.10 | 2.05 | 2.01 | 1.95 | 1.91 | 1.87 | 1.82 | 1.78 | 1.76 |
| 60 | 2.10 | 2.03 | 1.98 | 1.94 | 1.88 | 1.84 | 1.79 | 1.75 | 1.70 | 1.68 |
| 70 | 2.05 | 1.98 | 1.93 | 1.89 | 1.83 | 1.78 | 1.74 | 1.70 | 1.65 | 1.62 |
| 80 | 2.01 | 1.94 | 1.89 | 1.85 | 1.79 | 1.75 | 1.70 | 1.65 | 1.61 | 1.58 |
| 90 | 1.99 | 1.92 | 1.86 | 1.82 | 1.76 | 1.72 | 1.67 | 1.62 | 1.57 | 1.55 |
| 100 | 1.97 | 1.89 | 1.84 | 1.80 | 1.74 | 1.69 | 1.65 | 1.60 | 1.55 | 1.52 |
| 120 | 1.93 | 1.86 | 1.81 | 1.76 | 1.70 | 1.66 | 1.61 | 1.56 | 1.51 | 1.48 |
| 150 | 1.90 | 1.83 | 1.77 | 1.73 | 1.66 | 1.62 | 1.57 | 1.52 | 1.46 | 1.43 |
| 200 | 1.87 | 1.79 | 1.74 | 1.69 | 1.63 | 1.58 | 1.53 | 1.48 | 1.42 | 1.39 |
| 250 | 1.85 | 1.77 | 1.72 | 1.67 | 1.61 | 1.56 | 1.51 | 1.46 | 1.40 | 1.36 |
| 300 | 1.84 | 1.76 | 1.70 | 1.66 | 1.59 | 1.55 | 1.50 | 1.44 | 1.38 | 1.35 |
| 400 | 1.82 | 1.75 | 1.69 | 1.64 | 1.58 | 1.53 | 1.48 | 1.42 | 1.36 | 1.32 |
| 500 | 1.81 | 1.74 | 1.68 | 1.63 | 1.57 | 1.52 | 1.47 | 1.41 | 1.34 | 1.31 |
| 600 | 1.80 | 1.73 | 1.67 | 1.63 | 1.56 | 1.51 | 1.46 | 1.40 | 1.34 | 1.30 |
| 750 | 1.80 | 1.72 | 1.66 | 1.62 | 1.55 | 1.50 | 1.45 | 1.39 | 1.33 | 1.29 |
| 1000 | 1.79 | 1.72 | 1.66 | 1.61 | 1.54 | 1.50 | 1.44 | 1.38 | 1.32 | 1.28 |

### TABLE A.3 (continued)

**F Distribution: Critical Values of $F$ (0.1% significance level)**

| $v_2$ \ $v_1$ | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 12 | 14 | 16 | 18 | 20 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 4.05e05 | 5.00e05 | 5.40e05 | 5.62e05 | 5.76e05 | 5.86e05 | 5.93e05 | 5.98e05 | 6.02e05 | 6.06e05 | 6.11e05 | 6.14e05 | 6.17e05 | 6.19e05 | 6.21e05 |
| 2 | 998.50 | 999.00 | 999.17 | 999.25 | 999.30 | 999.33 | 999.36 | 999.37 | 999.39 | 999.40 | 999.42 | 999.43 | 999.44 | 999.44 | 999.45 |
| 3 | 167.03 | 148.50 | 141.11 | 137.10 | 134.58 | 132.85 | 131.58 | 130.62 | 129.86 | 129.25 | 128.32 | 127.64 | 127.14 | 126.74 | 126.42 |
| 4 | 74.14 | 61.25 | 56.18 | 53.44 | 51.71 | 50.53 | 49.66 | 49.00 | 48.47 | 48.05 | 47.41 | 46.95 | 46.60 | 46.32 | 46.10 |
| 5 | 47.18 | 37.12 | 33.20 | 31.09 | 29.75 | 28.83 | 28.16 | 27.65 | 27.24 | 26.92 | 26.42 | 26.06 | 25.78 | 25.57 | 25.39 |
| 6 | 35.51 | 27.00 | 23.70 | 21.92 | 20.80 | 20.03 | 19.46 | 19.03 | 18.69 | 18.41 | 17.99 | 17.68 | 17.45 | 17.27 | 17.12 |
| 7 | 29.25 | 21.69 | 18.77 | 17.20 | 16.21 | 15.52 | 15.02 | 14.63 | 14.33 | 14.08 | 13.71 | 13.43 | 13.23 | 13.06 | 12.93 |
| 8 | 25.41 | 18.49 | 15.83 | 14.39 | 13.48 | 12.86 | 12.40 | 12.05 | 11.77 | 11.54 | 11.19 | 10.94 | 10.75 | 10.60 | 10.48 |
| 9 | 22.86 | 16.39 | 13.90 | 12.56 | 11.71 | 11.13 | 10.70 | 10.37 | 10.11 | 9.89 | 9.57 | 9.33 | 9.15 | 9.01 | 8.90 |
| 10 | 21.04 | 14.91 | 12.55 | 11.28 | 10.48 | 9.93 | 9.52 | 9.20 | 8.96 | 8.75 | 8.45 | 8.22 | 8.05 | 7.91 | 7.80 |
| 11 | 19.69 | 13.81 | 11.56 | 10.35 | 9.58 | 9.05 | 8.66 | 8.35 | 8.12 | 7.92 | 7.63 | 7.41 | 7.24 | 7.11 | 7.01 |
| 12 | 18.64 | 12.97 | 10.80 | 9.63 | 8.89 | 8.38 | 8.00 | 7.71 | 7.48 | 7.29 | 7.00 | 6.79 | 6.63 | 6.51 | 6.40 |
| 13 | 17.82 | 12.31 | 10.21 | 9.07 | 8.35 | 7.86 | 7.49 | 7.21 | 6.98 | 6.80 | 6.52 | 6.31 | 6.16 | 6.03 | 5.93 |
| 14 | 17.14 | 11.78 | 9.73 | 8.62 | 7.92 | 7.44 | 7.08 | 6.80 | 6.58 | 6.40 | 6.13 | 5.93 | 5.78 | 5.66 | 5.56 |
| 15 | 16.59 | 11.34 | 9.34 | 8.25 | 7.57 | 7.09 | 6.74 | 6.47 | 6.26 | 6.08 | 5.81 | 5.62 | 5.46 | 5.35 | 5.25 |
| 16 | 16.12 | 10.97 | 9.01 | 7.94 | 7.27 | 6.80 | 6.46 | 6.19 | 5.98 | 5.81 | 5.55 | 5.35 | 5.20 | 5.09 | 4.99 |
| 17 | 15.72 | 10.66 | 8.73 | 7.68 | 7.02 | 6.56 | 6.22 | 5.96 | 5.75 | 5.58 | 5.32 | 5.13 | 4.99 | 4.87 | 4.78 |
| 18 | 15.38 | 10.39 | 8.49 | 7.46 | 6.81 | 6.35 | 6.02 | 5.76 | 5.56 | 5.39 | 5.13 | 4.94 | 4.80 | 4.68 | 4.59 |
| 19 | 15.08 | 10.16 | 8.28 | 7.27 | 6.62 | 6.18 | 5.85 | 5.59 | 5.39 | 5.22 | 4.97 | 4.78 | 4.64 | 4.52 | 4.43 |
| 20 | 14.82 | 9.95 | 8.10 | 7.10 | 6.46 | 6.02 | 5.69 | 5.44 | 5.24 | 5.08 | 4.82 | 4.64 | 4.49 | 4.38 | 4.29 |
| 21 | 14.59 | 9.77 | 7.94 | 6.95 | 6.32 | 5.88 | 5.56 | 5.31 | 5.11 | 4.95 | 4.70 | 4.51 | 4.37 | 4.26 | 4.17 |
| 22 | 14.38 | 9.61 | 7.80 | 6.81 | 6.19 | 5.76 | 5.44 | 5.19 | 4.99 | 4.83 | 4.58 | 4.40 | 4.26 | 4.15 | 4.06 |
| 23 | 14.20 | 9.47 | 7.67 | 6.70 | 6.08 | 5.65 | 5.33 | 5.09 | 4.89 | 4.73 | 4.48 | 4.30 | 4.16 | 4.05 | 3.96 |
| 24 | 14.03 | 9.34 | 7.55 | 6.59 | 5.98 | 5.55 | 5.23 | 4.99 | 4.80 | 4.64 | 4.39 | 4.21 | 4.07 | 3.96 | 3.87 |
| 25 | 13.88 | 9.22 | 7.45 | 6.49 | 5.89 | 5.46 | 5.15 | 4.91 | 4.71 | 4.56 | 4.31 | 4.13 | 3.99 | 3.88 | 3.79 |
| 26 | 13.74 | 9.12 | 7.36 | 6.41 | 5.80 | 5.38 | 5.07 | 4.83 | 4.64 | 4.48 | 4.24 | 4.06 | 3.92 | 3.81 | 3.72 |
| 27 | 13.61 | 9.02 | 7.27 | 6.33 | 5.73 | 5.31 | 5.00 | 4.76 | 4.57 | 4.41 | 4.17 | 3.99 | 3.86 | 3.75 | 3.66 |
| 28 | 13.50 | 8.93 | 7.19 | 6.25 | 5.66 | 5.24 | 4.93 | 4.69 | 4.50 | 4.35 | 4.11 | 3.93 | 3.80 | 3.69 | 3.60 |
| 29 | 13.39 | 8.85 | 7.12 | 6.19 | 5.59 | 5.18 | 4.87 | 4.64 | 4.45 | 4.29 | 4.05 | 3.88 | 3.74 | 3.63 | 3.54 |
| 30 | 13.29 | 8.77 | 7.05 | 6.12 | 5.53 | 5.12 | 4.82 | 4.58 | 4.39 | 4.24 | 4.00 | 3.82 | 3.69 | 3.58 | 3.49 |
| 35 | 12.90 | 8.47 | 6.79 | 5.88 | 5.30 | 4.89 | 4.59 | 4.36 | 4.18 | 4.03 | 3.79 | 3.62 | 3.48 | 3.38 | 3.29 |
| 40 | 12.61 | 8.25 | 6.59 | 5.70 | 5.13 | 4.73 | 4.44 | 4.21 | 4.02 | 3.87 | 3.64 | 3.47 | 3.34 | 3.23 | 3.14 |
| 50 | 12.22 | 7.96 | 6.34 | 5.46 | 4.90 | 4.51 | 4.22 | 4.00 | 3.82 | 3.67 | 3.44 | 3.27 | 3.14 | 3.04 | 2.95 |
| 60 | 11.97 | 7.77 | 6.17 | 5.31 | 4.76 | 4.37 | 4.09 | 3.86 | 3.69 | 3.54 | 3.32 | 3.15 | 3.02 | 2.91 | 2.83 |
| 70 | 11.80 | 7.64 | 6.06 | 5.20 | 4.66 | 4.28 | 3.99 | 3.77 | 3.60 | 3.45 | 3.23 | 3.06 | 2.93 | 2.83 | 2.74 |
| 80 | 11.67 | 7.54 | 5.97 | 5.12 | 4.58 | 4.20 | 3.92 | 3.70 | 3.53 | 3.39 | 3.16 | 3.00 | 2.87 | 2.76 | 2.68 |
| 90 | 11.57 | 7.47 | 5.91 | 5.06 | 4.53 | 4.15 | 3.87 | 3.65 | 3.48 | 3.34 | 3.11 | 2.95 | 2.82 | 2.71 | 2.63 |
| 100 | 11.50 | 7.41 | 5.86 | 5.02 | 4.48 | 4.11 | 3.83 | 3.61 | 3.44 | 3.30 | 3.07 | 2.91 | 2.78 | 2.68 | 2.59 |
| 120 | 11.38 | 7.32 | 5.78 | 4.95 | 4.42 | 4.04 | 3.77 | 3.55 | 3.38 | 3.24 | 3.02 | 2.85 | 2.72 | 2.62 | 2.53 |
| 150 | 11.27 | 7.24 | 5.71 | 4.88 | 4.35 | 3.98 | 3.71 | 3.49 | 3.32 | 3.18 | 2.96 | 2.80 | 2.67 | 2.56 | 2.48 |
| 200 | 11.15 | 7.15 | 5.63 | 4.81 | 4.29 | 3.92 | 3.65 | 3.43 | 3.26 | 3.12 | 2.90 | 2.74 | 2.61 | 2.51 | 2.42 |
| 250 | 11.09 | 7.10 | 5.59 | 4.77 | 4.25 | 3.88 | 3.61 | 3.40 | 3.23 | 3.09 | 2.87 | 2.71 | 2.58 | 2.48 | 2.39 |
| 300 | 11.04 | 7.07 | 5.56 | 4.75 | 4.22 | 3.86 | 3.59 | 3.38 | 3.21 | 3.07 | 2.85 | 2.69 | 2.56 | 2.46 | 2.37 |
| 400 | 10.99 | 7.03 | 5.53 | 4.71 | 4.19 | 3.83 | 3.56 | 3.35 | 3.18 | 3.04 | 2.82 | 2.66 | 2.53 | 2.43 | 2.34 |
| 500 | 10.96 | 7.00 | 5.51 | 4.69 | 4.18 | 3.81 | 3.54 | 3.33 | 3.16 | 3.02 | 2.81 | 2.64 | 2.52 | 2.41 | 2.33 |
| 600 | 10.94 | 6.99 | 5.49 | 4.68 | 4.16 | 3.80 | 3.53 | 3.32 | 3.15 | 3.01 | 2.80 | 2.63 | 2.51 | 2.40 | 2.32 |
| 750 | 10.91 | 6.97 | 5.48 | 4.67 | 4.15 | 3.79 | 3.52 | 3.31 | 3.14 | 3.00 | 2.78 | 2.62 | 2.49 | 2.39 | 2.31 |
| 1000 | 10.89 | 6.96 | 5.46 | 4.65 | 4.14 | 3.78 | 3.51 | 3.30 | 3.13 | 2.99 | 2.77 | 2.61 | 2.48 | 2.38 | 2.30 |

## TABLE A.3 (continued)

### F Distribution: Critical Values of F (0.1% significance level)

| $v_2$ \ $v_1$ | 25 | 30 | 35 | 40 | 50 | 60 | 75 | 100 | 150 | 200 |
|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 6.24e05 | 6.26e05 | 6.28e05 | 6.29e05 | 6.30e05 | 6.31e05 | 6.32e05 | 6.33e05 | 6.35e05 | 6.35e05 |
| 2 | 999.46 | 999.47 | 999.47 | 999.47 | 999.48 | 999.48 | 999.49 | 999.49 | 999.49 | 999.49 |
| 3 | 125.84 | 125.45 | 125.17 | 124.96 | 124.66 | 124.47 | 124.27 | 124.07 | 123.87 | 123.77 |
| 4 | 45.70 | 45.43 | 45.23 | 45.09 | 44.88 | 44.75 | 44.61 | 44.47 | 44.33 | 44.26 |
| 5 | 25.08 | 24.87 | 24.72 | 24.60 | 24.44 | 24.33 | 24.22 | 24.12 | 24.01 | 23.95 |
| 6 | 16.85 | 16.67 | 16.54 | 16.44 | 16.31 | 16.21 | 16.12 | 16.03 | 15.93 | 15.89 |
| 7 | 12.69 | 12.53 | 12.41 | 12.33 | 12.20 | 12.12 | 12.04 | 11.95 | 11.87 | 11.82 |
| 8 | 10.26 | 10.11 | 10.00 | 9.92 | 9.80 | 9.73 | 9.65 | 9.57 | 9.49 | 9.45 |
| 9 | 8.69 | 8.55 | 8.46 | 8.37 | 8.26 | 8.19 | 8.11 | 8.04 | 7.96 | 7.93 |
| 10 | 7.60 | 7.47 | 7.37 | 7.30 | 7.19 | 7.12 | 7.05 | 6.98 | 6.91 | 6.87 |
| 11 | 6.81 | 6.68 | 6.59 | 6.52 | 6.42 | 6.35 | 6.28 | 6.21 | 6.14 | 6.10 |
| 12 | 6.22 | 6.09 | 6.00 | 5.93 | 5.83 | 5.76 | 5.70 | 5.63 | 5.56 | 5.52 |
| 13 | 5.75 | 5.63 | 5.54 | 5.47 | 5.37 | 5.30 | 5.24 | 5.17 | 5.10 | 5.07 |
| 14 | 5.38 | 5.25 | 5.17 | 5.10 | 5.00 | 4.94 | 4.87 | 4.81 | 4.74 | 4.71 |
| 15 | 5.07 | 4.95 | 4.86 | 4.80 | 4.70 | 4.64 | 4.57 | 4.51 | 4.44 | 4.41 |
| 16 | 4.82 | 4.70 | 4.61 | 4.54 | 4.45 | 4.39 | 4.32 | 4.26 | 4.19 | 4.16 |
| 17 | 4.60 | 4.48 | 4.40 | 4.33 | 4.24 | 4.18 | 4.11 | 4.05 | 3.98 | 3.95 |
| 18 | 4.42 | 4.30 | 4.22 | 4.15 | 4.06 | 4.00 | 3.93 | 3.87 | 3.80 | 3.77 |
| 19 | 4.26 | 4.14 | 4.06 | 3.99 | 3.90 | 3.84 | 3.78 | 3.71 | 3.65 | 3.61 |
| 20 | 4.12 | 4.00 | 3.92 | 3.86 | 3.77 | 3.70 | 3.64 | 3.58 | 3.51 | 3.48 |
| 21 | 4.00 | 3.88 | 3.80 | 3.74 | 3.64 | 3.58 | 3.52 | 3.46 | 3.39 | 3.36 |
| 22 | 3.89 | 3.78 | 3.70 | 3.63 | 3.54 | 3.48 | 3.41 | 3.35 | 3.28 | 3.25 |
| 23 | 3.79 | 3.68 | 3.60 | 3.53 | 3.44 | 3.38 | 3.32 | 3.25 | 3.19 | 3.16 |
| 24 | 3.71 | 3.59 | 3.51 | 3.45 | 3.36 | 3.29 | 3.23 | 3.17 | 3.10 | 3.07 |
| 25 | 3.63 | 3.52 | 3.43 | 3.37 | 3.28 | 3.22 | 3.15 | 3.09 | 3.03 | 2.99 |
| 26 | 3.56 | 3.44 | 3.36 | 3.30 | 3.21 | 3.15 | 3.08 | 3.02 | 2.95 | 2.92 |
| 27 | 3.49 | 3.38 | 3.30 | 3.23 | 3.14 | 3.08 | 3.02 | 2.96 | 2.89 | 2.86 |
| 28 | 3.43 | 3.32 | 3.24 | 3.18 | 3.09 | 3.02 | 2.96 | 2.90 | 2.83 | 2.80 |
| 29 | 3.38 | 3.27 | 3.18 | 3.12 | 3.03 | 2.97 | 2.91 | 2.84 | 2.78 | 2.74 |
| 30 | 3.33 | 3.22 | 3.13 | 3.07 | 2.98 | 2.92 | 2.86 | 2.79 | 2.73 | 2.69 |
| 35 | 3.13 | 3.02 | 2.93 | 2.87 | 2.78 | 2.72 | 2.66 | 2.59 | 2.52 | 2.49 |
| 40 | 2.98 | 2.87 | 2.79 | 2.73 | 2.64 | 2.57 | 2.51 | 2.44 | 2.38 | 2.34 |
| 50 | 2.79 | 2.68 | 2.60 | 2.53 | 2.44 | 2.38 | 2.31 | 2.25 | 2.18 | 2.14 |
| 60 | 2.67 | 2.55 | 2.47 | 2.41 | 2.32 | 2.25 | 2.19 | 2.12 | 2.05 | 2.01 |
| 70 | 2.58 | 2.47 | 2.39 | 2.32 | 2.23 | 2.16 | 2.10 | 2.03 | 1.95 | 1.92 |
| 80 | 2.52 | 2.41 | 2.32 | 2.26 | 2.16 | 2.10 | 2.03 | 1.96 | 1.89 | 1.85 |
| 90 | 2.47 | 2.36 | 2.27 | 2.21 | 2.11 | 2.05 | 1.98 | 1.91 | 1.83 | 1.79 |
| 100 | 2.43 | 2.32 | 2.24 | 2.17 | 2.08 | 2.01 | 1.94 | 1.87 | 1.79 | 1.75 |
| 120 | 2.37 | 2.26 | 2.18 | 2.11 | 2.02 | 1.95 | 1.88 | 1.81 | 1.73 | 1.68 |
| 150 | 2.32 | 2.21 | 2.12 | 2.06 | 1.96 | 1.89 | 1.82 | 1.74 | 1.66 | 1.62 |
| 200 | 2.26 | 2.15 | 2.07 | 2.00 | 1.90 | 1.83 | 1.76 | 1.68 | 1.60 | 1.55 |
| 250 | 2.23 | 2.12 | 2.03 | 1.97 | 1.87 | 1.80 | 1.72 | 1.65 | 1.56 | 1.51 |
| 300 | 2.21 | 2.10 | 2.01 | 1.94 | 1.85 | 1.78 | 1.70 | 1.62 | 1.53 | 1.48 |
| 400 | 2.18 | 2.07 | 1.98 | 1.92 | 1.82 | 1.75 | 1.67 | 1.59 | 1.50 | 1.45 |
| 500 | 2.17 | 2.05 | 1.97 | 1.90 | 1.80 | 1.73 | 1.65 | 1.57 | 1.48 | 1.43 |
| 600 | 2.16 | 2.04 | 1.96 | 1.89 | 1.79 | 1.72 | 1.64 | 1.56 | 1.46 | 1.41 |
| 750 | 2.15 | 2.03 | 1.95 | 1.88 | 1.78 | 1.71 | 1.63 | 1.55 | 1.45 | 1.40 |
| 1000 | 2.14 | 2.02 | 1.94 | 1.87 | 1.77 | 1.69 | 1.62 | 1.53 | 1.44 | 1.38 |

## TABLE A.4

### $\chi^2$ (Chi-Squared) Distribution: Critical Values of $\chi^2$

| Degrees of freedom | Significance level | | |
|---|---|---|---|
| | 5% | 1% | 0.1% |
| 1 | 3.841 | 6.635 | 10.828 |
| 2 | 5.991 | 9.210 | 13.816 |
| 3 | 7.815 | 11.345 | 16.266 |
| 4 | 9.488 | 13.277 | 18.467 |
| 5 | 11.070 | 15.086 | 20.515 |
| 6 | 12.592 | 16.812 | 22.458 |
| 7 | 14.067 | 18.475 | 24.322 |
| 8 | 15.507 | 20.090 | 26.124 |
| 9 | 16.919 | 21.666 | 27.877 |
| 10 | 18.307 | 23.209 | 29.588 |