UNIVERSITY OF SWAZILAND

MAIN EXAMINATION PAPER 2017

TITLE OF PAPER : PROBABILITY THEORY II

COURSE CODE : STA 212

TIME ALLOWED : 2 HOURS

INSTRUCTIONS : ANSWER ANY THREE QUESTIONS.

REQUIREMENTS : SCIENTIFIC CALCULATOR AND
STATISTICAL TABLES.

Question 1

If the joint moment generating function of the random variable X and Y is
$$M_{XY}(s,t) = \exp(s + 3t + 2s^2 + 18t^2 + 12st)$$
What is the Covariance of X and Y?

(20 Marks)

Question 2

a) Let X and Y be random variables such that X has density function
$$f_X(x) = 24x^2 \quad , \quad 0 < x < \frac{1}{2}$$
and the conditional density of Y given X = x is
$$p(y|x) = \frac{y}{2x^2} \quad , \quad 0 < y < 2x$$
What is the conditional density of X given Y = y over the appropriate domain?

(10 Marks)

b) Let the joint density of two random variables x and y be given by
$$f(x,y) = \frac{1}{6}(x + 4y), \quad 0 < x < 2, 0 < y < 1$$
Find the probability of $X \leq 1$ given that $y = \frac{1}{2}$.

(10 Marks)

Question 3

a) Let X and Y be discrete random variables with joint density

$$p(x,y) = \frac{x + 2y}{18} \quad , x = 1,2; y = 1,2$$

What is the covariance σ_{XY} between X and Y.

(15 Marks)

b) If $Var(X + Y) = 3, \quad Var(X - Y) = 1, \quad E(X) = 1,$ and $E(Y) = 2,$ the what is $E(XY)$?

(5 Marks)

Question 4

a) Let X and Y be discrete random variables with joint probability mass function

$$p(x,y) = \frac{1}{21}(x + y), \quad x = 1,2,3; y = 1,2$$

What is the conditional mean of X given Y= y, that is $E(X|y)$?

(10 Marks)

b) Let X and Y be continuous random variables with joint probability density function

$$f(x,y) = e^{-y}, \quad 0 < x < y < \infty$$

What is the conditional variance of Y *given that* $X = x$?

(10 Marks)

Question 5

a) Let each of the independent random variables X and Y have the density function

$$f(x) = e^{-x}, \quad 0 < x < \infty$$

What is the joint density of $U = X$ and $V = 2X + 3Y$ and the domain on which this density is positive?

(10 Marks)

b) Let X and Y be independent random variables, each with density function

$$f(x) = \lambda e^{-\lambda x}, \quad 0 < x < \infty$$

where $\lambda > 0$. Let $U = X + 2Y$ and $V = 2X + Y$. What is the joint density of U and V ?

(10 Marks)