UNIVERSITY OF SWAZILAND

SUPPLEMENTARY EXAMINATION PAPER 2017

TITLE OF PAPER : PROBABILITY THEORY II

COURSE CODE : STA 212

TIME ALLOWED : 2 HOURS

INSTRUCTIONS : ANSWER ANY THREE QUESTIONS.

REQUIREMENTS : SCIENTIFIC CALCULATOR

Question 1

a) The random variables X_1, ..., X_4 have a joint p.d.f. given by

$$f(x_1, x_2, x_3, x_4) = \frac{3}{4}(x_1^2 + x_2^2 + x_3^2 + x_4^2), \quad 0 < x_i < 1, i = 1,2,3,4$$

i) Use the joint probability density function to compute $P(X_1 < 1/2, X_2 < 3/4, X_3 < 1, X_4 > 1/2)$

(8 Marks)

ii) Compute the marginal probability density function of (X_1, X_2)

(7 Marks)

b) If X and Y have the joint p.d.f. $f(x, y) = ce^{-x}e^{-2y}$ for $0 < x < 1$ and $x < y < 1$, what the value of the constant is c?

(5 Marks)

Question 2

a) Let the joint density of two random variables X and Y be given by

$$f(x, y) = (0.6x + 0.6 \times 4y), \quad 0 < x < 2, \qquad 0 < y < 1$$

Find the conditional distribution of X given y.

b) Let the joint density of two random variables X and y be given by

$$f(x, y) = \frac{-3x^2 \log(y)}{2(1 + \log(2) - 2\log(4))}, \quad 0 \le x \le 1, \qquad 2 \le y \le 4$$

Are X and Y independent random variables?

Question 3

a) Let $X \sim U(-1,1)$ and let $Y = X^2$. Find Cov(X,Y).

b) Let the joint *pdf* of X and Y be

$$f(x, y) = 10, \quad for\ x < 1; x < y < x + 0.1$$

Find the correlation coefficient between X and Y

Question 4

Suppose the random variable X and Y are independent and are Gamma distributed with parameters (α, λ) and (β, λ) respectively

i.e. $f_X(x) = \frac{\lambda^\alpha x^{\alpha-1} \exp(-\lambda x)}{\Gamma(\alpha)}, \quad x \geq 0, \ \alpha, \lambda > 0$

with a similar expression for $f_Y(y)$.

By calculating the joint density function of (X+Y and X/Y, show that these random variables are independent and that X + Y has the Gamma $(\alpha + \beta, \lambda)$ distribution. Find the probability density function of X/Y. Why are X = y and Y/X independent? What is the probability density function of Y/X?

(20 Marks)

Question 5

Two tennis players, A and B, are playing a match. Let X be the number of serves faster than 125 km/h served by A in one of his service games and let Y be the number of these serves returned by B. The following probability model is proposed:

$P(X = 0) = 0.4$, $P(X = 1) = 0.3$, $P(X = 2) = 0.2$ and $P(X = 3) = 0.1$.

The conditional distribution of Y (given that $X = x > 0$) is binomial with parameters x and 0.4, and $P(Y = 0 \mid X = 0) = 1$. Assume that this model is correct when answering the following questions.

(a) Find the joint probability distribution of X and Y and display it in the form of a two-way table.

(7 Marks)

(b) Find the marginal distribution of Y and evaluate E(Y).

(4 Marks)

(c) Find Cov(X, Y).

(4 Marks)

(d) Use your joint probability distribution table to find the probability distribution of the number of serves faster than 125 km/h that are not returned by B in a game.

(5 Marks)