

DEPARTMENT OF STATISTICS AND DEMOGRAPHY

MAIN EXAMINATION, 2016/17

COURSE TITLE: MATHHEMATICS FOR STATISTICS

COURSE CODE: STA 213/ ST 202

TIME ALLOWED: TWO (2) HOURS

INSTRUCTION: ANSWER ANY THREE QUESTIONS
ALL QUESTIONS CARRY EQUAL MARKS (25 MARKS)

SPECIAL REQUIREMENTS: SCIENTIFIC CALCULATORS AND GRAPH PAPER

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INVIGILATOR

Question 1

Jim, the owner of Jim's Boat shop has added a new line of bassboats in his boat shop. He now wants to make a decision concerning his monthly advertisement and promotional efforts with respect to this new line of bassboats. He has to decide whether to advertise in radio commercials or use local Newspapers. He currently has E1000 to spend on advertising and radio commercials cost E300 (if presented 10 times each month) whilst Newspaper advertisement costs E200 (if advertised for 4 consecutive days). He estimates that his monthly bassboat sales (in thousands of Emalangeni) is approximated by the following function:

Bassboat sales $S = 4300x_1 - 300x_1^2 + 2500x_2 - 100x_2^2$, Where x_1 = number of radio commercials; x_2 =

number of newspaper advertisement.

The problem that describes this situation can be written as"

Maximise

$$f(x_1, x_2) = 4300x_1 - 300x_1^2 + 2500x_2 - 100x_2^2$$

Subject to $300x_1 + 200x_2 = 1000$

With $x_1, x_2 \geq 0$

(25 marks)

Question 2

Consider a quadratic model of the form: $f(x) = ax^2 + bx + c$, with the sum of squared errors for this

model given by $S = \sum_{i=1}^n [f(x_i) - y_i]^2 = \sum_{i=1}^n (ax_i^2 + bx_i + c - y_i)^2$. Find the values of a, b and c that minimise

S for the following points: (1,0), (2,1), (3,7), (4,13)

(25 marks)

Question 3

(a) The table below shows the number of deaths due to cancer by age group in a given country:

Age	14 and under	15-24	25-34	35-44	45-54	55-64	65-74	75-84	85 and over
Number	28	30	85	317	841	1599	2904	2897	1295

- (i) Determine the probability distribution $P(x)$ of the number of deaths due to cancer.
(ii) Find the probability that an individual who dies of cancer was between 35 and 74 years of age.
(iii) Find the probability that an individual who dies of cancer was over 44 years of age.
(iv) Find the probability that an individual who dies of cancer was under 25 years of age.

(b) The weekly demand for a product is modelled by the probability density function:

$$f(x) = \frac{1}{36}(-x^2 + 6x), 0 \leq x \leq 6$$

Where x is the number of units sold (in thousands). What are the minimum and maximum weekly sales? Find the probability that the sales for a randomly chosen week will be between 2000 and 4000 units.

(8+4+3+3+7 marks)

Question 4

(a) Find $\int \frac{e^{3x}}{1+e^{3x}} dx$

(b) Evaluate $\int_1^5 \frac{x}{\sqrt{2x-1}} dx$

(c) A psychologist finds that the probability that a participant in a memory experiment will recall between a and b percent (in decimal form) of the material is:

$$P(a \leq x \leq b) = \int_a^b \frac{28}{9} x^3 \sqrt{1-x} dx, 0 \leq a \leq b \leq 1.$$

Find the probability that a randomly chosen participant will recall between 0% and 87.5% of the material.

(d) Find $\int x^2 e^2 dx$

(5+8+8+4 marks)

Question 5

(a) Solve the following system of equations using the Gauss-Jordan elimination method:

$$3x_1 - x_2 + 2x_3 = 18$$

$$2x_1 + x_2 + x_3 = 6$$

$$x_1 - 3x_2 = 12$$

(15 marks)

(b) Use row operations and the Gauss-Jordan elimination method to transform $[A|I]$ to $[I|A^{-1}]$ where

$$A = \begin{bmatrix} 3 & -1 & 2 \\ 2 & 1 & 1 \\ 1 & -3 & 0 \end{bmatrix}$$

(10 marks)

END OF EXAM!!!!