## UNIVERSITY OF SWAZILAND

## FINAL EXAMINATION PAPER 2017

TITLE OF PAPER:<br>COURSE CODE:<br>TIME ALLOCATED:<br>2゙(TWO) HOURS<br>REQUIREMENTS: STATISTICAL TABLES AND CALCULATOR<br>INSTRUCTION:<br>ANSWER ALL QUESTIONS. THE QUESTIONS CARRY THE MARKS AS INDICATED WITHIN THE PARENTHESIS

THIS PAPER IS NOT TO BE OPENED UNTIL PERMISSION HAS BEEN GRANTED BY THE INVIGILATOR.

1. A multiple choice test consists of ten questions and three answers to each question, only one of which is correct. A person answers each question by tossing a balanced die and checking the first answer if the top face shows 1 or 2 , checking the second answer if it shows 3 or 4 , and checking the third answer if it shows 5 or 6 . Let $Y$ denote the number of correct answers the person chooses.
a. Explain why the random variable $Y$ has a binomial distribution, and give its parameters, $n$ and $p$.
b. What is the probability that the person chooses exactly 2 correct answers?
c. What is the probability that the person chooses at least one correct answer?
d. What is the expected number of correct answers the person chooses?
e. What is the variance of $Y$ ?
2. In a random sample of 150 mathematics lecturers, $63.5 \%$ say that they are employed part-time on their lecturing jobs. Determine the $99 \%$ confidence interval for the proportion of mathematics lecturers with part-time jobs.
3. A machine in a factory must be repaired if it produces more than $10 \%$ defectives among the large lot of items that it produces in a day. A random sample of 100 items from the day's production contains 15 defectives and the manager says that the machine must be repaired. Use a test with $\alpha=0.01$ level of significance.
a. Select the appropriate test statistic and calculate the chosen test statistic.
b. Determine the critical value at $\alpha=0.01$.
c. On the basis of the results in (a) and (b), what will be the conclusion?
4. An experiment was planned to compare the mean time (in days) required to recover from a common cold for persons given a daily dose of 4 milligrams ( mg ) of vitamin $C$ versus those who were not given a vitamin supplement. Suppose that 35 adults were randomly selected for each treatment category and that the mean recovery times and standard deviations for the two groups were as follows:

|  | No Vitamin Supplement | 4 mg Vitamin C |
| :--- | :--- | :--- |
| Sample Size | 35 | 35 |
| Sample Mean | 6.9 | 5.8 |
| Sample <br> Deviation | Standard | 2.9 |

a. If your research objective is to show that the use of vitamin $C$ reduces the mean time required to recover from a common cold and its complications, give the null and alternative hypotheses for the test. Is this a one- or a two-tailed test?
b. Conduct the statistical test of the null hypothesis in part a and state your conclusion. Test using $\alpha=0.05$.
5. A freeway with four lanes in each direction was studied to see whether drivers prefer to drive on the inside lanes. A total of 1000 automobiles were observed during heavy early morning traffic, and the number of cars in each lane was recorded:

| Lane | 1 | 2 | 3 | 4 |
| :--- | :--- | :--- | :--- | :--- |
| Observed <br> Count | 294 | 276 | 238 | 192 |

a. Do the data present sufficient evidence to indicate that some lanes are preferred over others? Test using $\alpha=0.05$.
b. Are there are any differences, discuss the nature of the differences.
6. A random sample of size $n=7$ from a normal population produced these measurements: 1.4, 3.6, 1.7, 2.0, 3.3, 2.8, 2.9.
a. Calculate the sample variance, $s^{2}$.
b. Construct a $95 \%$ confidence interval for the population variance, $\sigma^{2}$.
c. Test $H_{0}: \sigma^{2}=0.8$ versus $H_{a}: \sigma^{2} \neq 0.8$ using $\alpha=0.05$. State your conclusions.

