# DEPARTMENT OF STATISTICS AND DEMOGRAPHY 

SUPPLEMENTARY EXAMINATION, 2017/18

## COURSE TITLE:

COURSE CODE:
ST 408

TIME ALLOWED:

INSTRUCTION:

SPECIAL REQUIREMENTS:

OPERATIONS RESEARCH II

THREE (3) HOURS

ANSWER SECTION A AND ANY THREE OUESTIONS IN SECTION B

SCIENTIFIC CALCULATORS AND STATISTICAL TABLES

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## SECTION A

## Question 1

Bill Holiday is not sure what she should do. He can either build quadplex (i.e a building with four apartments), build a duplex, gather additional information or simply do nothing. If he gathers additional information, the results could either be favourable or unfavourable, but it would cost him $\$ 3,000$ to gather the information. Bill believes that there is a $50-50$ chance that the information will be favourable. If the rental market is favourable, Bill will earn $\$ 15,000$ with the quadplex or $\$ 5.000$ with the duplex. Bill doesn't have the financial resources to do both. With an unfavourable rental market, however, Bill could loose $\$ 20,000$ with the quadplex or $\$ 10,000$ with the duplex. Without gathering additional information. Bill estimates that the probability of a favourable rental market is 0.7 . A favourable report from the study would increase the probability of a favourable rental market to 0.9 . Furthermore, an unfavourable report from the additional information would decrease the probability of a favourable rental market to 0.4 . Of course, Bill could forget all of these numbers and do nothing. What is your advice to Bill?
(25 marks)

## SECTION B

## Question 2

(a). Accounts receivable, A manager has developed the following transition matrix for a firm's accounts receivable:
$\left.\begin{array}{c}P \\ 1 \\ 2 \\ b\end{array} \begin{array}{cccc}P & 1 & 2 & b \\ 1 & 0 & 0 & 0 \\ .5 & .3 & .2 & 0 \\ .3 & 0 & .4 & .3 \\ 0 & 0 & 0 & 1\end{array}\right]$

Where
$P=$ paid
$1=1$ to 30 days overdue
$2=31$ to 60 days overdue
$b=\mathrm{bad}$ debt
(Note: Accounts are billed but classified in items of months overdue. Consequently, it is possible for an account to remain in either the 1 or 2 categories for several periods.
Therefore, there is a nonzero probability of remaining in either 1 or 2 )
(i) Obtain the fundamental matrix.
(ii) If there is currently $\$ 10,000$ in accounts in the 1 category and $\$ 6,000$ in the 2 category, determine the expected amount of bad debt.
(15 marks)
(b). The following table gives a breakdown of customers staying and switching brands for two periods:

|  |  | This period |  |  |  |
| :--- | :--- | :--- | :--- | :--- | :---: |
|  |  | Brand A | Brand B | Brand C |  |
|  | Last Period | A | 350 | 80 |  |
|  | B | 240 | 480 | 80 |  |
|  | C | 210 | 140 | 350 |  |
|  |  |  |  | 2 |  |

Assume that these figures accurately, they reflect period-to-period behaviours of brand switching. Find the transition probabilities.
(10 marks)

## Question 3

(a) Trucks arrive at a warehouse at a rate of 15 per hour during business hours. Crews can unload the trucks at a rate of 5 per hour. The high unloading rate is due to cargo being containerized. Recent changes in wage rates have caused the warehouse manager to re-examine the question of how many crews to use. The new rates for crews and dock costs is $\$ 100$ per hour: truck and driver cost is $\$ 120$ per hour.
Determine the optimal crew size.
(15 marks)
(b) A video arcade game is designed to operate for exactly three minutes, during which time a player attempts to capture as many purple monkeys as possible. Customer player arrivals can be described by a Poisson distribution with a mean arrival rate of 12 per hour. Compute each of the performance measures: namely:
(i) Average number waiting in the line and in the system.
(ii) Average time waiting in the line and in the system.
(iii)The system utilization.
(10 marks)

## Question 4

Building a backyard swimming pool consists of nine major activities. The activities and their immediate predecessors are shown.
a. Develop the project network.

(5 marks)
b. Assume that the activity time estimates (in days) for the swimming pool construction project are as follows:

| Activity | Optimistic | Most Probable | Pessimistic |
| :--- | :---: | :---: | :---: |
| A | 3 | 5 | 6 |
| B | 2 | 4 | 6 |
| C | 5 | 6 | 7 |
| D | 7 | 9 | 10 |
| E | 2 | 4 | 6 |
| F | 1 | 2 | 3 |
| G | 5 | 8 | 10 |
| H | 6 | 8 | 10 |
| I | 3 | 4 | 5 |

(i) What are the critical activities?
(ii) What is the expected time to complete the project?
(iii) What is the probability that project can be completed in 25 or fewer days? ( 20 marks)

## Question 5

(a). Economic production quantity. The Dine Corporation is both a producer and a user of brass
couplings. The firm operates 220 days a year and uses the couplings at a steady rate of 50 per day.
Couplings can be produced at a rate of 200 per day. Annual storage cost is $\$ 2$ per coupling, and machine setup cost is $\$ 70$ per run.
(i) Determine the economic run quantity.
(ii) Approximately how many runs per year will there be?
(iii) Compute the maximum inventory level.
(iv) Determine the length of the pure consumption portion of the cycle.
(b). Single-period model. A firm that installs cable TV systems uses a certain piece of equipment for which it carries two spare parts cost $\$ 500$ each and have no salvage value. Part failures can be modelled by a Poisson distribution with a mean of two failures during the useful life of the equipment. Holding and disposal costs are negligible. Estimate the apparent rage of shortage cost.
(9 marks)

## END OF EXAM:!

## Standard Normal Probabilities



Table entry for $z$ is the area under the standard normal curve to the left of $z$.

| $z$ | . 00 | . 01 | . 02 | . 03 | . 04 | . 05 | . 06 | . 07 | . 08 | . 09 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0.0 | . 5000 | . 5040 | . 5080 | . 5120 | . 5160 | . 5199 | . 5239 | . 5279 | . 5319 | . 5359 |
| 0.1 | . 5398 | . 5438 | . 5478 | . 5517 | . 5557 | . 5596 | . 5636 | . 5675 | . 5714 | . 5753 |
| 0.2 | . 5793 | . 5832 | . 5871 | . 5910 | . 5948 | . 5987 | . 6026 | . 6064 | . 6103 | . 6141 |
| 0.3 | . 6179 | . 6217 | . 6255 | . 6293 | . 6331 | . 6368 | . 6406 | . 6443 | . 6480 | . 6517 |
| 0.4 | . 6554 | . 6591 | . 6628 | . 6664 | . 6700 | . 6736 | . 6772 | . 6808 | . 6844 | . 6879 |
| 0.5 | . 6915 | . 6950 | . 6985 | . 7019 | . 7054 | . 7088 | . 7123 | . 7157 | . 7190 | . 7224 |
| 0.6 | . 7257 | . 7291 | . 7324 | . 7357 | . 7389 | . 7422 | . 7454 | . 7486 | . 7517 | . 7549 |
| 0.7 | . 7580 | . 7611 | . 7642 | . 7673 | . 7704 | . 7734 | . 7764 | . 7794 | . 7823 | . 7852 |
| 0.8 | . 7881 | . 7910 | . 7939 | . 7967 | . 7995 | . 8023 | . 8051 | . 8078 | . 8106 | . 8133 |
| 0.9 | . 8159 | . 8186 | . 8212 | . 8238 | . 8264 | . 8289 | . 8315 | . 8340 | . 8365 | . 8389 |
| 1.0 | . 8413 | . 8438 | . 8461 | . 8485 | . 8508 | . 8531 | . 8554 | . 8577 | . 8599 | . 8621 |
| 1.1 | . 8643 | . 8665 | . 8686 | . 8708 | . 8729 | . 8749 | . 8770 | . 8790 | . 8810 | . 8830 |
| 1.2 | . 8849 | . 8869 | . 8888 | . 8907 | . 8925 | . 8944 | . 8962 | . 8980 | . 8997 | . 9015 |
| 1.3 | . 9032 | . 9049 | . 9066 | . 9082 | . 9099 | . 9115 | . 9131 | . 9147 | . 9162 | . 9177 |
| 1.4 | . 9192 | . 9207 | . 9222 | . 9236 | . 9251 | . 9265 | . 9279 | . 9292 | . 9306 | . 9319 |
| 1.5 | . 9332 | . 9345 | . 9357 | . 9370 | . 9382 | . 9394 | . 9406 | . 9418 | . 9429 | . 9441 |
| 1.6 | . 9452 | . 9463 | . 9474 | . 9484 | . 9495 | . 9505 | . 9515 | . 9525 | . 9535 | . 9545 |
| 1.7 | . 9554 | . 9564 | . 9573 | . 9582 | . 9591 | . 9599 | . 9608 | . 9616 | . 9625 | . 9633 |
| 1.8 | . 9641 | . 9649 | . 9656 | . 9664 | . 9671 | . 9678 | . 9686 | . 9693 | . 9699 | . 9706 |
| 1.9 | . 9713 | . 9719 | . 9726 | . 9732 | . 9738 | . 9744 | . 9750 | . 9756 | . 9761 | . 9767 |
| 2.0 | . 9772 | . 9778 | . 9783 | . 9788 | . 9793 | . 9798 | . 9803 | . 9808 | . 9812 | . 9817 |
| 2.1 | . 9821 | . 9826 | . 9830 | . 9834 | . 9838 | . 9842 | . 9846 | . 9850 | . 9854 | . 9857 |
| 2.2 | . 9861 | . 9864 | . 9868 | . 9871 | . 9875 | . 9878 | . 9881 | . 9884 | . 9887 | . 9890 |
| 2.3 | . 9893 | . 9896 | . 9898 | . 9901 | . 9904 | . 9906 | . 9909 | . 9911 | . 9913 | . 9916 |
| 2.4 | . 9918 | . 9920 | . 9922 | . 9925 | . 9927 | . 9929 | . 9931 | . 9932 | . 9934 | . 9936 |
| 2.5 | . 9938 | . 9940 | . 9941 | . 9943 | . 9945 | . 9946 | . 9948 | . 9949 | . 9951 | . 9952 |
| 2.6 | . 9953 | . 9955 | . 9956 | . 9957 | . 9959 | . 9960 | . 9961 | . 9962 | . 9963 | . 9964 |
| 2.7 | . 9965 | . 9966 | . 9967 | . 9968 | . 9969 | . 9970 | . 9971 | . 9972 | . 9973 | . 9974 |
| 2.8 | . 9974 | . 9975 | . 9976 | . 9977 | . 9977 | . 9978 | . 9979 | . 9979 | . 9980 | . 9981 |
| 2.9 | . 9981 | . 9982 | . 9982 | . 9983 | . 9984 | . 9984 | . 9985 | . 9985 | . 9986 | . 9986 |
| 3.0 | . 9987 | . 9987 | . 9987 | . 9988 | . 9988 | . 9989 | . 9989 | . 9989 | . 9990 | . 9990 |
| 3.1 | . 9990 | . 9991 | . 9991 | . 9991 | . 9992 | . 9992 | . 9992 | . 9992 | . 9993 | . 9993 |
| 3.2 | . 9993 | . 9993 | . 9994 | . 9994 | . 9994 | . 9994 | . 9994 | . 9995 | . 9995 | . 9995 |
| 3.3 | . 9995 | . 9995 | . 9995 | . 9996 | . 9996 | . 9996 | . 9996 | . 9996 | . 9996 | . 9997 |
| 3.4 | . 9997 | . 9997 | . 9997 | . 9997 | . 9997 | . 9997 | . 9997 | . 9997 | . 9997 | . 9998 |

## Standard Normal Probabilities

| Table entry for $z$ is the area under the standard normal curve to the left of $z$. |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $z$ | . 00 | . 01 | . 02 | . 03 | . 04 | . 05 | . 06 | . 07 | . 08 | . 09 |
| -3.4 | . 0003 | . 0003 | . 0003 | . 0003 | . 0003 | . 0003 | . 0003 | . 0003 | . 0003 | . 0002 |
| -3.3 | . 0005 | . 0005 | . 0005 | . 0004 | . 0004 | . 0004 | . 0004 | . 0004 | . 0004 | . 0003 |
| -3.2 | . 0007 | . 0007 | . 0006 | . 0006 | . 0006 | . 0006 | . 0006 | . 0005 | . 0005 | . 0005 |
| -3.1 | . 0010 | . 0009 | . 0009 | . 0009 | . 0008 | . 0008 | . 0008 | . 0008 | . 0007 | . 0007 |
| -3.0 | . 0013 | . 0013 | . 0013 | . 0012 | . 0012 | . 0011 | . 0011 | . 0011 | . 0010 | . 0010 |
| -2.9 | . 0019 | . 0018 | . 0018 | . 0017 | . 0016 | . 0016 | . 0015 | . 0015 | . 0014 | . 0014 |
| -2.8 | . 0026 | . 0025 | . 0024 | . 0023 | . 0023 | . 0022 | . 0021 | . 0021 | . 0020 | . 0019 |
| -2.7 | . 0035 | . 0034 | . 0033 | . 0032 | . 0031 | . 0030 | . 0029 | . 0028 | . 0027 | . 0026 |
| -2.6 | . 0047 | . 0045 | . 0044 | . 0043 | . 0041 | . 0040 | . 0039 | . 0038 | . 0037 | . 0036 |
| -2.5 | . 0062 | . 0060 | . 0059 | . 0057 | . 0055 | . 0054 | . 0052 | . 0051 | . 0049 | . 0048 |
| -2.4 | . 0082 | . 0080 | . 0078 | . 0075 | . 0073 | . 0071 | . 0069 | . 0068 | . 0066 | . 0064 |
| -2.3 | . 0107 | . 0104 | . 0102 | . 0099 | . 0096 | . 0094 | . 0091 | . 0089 | . 0087 | . 0084 |
| -2.2 | . 0139 | . 0136 | . 0132 | . 0129 | . 0125 | . 0122 | . 0119 | . 0116 | . 0113 | . 0110 |
| -2.1 | . 0179 | . 0174 | . 0170 | . 0166 | . 0162 | . 0158 | . 0154 | . 0150 | . 0146 | . 0143 |
| -2.0 | . 0228 | . 0222 | . 0217 | . 0212 | . 0207 | . 0202 | . 0197 | . 0192 | . 0188 | . 0183 |
| -1.9 | . 0287 | . 0281 | . 0274 | . 0268 | . 0262 | . 0256 | . 0250 | . 0244 | . 0239 | . 0233 |
| -1.8 | . 0359 | . 0351 | . 0344 | . 0336 | . 0329 | . 0322 | . 0314 | . 0307 | . 0301 | . 0294 |
| -1.7 | . 0446 | . 0436 | . 0427 | . 0418 | . 0409 | . 0401 | . 0392 | . 0384 | . 0375 | . 0367 |
| -1.6 | . 0548 | . 0537 | . 0526 | . 0516 | . 0505 | . 0495 | . 0485 | . 0475 | . 0465 | . 0455 |
| -1.5 | . 0668 | . 0655 | . 0643 | . 0630 | . 0618 | . 0606 | . 0594 | . 0582 | . 0571 | . 0559 |
| -1.4 | . 0808 | . 0793 | . 0778 | . 0764 | . 0749 | . 0735 | . 0721 | . 0708 | . 0694 | . 0681 |
| -1.3 | . 0968 | . 0951 | . 0934 | . 0918 | . 0901 | . 0885 | . 0869 | . 0853 | . 0838 | . 0823 |
| -1.2 | . 1151 | . 1131 | . 1112 | . 1093 | . 1075 | . 1056 | . 1038 | . 1020 | . 1003 | . 0985 |
| -1.1 | . 1357 | . 1335 | . 1314 | . 1292 | . 1271 | . 1251 | . 1230 | . 1210 | . 1190 | . 1170 |
| -1.0 | . 1587 | . 1562 | . 1539 | . 1515 | . 1492 | . 1469 | . 1446 | . 1423 | . 1401 | . 1379 |
| -0.9 | . 1841 | . 1814 | . 1788 | . 1762 | . 1736 | . 1711 | . 1685 | . 1660 | . 1635 | . 1611 |
| -0.8 | . 2119 | . 2090 | . 2061 | . 2033 | . 2005 | . 1977 | . 1949 | . 1922 | . 1894 | . 1867 |
| -0.7 | . 2420 | . 2389 | . 2358 | . 2327 | . 2296 | . 2266 | . 2236 | . 2206 | . 2177 | . 2148 |
| -0.6 | . 2743 | . 2709 | . 2676 | . 2643 | . 2611 | . 2578 | . 2546 | . 2514 | . 2483 | . 2451 |
| -0.5 | . 3085 | . 3050 | . 3015 | . 2981 | . 2946 | . 2912 | . 2877 | . 2843 | . 2810 | . 2776 |
| -0.4 | . 3446 | . 3409 | . 3372 | . 3336 | . 3300 | . 3264 | . 3228 | . 3192 | . 3156 | . 3121 |
| -0.3 | . 3821 | . 3783 | . 3745 | . 3707 | . 3669 | . 3632 | . 3594 | . 3557 | . 3520 | . 3483 |
| -0.2 | . 4207 | . 4168 | . 4129 | . 4090 | . 4052 | . 4013 | . 3974 | . 3936 | . 3897 | . 3859 |
| -0.1 | . 4602 | . 4562 | . 4522 | . 4483 | . 4443 | . 4404 | . 4364 | . 4325 | . 4286 | . 4247 |
| -0.0 | . 5000 | . 4960 | . 4920 | . 4880 | . 4840 | . 4801 | . 4761 | . 4721 | . 4681 | . 4641 |

