UNIVERSITY OF SWAZILAND

UMCULU SISEKELO SESIVE

MAIN EXAMINATION PAPER 2017

TITLE OF PAPER	:	PROBABILITY THEORY I
COURSE CODE	:	STA 211
TIME ALLOWED	:	2 HOURS
INSTRUCTIONS	:	ANSWER ANY THREE QUESTIONS.
REQUIREMENTS	:	SCIENTIFIC CALCULATOR AND STATISTICAL TABLES.

Question 1

a) How many arrangements are there of the word PROBABILITY?

(8 Marks)

b) Let C and D be two events with $P(C) = 0.25$, $P(D) = 0.45$, and $P(C \cap D) = 0.1$.
What is $P(C^c \cap D)$?

(6 Marks)

c) A multiple choice exam has 4 choices for each question. A student has studied enough so that the probability they will know the answer to a question is 0.5, the probability that they will be able to eliminate one choice is 0.25, otherwise all 4 choices seem equally plausible. If they know the answer they will get the question right. If not they have to guess from the 3 or 4 choices. As the teacher you want the test to measure what the student knows. If the student answers a question correctly what's the probability they knew the answer?

(6 Marks)

Question 2

a) Let R be the rate at which customers are served in a queue. Suppose that R is a random variable with pdf $f(r) = 2e^{-2r}$ on $[0,\infty)$. Find the pdf of the waiting time per customer $T = 1/R$.

(5 Marks)

b) Suppose that X is a random variable that takes on values 0, 2 and 3 with probabilities 0.3, 0.1, 0.6 respectively. Let $Y = 3(X - 1)^2$.

i. What is the expectation of X?
ii. What is the variance of X?
iii. What is the expectation of Y?

(5 + 5 + 5 Marks)

Question 3

Let X have range [0, 3] and density $f(x) = kx^2$. Let $Y = X^3$.

a) Find k and the cumulative distribution function of X.

(4 Marks)

b) Find the 30th percentile of X.

(3 Marks)

c) Use moment generating functions to compute E(Y) and Var(Y).

(5+4+4 Marks)

Question 4

Suppose that buses are scheduled to arrive at a bus stop at noon but are always X minutes late, Suppose that you arrive at the bus stop precisely at noon.

a) Compute the probability that you have to wait for more than five minutes for the bus to arrive.

(10 Marks)

b) Suppose that you have already waiting for 10 minutes. Compute the probability that you have to wait an additional five minutes or more.

(10 Marks)

Question 5

a) Using a moment generating function to derive the mean and variance of a probability density function of this form;

$$f(x;\alpha,\beta) = \beta x^{\beta-1}e^{-\alpha x^{\beta}}, x > 0$$

(10 Marks)

b) Suppose that the service life, in years, of a hearing aid battery is a random variable having parameters $\alpha = 1/2$ and $\beta = 2$.

(i) How long can such a battery be expected to last?
(ii) What is the probability that such a battery will be operating after 2 years?

(5+5 Marks)