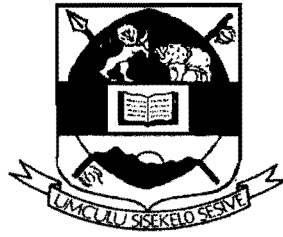


UNIVERSITY OF SWAZILAND



MAIN EXAMINATION PAPER 2017

- TITLE OF PAPER :** **PROBABILITY THEORY I**
- COURSE CODE :** **STA 211**
- TIME ALLOWED :** **2 HOURS**
- INSTRUCTIONS :** **ANSWER ANY THREE QUESTIONS.**
- REQUIREMENTS :** **SCIENTIFIC CALCULATOR AND
STATISTICAL TABLES.**

Question 1

- a) How many arrangements are there of the word PROBABILITY? (8 Marks)
- b) Let C and D be two events with $P(C) = 0.25$, $P(D) = 0.45$, and $P(C \cap D) = 0.1$. What is $P(C^c \cap D)$? (6 Marks)
- c) A multiple choice exam has 4 choices for each question. A student has studied enough so that the probability they will know the answer to a question is 0.5, the probability that they will be able to eliminate one choice is 0.25, otherwise all 4 choices seem equally plausible. If they know the answer they will get the question right. If not they have to guess from the 3 or 4 choices. As the teacher you want the test to measure what the student knows. If the student answers a question correctly what's the probability they knew the answer? (6 Marks)

Question 2

- a) Let R be the rate at which customers are served in a queue. Suppose that R is a random variable with pdf $f(r) = 2e^{-2r}$ on $[0, \infty)$. Find the pdf of the waiting time per customer $T = 1/R$. (5 Marks)
- b) Suppose that X is a random variable that takes on values 0, 2 and 3 with probabilities 0.3, 0.1, 0.6 respectively. Let $Y = 3(X - 1)^2$.
- What is the expectation of X?
 - What is the variance of X?
 - What is the expectation of Y?

(5 + 5 + 5 Marks)

Question 3

Let X have range $[0, 3]$ and density $f(x) = kx^2$. Let $Y = X^3$.

- a) Find k and the cumulative distribution function of X. (4 Marks)
- b) Find the 30th percentile of X. (3 Marks)
- c) Use moment generating functions to compute $E(Y)$ and $\text{Var}(Y)$. (5+4+4 Marks)

Question 4

Suppose that buses are scheduled to arrive at a bus stop at noon but are always X minutes late, Suppose that you arrive at the bus stop precisely at noon.

- a) Compute the probability that you have to wait for more than five minutes for the bus to arrive. (10 Marks)
- b) Suppose that you have already waiting for 10 minutes. Compute the probability that you have to wait an additional five minutes or more. (10 Marks)

Question 5

- a) Using a moment generating function to derive the mean and variance of a probability density function of this form;

$$f(x; \alpha, \beta) = \beta x^{\beta-1} e^{-\alpha x^\beta}, x > 0$$

(10 Marks)

- b) Suppose that the service life, in years, of a hearing aid battery is a random variable having parameters $\alpha = 1/2$ and $\beta = 2$.

- (i) How long can such a battery be expected to last?
(ii) What is the probability that such a battery will be operating after 2 years?

(5+5 Marks)