UNIVERSITY OF SWAZILAND

MAIN EXAMINATION PAPER 2018

TITLE OF PAPER	**:**	**PROBABILITY THEORY II**
COURSE CODE	**:**	**STA 212**
TIME ALLOWED	**:**	**2 HOURS**
INSTRUCTIONS	**:**	**ANSWER ANY THREE QUESTIONS.**
REQUIREMENTS	**:**	**SCIENTIFIC CALCULATOR AND STATISTICAL TABLES.**

Question 1

The joint probability mass function of the lifetimes X and Y of two connected components in a machine can be modelled by $p(x,y) = \frac{e^{-2}}{x!(y-x)!}$ for $x = 0, 1, \ldots$ and $y = x, x+1, \ldots$

a) What is the marginal distributions of X and Y?
b) What is the joint probability mass function of X and Y – X? Are X and Y – x independent?
c) What is the correlation between X and Y?

(7 + 7 + 6 Marks)

Question 2

The continuous random variables X and Y satisfy $f(y|x) = \frac{1}{x}$ for 0 < y < x, and $f(y|x) = 0$ otherwise. The marginal density function of X is given by $f_X(x) = 2x$ for $0 < x < 1$ and $f_X(x) = 0$ otherwise.

a) What is the conditional density? $f(x|y)$?
b) What is $E(X|Y = y)$?

(10 + 10 Marks)

Question 3

a) Let $X \sim N(\mu_1, \sigma_1^2)$, and $Y \sim N(\mu_2, \sigma_2^2)$. Furthermore X and Y are independent to each other. Derive the distribution of 2X - 3Y – 5.

b) A company insures homes in three cities J, K, L. The losses occurring in these cities are independent. The moment generating functions for the loss distributions of the cities are

$$M_J(t) = (1-2t)^{-3}, \quad M_K(t) = (1-2t)^{-2.5}, \quad M_L(t) = (1-2t)^{-4.5}$$

Let X represent the combined losses from the three cities. Calculate $E(X^3)$.

(10 + 10 Marks)

Question 4

a) An insurance policy pays a total medical benefit consisting of a part paid to the surgeon, X, and a part paid to the hospital, Y, so that the total benefit is X+Y. Suppose that Var(X) = E5, 000, Var(Y) = E10,000, and Var(X + Y) = E17, 000.

If X is the increased by a flat amount of E100, and Y is increased by 10%, what is the variance of the total benefit after these increases?

b) Given that E(X) = 5, $E(X^2)$ = 27.4, E(Y) = 7, E(Y2) = 51.4, and Var(X + Y) =8, find Cov(X + Y, X + 1.2Y).

(10 + 10 Marks)

Question 5

a) A diagnostic test for the presence of disease has two possible outcomes: 1 for disease present and 0 for disease not present. Let X denote the disease state of a patient, and let Y denote the outcome of a diagnostic test. The joint probability function of X and Y is given by:

$$P(X = x, Y = y) = \begin{cases} 0.800 & for\ (x, y) = (0,0) \\ 0.050 & for\ (x, y) = (1,0) \\ 0.025 & for\ (x, y) = (0,1) \\ 0.125 & for\ (x, y) = (1,1) \end{cases}$$

Calculate Var(Y|X=1).

b) The stock prices of two companies at the end of any given year are modeles with random variables X and Y that follow a distribution with joint density function

$$f(x, y) = \begin{cases} 2x & for\ 0 < x < 1, x < y < x + 1 \\ 0 & otherwise \end{cases}$$

What is the conditional variance of Y given that X = x?

(8 + 12 Marks)