UNIVERSITY OF SWAZILAND

MAIN EXAMINATION PAPER 2018

TITLE OF PAPER	: STATISTICAL INFERENCE II
COURSE CODE	: STA 302 / ST 303
TIME ALLOWED	: 2 HRS
REQUIREMENTS	: CALCULATOR
INSTRUCTIONS	: ANSWER ANY THREE QUESTIONS

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Question 1

Let $X_1, X_2, \ldots, X_n$ be a random sample from a population density function $f(x|\theta)$, where θ is a parameter. Let $T(\mathbf{X}) = T(X_1, X_2, \ldots, X_n)$ be a sufficient statistic.

a) What can be said about the conditional distribution of $X_1, X_2, \ldots, X_n$ given $T(\mathbf{X}) = T(x)$.

b) State the factorization theorem for sufficient statistics.

c) Suppose now that

$$f(x|\theta) = \frac{x^{\theta-1}e^{-x}}{(\theta-1)!} \quad x > 0, \text{ and } \theta > 0.$$

Show that $T(\mathbf{X}) = \sum_{i=1}^{n} lnX_i$ is a sufficient statistic for θ.

(5+5+10 Marks)

Question 2

Let $X_1, X_2, \ldots, X_n$ be a random sample from a distribution with probability density function given by

$$f(x;\theta) = \frac{1}{2\theta}e^{-\frac{|x|}{\theta}}, -\infty < x < \infty, \theta > 0.$$

a) Find the UMVUE of θ.
b) Show that the UMVUE of θ achieves the Crame'r-Rao lower bound.

(10 + 10 Marks)

Question 3

Suppose $X_1, X_2, \ldots, X_n$ is a i.i.d. sample from Uniform $[0,\theta]$, $\theta > 0$.

a) Find the MLE of θ^2.
b) Show that the MLE obtained in a) is biased for θ^2.
c) Show that for any fixed $\theta > 0$, the bias goes to 0 as $n \to \infty$.

(5+10+5 Marks)

Question 4

a) Suppose X is a single observation from a population with probability density function given by:

$$f(x|\theta) = \theta x^{\theta-1}, 0 < x < 1.$$

where $\theta > 0$ is the parameter of interest. Find the rejection region for the most powerful test of level 0.05, for testing the simple hypothesis $H_0: \theta = 3$ against the simple alternative hypothesis $H_0: \theta = 2$.

b) Let $X_1, X_2, \ldots, X_n$ be a random sample from a Uniform$[0,\theta]$ distribution, where $\theta > 0$ is the population parameter. Find the uniformly most powerful rejection region of size α for testing the hypothesis $H_0: \theta = 2$ against $H_1: \theta \leq 2$.

(10 + 10 Marks)

Question 5

a) Suppose the interest is in the true mortality risk θ in a hospital H which is about to try a new operation. On average in the country around 10% of people die, but mortality rates differ in different hospitals vary from 3% to around 20%. Hospital H has no deaths in their first 10 operations. What should be the belief about θ?

b) Suppose $X_1, X_2, \ldots, X_n$ are iid Poisson(λ) random variables, and that λ has an exponential distribution with mean 1, so that $\pi(\lambda) = e^{-\lambda}$, $\lambda > 0$. Find $\hat{\theta}$ under quadratic loss and absolute error loss.

(10 + 10 Marks)