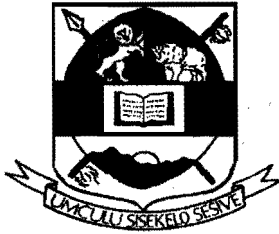


UNIVERSITY OF SWAZILAND



MAIN EXAMINATION PAPER 2018

TITLE OF PAPER : STATISTICAL INFERENCE II

COURSE CODE : STA 302 / ST 303

TIME ALLOWED : 2 HRS

REQUIREMENTS : CALCULATOR

INSTRUCTIONS : ANSWER ANY THREE QUESTIONS

**DO NOT OPEN THIS PAPER UNTIL PERMISSION IS GIVEN BY THE
INVIGILATOR**

Question 1

Let X_1, X_2, \dots, X_n be a random sample from a population density function $f(x|\theta)$, where θ is a parameter. Let $T(\mathbf{X}) = T(X_1, X_2, \dots, X_n)$ be a sufficient statistic.

- What can be said about the conditional-distribution of X_1, X_2, \dots, X_n given $T(\mathbf{X}) = T(x)$.
- State the factorization theorem for sufficient statistics.
- Suppose now that

$$f(x|\theta) = \frac{x^{\theta-1}e^{-x}}{(\theta-1)!} \quad x > 0, \text{ and } \theta > 0.$$

Show that $T(\mathbf{X}) = \sum_{i=1}^n \ln X_i$ is a sufficient statistic for θ .

(5+5+10 Marks)

Question 2

Let X_1, X_2, \dots, X_n be a random sample from a distribution with probability density function given by

$$f(x; \theta) = \frac{1}{2\theta} e^{-\frac{|x|}{\theta}}, \quad -\infty < x < \infty, \theta > 0.$$

- Find the UMVUE of θ .
- Show that the UMVUE of θ achieves the Crame'r-Rao lower bound.

(10 + 10 Marks)

Question 3

Suppose X_1, X_2, \dots, X_n is a i.i.d. sample from Uniform $[0, \theta]$, $\theta > 0$.

- Find the MLE of θ^2 .
- Show that the MLE obtained in a) is biased for θ^2 .
- Show that for any fixed $\theta > 0$, the bias goes to 0 as $n \rightarrow \infty$.

(5+10+5 Marks)

Question 4

- Suppose X is a single observation from a population with probability density function given by:

$$f(x|\theta) = \theta x^{\theta-1}, \quad 0 < x < 1.$$

where $\theta > 0$ is the parameter of interest. Find the rejection region for the most powerful test of level 0.05, for testing the simple hypothesis $H_0: \theta = 3$ against the simple alternative hypothesis $H_1: \theta = 2$.

- Let X_1, X_2, \dots, X_n be a random sample from a Uniform $[0, \theta]$ distribution, where $\theta > 0$ is the population parameter. Find the uniformly most powerful rejection region of size α for testing the hypothesis $H_0: \theta = 2$ against $H_1: \theta \leq 2$.

(10 + 10 Marks)

Question 5

- a) Suppose the interest is in the true mortality risk θ in a hospital H which is about to try a new operation. On average in the country around 10% of people die, but mortality rates differ in different hospitals vary from 3% to around 20%. Hospital H has no deaths in their first 10 operations. What should be the belief about θ ?
- b) Suppose X_1, X_2, \dots, X_n are iid Poisson(λ) random variables, and that λ has an exponential distribution with mean 1, so that $\pi(\lambda) = e^{-\lambda}$, $\lambda > 0$. Find $\hat{\theta}$ under quadratic loss and absolute error loss.

(10 + 10 Marks)