UNIVERSITY OF SWAZILAND

RE-SIT AND SUPPLEMENTARY EXAMINATION PAPER 2018

TITLE OF PAPER	: STATISTICAL INFERENCE II
COURSE CODE	: STA302 / ST303
TIME ALLOWED	: 2 HRS
REQUIREMENTS	: CALCULATOR
INSTRUCTIONS	: ANSWER ANY THREE QUESTIONS

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Question 1

Suppose $X_1, X_2, \ldots, X_n$ are i.i.d. random variables with probability density function

$$f(x|\sigma) = \frac{1}{2\sigma} exp\left(-\frac{|x|}{\sigma}\right), \quad \sigma > 0, -\infty < x < \infty$$

Use the method of moments and maximum likelihood estimation to find the estimator of σ.

(8 + 12 Marks)

Question 2

Suppose that $X_1, X_2, \ldots, X_n$ are i.i.d. random variables on the interval [0,1] with a density function

$$f(x|\alpha) = \frac{\Gamma(2\alpha)}{\Gamma(\alpha)^2}[x(1-\alpha)]^{\alpha-1}$$

where α > 0 is the parameter to be estimated from the sample. Find the sufficient statistic for α by verifying that this distribution belongs to the exponential family.

(20 Marks)

Question 3

$X_1, X_2, \ldots, X_n \sim N(\theta_1, \theta_2)$, where $\theta_2 > 0$ is the variance.

a) Argue that $S^2 = \frac{1}{n-1}\sum_{i=1}^{n}(X_i - \bar{X})^2$ is the MVUE of θ_2.

(10 Marks)

b) Show that the MVUE is not efficient, i.e. the variance of S^2 is greater than the Cramer-Rao lower bound.

(10 Marks)

Question 4

a) Let $X_{(n)}$ be the largest value in a sample of size n drawn from the uniform distribution on [0,θ]. Show that $X_{(n)}/\theta$ is a pivot. Using the pivot, find a 100(1-α)% confidence interval for θ. Discuss how you would test the hypothesis that θ takes a specific value θ0 for such a sample.

(10 Marks)

b) Suppose you have a sample of size n from an exponential distribution with mean μ. Find the best size-α test of $H_0: \mu = \mu_0$ against the alternative$H_1: \mu = \mu_1$, where $\mu_1 > \mu_0$.

(10 Marks)

Question 5

Suppose that $X_1, X_2, \ldots, X_n$ is a random sample from a Poisson distribution with unknown mean θ. Two models for the prior distribution of θ are contemplated;

$$\pi_1(\theta) = e^{-\theta},\ \theta > 0, \text{ and } \ \pi_2(\theta) = \theta e^{-\theta},\ \theta > 0$$

a) Calculate the Bayes estimator of θ under both models, with quadratic loss function.
(10 Marks)

b) The prior probabilities of model 1 and model 2 are assessed at probability ½ each. Calculate the Bayes factor for H_0: *model 1 applies* against H_1:*model 2 applies*.
(10 Marks)