UNIVERSITY OF SWAZILAND

FINAL EXAMINATION PAPER 2017/8

TITLE OF PAPER	:	TIME SERIES ANALYSIS
COURSE CODE	:	STA306
TIME ALLOWED	:	TWO (2) HOURS
REQUIREMENTS	:	CALCULATOR
INSTRUCTIONS	:	ANSWER ANY THREE QUESTIONS

Question 1 [20 marks, 6+7+2+5]

Lets $\{X_t\}_{t=1,2,\cdots}$ be a time series such that

$$X_t = \beta_0 + \beta_1 t + \beta_2 t^2 + w_t,$$

where β_0, β_1, β_2 are unknown constant parameters and w_t denotes a zero mean weakly stationary process.

(a) Describe briefly two methods of removing trend from such a time series: the differencing method and the method of least squares estimation of the unknown parameters. What is the main advantage and disadvantage of each of these two methods?

(b) Show that is a $\nabla^2 X_t$ weakly stationary process and give its autocovariance function in terms of the process w_t

(c) Give the definition of the convolution operation on the linear filters $\{a_j\}$ and $\{b_k\}$.

(d) Show that the operator ∇^2 is a convolution of two filters of the form $(-1, 1)$.

Question 2 [20 marks, 4+4+4+4+4]

Write down a general model for each of the following series. Use the backshift operator and explain your notation.

(a) ARMA$(1, 1)_{12}$

(b) MA$(2)_4$

(c) AR(1)

(d) ARIMA$(2, 0, 1) \times (0, 1, 0)_{12}$

(e) ARIMA$(2, 1, 0)$

Question 3 [20 marks, 6+6+3+5]

(a) Define the sample autocorrelation function (ACF) of a time series with n observations. Explain briefly how the sample partial autocorrelation function (PACF) can be calculated.

(b) Describe briefly the expected behaviour of the ACF and PACF for autoregressive (AR(p)), moving average (MA(q)) and autoregressive moving average (ARMA(p, q)) processes.

(c) Values of the sample ACF and of the sample PACF for lags $t = 1, \cdots, 5$ of an observed time series are given in the following tables. What kind of model is the time series most likely to follow? Explain your answer.

	ACF				
t	1	2	3	4	5
$\hat{\rho}(t)$	0.9	0.8	0.5	0.2	0.1

	PACF				
t	1	2	3	4	5
$\hat{\phi}_{tt}$	0.9	0.01	-0.02	0.03	-0.02

(d) Consider an AR(2) process of the form

$$X_t = 0.9X_{t-1} - 0.2X_{t-1} + w_t,$$

where $\{w_t\} \sim WN(0, \sigma^2)$. Show that there is a stationary solution to this process.

Question 4 [20 marks, 7+3+5+5]

(a) Consider the following three time series models where the error terms w_t are uncorrelated random errors with zero mean and constant variance.

(i)

$$X_t - 0.3X_{t-1} = w_t.$$

(ii)

$$X_t - 1.2X_{t-1} - 0.2X_{t-2} = w_t - 0.5w_{t-1}.$$

(iii)

$$X_t - X_{t-1} = w_t.$$

For each model, say whether it is stationary or not and specify p and q in the standard ARMA(p, q) framework.

(b) Show that the model

$$X_t - X_{t-1} = w_t.$$

can be written in the ARIMA(p, d, q) framework and specify the values of p, d and q.

(c) Consider the time series

$$X_t + \frac{1}{6}X_{t-1} - \frac{1}{3}X_{t-2} = w_t - \frac{3}{4}w_{t-1} + \frac{1}{8}w_{t-2},$$

where w_t is a white noise random variable.

(i) Determine the values of p and q so that there is no parameter redundancy in the model.

(ii) Check whether the process is causal and invertible.

Question 5 [20 marks, 5+5+4+6]

Assume you want to fit an ARIMA(p, d, q) model to represent some time series data $\{x_t\}_{t=1,2,\cdots,n}$ that you have been given.

(a) Give the definition of a general ARIMA(p, d, q) model.

(b) Explain how the orders p, d and q can be identified

(c) Assume $d = 2$, $p = 1$ and $q = 2$ have been fitted. Write down an explicit model for the resulting ARIMA(1,2,2) model using polynomials of the backward shift operator B.

(d) Having fitted an ARIMA(p, d, q) model to the data, what residual diagnostics should be looked at, and why?