UNIVERSITY OF SWAZILAND

RE-SIT EXAMINATION PAPER 2017/8

TITLE OF PAPER	:	TIME SERIES ANALYSIS
COURSE CODE	:	STA306
TIME ALLOWED	:	TWO (2) HOURS
REQUIREMENTS	:	CALCULATOR
INSTRUCTIONS	:	ANSWER ANY THREE QUESTIONS

Question 1 [20 marks, 2+5+6+2+5]

An MA(1) process with parameter θ is defined by the equation

$$X_t = w_t + \theta w_{t-1},$$

where $\{w_t\}$ is a white noise process, that is, a sequence of uncorrelated random variables with mean zero and constant variance σ^2.

(a) Define what it means for a moving average process to be **invertible**.

(b) Show that an MA(1) process is invertible if the parameter θ satisfies a condition which you should state.

(c) The autocovariance of X_t and X_{t+h} is defined to be $\rho(h)$. For the MA(1) process with parameter find $\rho(0)$ and $\rho(1)$ and write down $\rho(h)$ for $h \geq 2$. Hence calculate the autocorrelation function (ACF) for the MA(1) process.

(d) Show that an MA(1) process with parameter θ^{-1} has the same ACF as an MA(1) process with parameter θ.

(e) Consider two MA(1) processes with parameters $\theta = 0.25$, $\sigma^2 = 16$ and $\theta = 4$, $\sigma^2 = 1$ respectively. Show that they have the same autocovariance function (ACVF). Explain how you can choose between these processes by considering the invertibility of the processes.

Question 2 [20 marks, 4+5+2+2+7]

(a) Consider a time series model

$$X_t = m_t + w_t$$

where the trend m_t is a polynomial function of t with degree k and coefficients $\theta_0, \theta_1, \cdots, \theta_k$. The first difference is defined as

$$\nabla X_t = X_t - X_{t-1}.$$

(i) Show that if m_t is a polynomial function of t with degree 1, then the first difference gives

$$\nabla X_t = \beta_1 \nabla w_t.$$

(ii) Similarly, assume that m_t is a polynomial with degree 2. Find the second difference as a function of the coefficients.

(b) (i) For a time series model

$$X_t = m_t + w_t$$

define a **linear filter**.

(ii) What does it mean to say that a linear filter **passes through without distortion**?

(iii) A time series is to be smoothed by fitting a quadratic polynomial to successive groups of 5 observations, thus obtaining a weighted moving average filter. Find the filter which passes through without distortion, if least squares fitting is used.

Question 3 [20 marks, 8+6+6]

(a) Let the ARMA(1, 2) process for a time series $\{X_t\}$ be given by

$$X_t - 0.9X_{t-1} = w_t + 0.3w_{t-1} - 0.4w_{t-2},$$

where $\{w_t\} \sim WN(0, \sigma^2)$. Write down the operator form of this process. Show that it is invertible.

(b) Suppose that an ARIMA(p, d, q) model is to be fitted to some time series data $\{x_t\}_{t=1,\cdots,n}$. Describe what important features of the data can be revealed by a time series plot.

(c) Consider an AR(2) process of the form

$$X_t = 0.9X_{t-1} - 0.2X_{t-1} + w_t,$$

where $\{w_t\} \sim WN(0, \sigma^2)$. Show that there is a stationary solution to this process.

Question 4 [20 marks, 5+6+7+2]

A time series data set of size n = 100 was modelled as an AR(2) process

$$X_t = \phi_1 X_{t-1} + \phi_2 X_{t-2} + w_t, \qquad w_t \sim WN(0, \sigma^2).$$

(a) Give the Yule-Walker equations for estimation of the AR parameters and of the variance of the white noise. Briefly explain your notation.

(b) Find the estimates of the model parameters ϕ_1, ϕ_2 and σ^2 knowing that the estimates of the series variance and the autocorrelation at lags 1 and 2 are

$$\hat{\gamma}(0) = 1.6, \qquad \hat{\rho}(1) = 0.6, \qquad \hat{\rho}(2) = 0.4.$$

(c) Calculate 95% confidence intervals for ϕ_1 and for ϕ_2.

Note that for a standard normal random variable U and for u_α such that $\mathbb{P}(|U| > u_\alpha) = \alpha$ we have $u_\alpha = 1.96$ for $\alpha = 0.05$.

(d) Predict value x_{101} of the series knowing that $x_{100} = 2.4$ and $x_{99} = 1.6$.

Question 5 [20 marks, 2+2+2+2+2+3+4+3]

(a) Define what it means for a time series to be stationary

(b) What does it mean for a time series to be causal.

(c) Define, precisely, what it means for a time series Yt to be an AR(1) series.

(d) What condition will ensure that an AR(1) series will be stationary and causal.

(e) Write down the operator form of the general ARIMA(p,d,q) series.

(f) The yearly real GDP (in Billions of Emalangeni) for Swaziland was read into an R data set called `rGDP`:

(i) The following statements are issued to R.

```
> rGDP <- ts(rGDP,start=1960))
> acf(diff(rGDP))
```

Describe in detail exactly what these statements do.

(ii) In order to investigate possible non-stationarity of the series, the following two plots are produced from the series. What kind of effects do the plots show and how would these effects influence the analysis of the series?

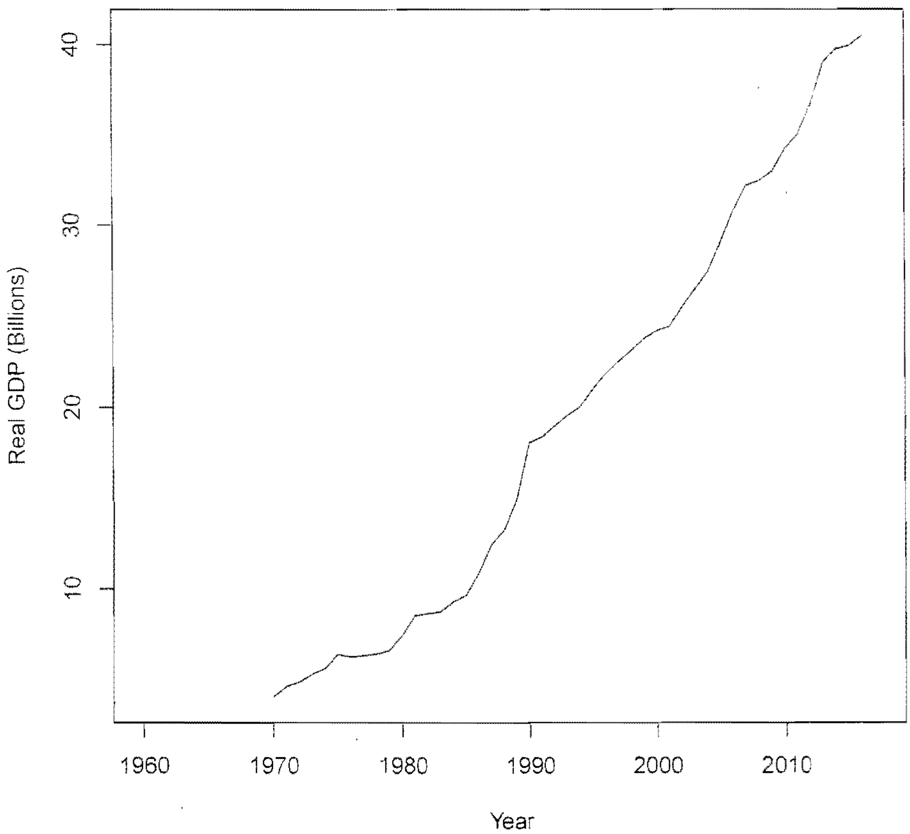

Figure 1: A plot of the original series

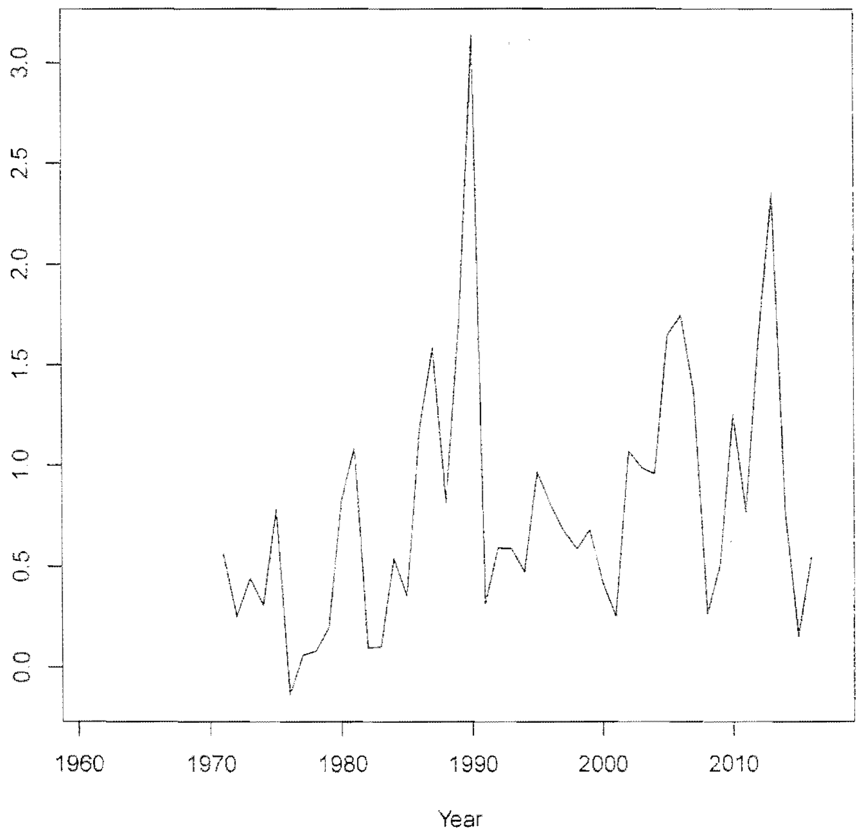

Figure 2: A plot of the differenced series

(iii) The following graphs are produced by running the following R statements.

```
> acf(diff(rGDP))
> pacf(diff(rGDP))
```

Explain what kind of modelling structure you think that the graphs indicate as appropriate for modelling the series.

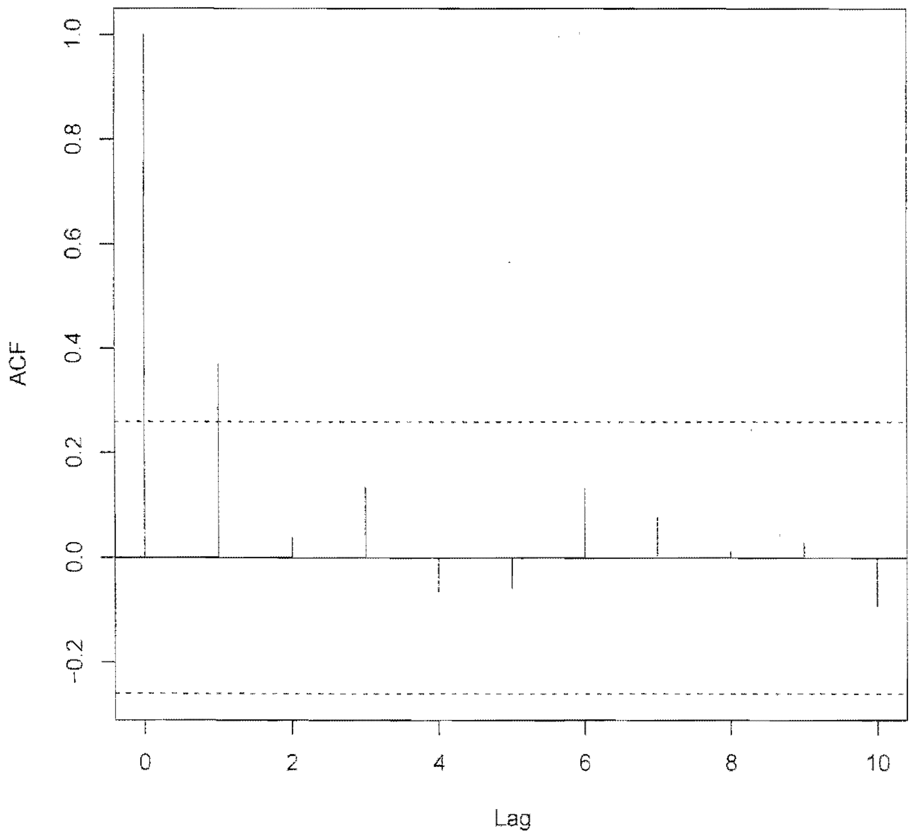

Figure 3: The first plot produced by the R statements

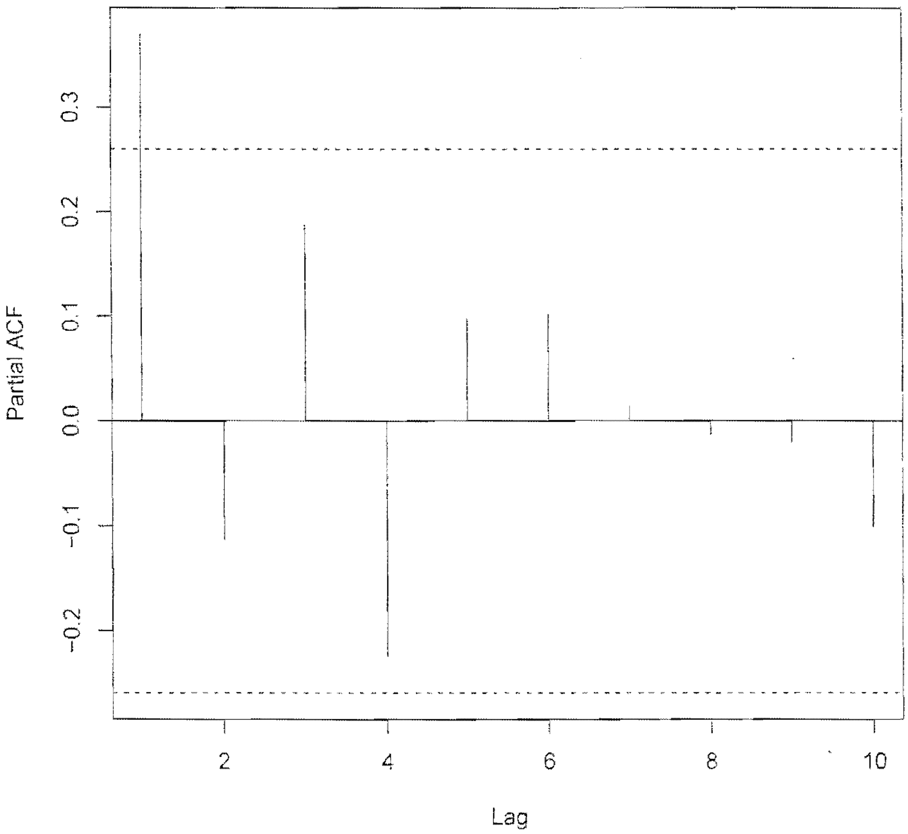

Figure 4: The second plot produced by the R statements

(iv) Using operator notation, write down the **complete** model which should be fitted to the GDP data series.