DEPARTMENT OF STATISTICS AND DEMOGRAPHY

SUPPLEMENTARY EXAMINATION, 2018/19

COURSE TITLE: INTRODUCTION TO STATISTICS

COURSE CODE: STA 141

TIME ALLOWED: TWO (2) HOURS

INSTRUCTION: ANSWER <u>ALL QUESTIONS IN SECTION A</u> AND <u>ANY TWO</u> QUESTIONS IN <u>SECTION B</u>

SPECIAL REQUIREMENTS: SCIENTIFIC CALCULATORS, GRAPH PAPER AND STATISTICAL TABLES

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SECTION A

Question 1

(a) Determine the following probabilities using the standard normal distribution:
(i) $P(z < -0.6)$
(ii) $P(0.17 \leq z \leq 2.49)$
(iii) $P(-1.57 \leq z \leq 0.93)$
(iv) $P(z > -1.85)$ **(2+3+3+2 marks)**

Question 2

9. In a certain region of the country it is known from past experience that the probability of selecting an adult over 40 years of age with cancer is 0.05. If the probability of a doctor correctly diagnosing a person with cancer as having the disease is 0.78 and the probability of incorrectly diagnosing a person without cancer as having cancer is 0.06.
(a) What is the probability that a person is diagnosed as having cancer?
(b) What is the probability that a person diagnosed as having cancer actually has the disease?
                                                         (10 marks)

Question 3

A shop has 11 video games to choose, 4 of them contain extreme violence. A customer picks 3 of these games at random. What is the probability that the number of extremely violent games among the three selected games is:
  (a) Exactly two
  (b) More than one
  (c) None   **(10 marks)**
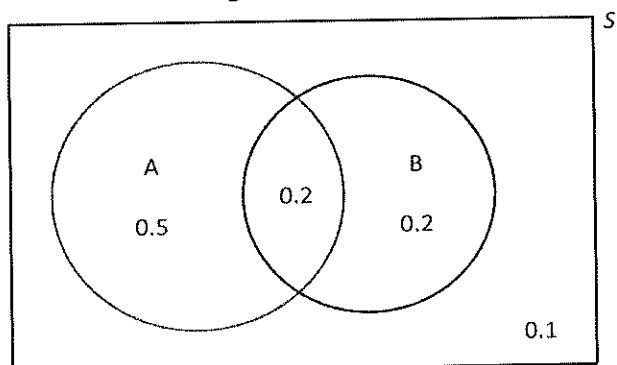
Question 4

On average, two new accounts are opened at a Savings Bank based in Manzini. Find the probability that on a given day, the number of new accounts opened at this bank will be: **(10 marks)**
  (a) Exactly six
  (b) At most three
  (c) At least 7

Question 5

Given the Venn diagram below, find the following probabilities:

i. $P(B|A)$
ii. $P(A|B)$
iii. $P(A \cap B)$ '
iv. $P(A \cup B)$ **(3+3+2+2 marks)**

Question 6

The scores on a national achievement test were approximately normally distributed, with a mean of 540 and a standard deviation of 110.

    i. If you achieved a score of 680, how far, in standard deviations, did your score depart from the mean?

    ii. What percentage of those who took the examination scored higher than you?   **(5+5 marks)**

SECTION B

Question 7

a. All radio manufactured by a certain factory are tested by one of three inspectors, Benade, Botha and Brink. Due to the extent of the work the testing cannot be done on a one hundred per cent basis. However, experience has shown that only 2%, 1% and 4% of the radios approved by Benade, Botha and Brink respectively have some or other defect. The performance of the three inspectors also differs so much that 40%, 30% and 25% of the total production is tested by the three inspectors respectively. If a newly sold radio is sent back to the factory because of a defect, what are the respective probabilities of is having been tested by these three inspectors?   **(10 marks)**

b. An insurance company approaches an enterprise with many employees with a view to short-term group insurance. Experience has shown the insurance company that enterprises such as this one can be divides into one of three categories, namely:

    B_1: low-risk group

    B_2: medium-risk group

    B_3: high-risk group

The insurance company bases its classification of enterprises on the probability that the total claim of an enterprise would be more than an amount K.

    Let D = enterprise's total claims more than an amount K

Furthermore, reports by insurance companies show that the distribution of this type of enterprise across the three categories (expressed as probabilities) is as follows:

    $P(B_1) = 0.25$

    $P(B_2) = 0.60$

    $P(B_3) = 0.15$

The probabilities of a specific group submitting a total claim of more than K, are given respectively by

    $P(D|B_1) = 0.01$

    $P(D|B_2) = 0.08$

    $P(D|B_3) = 0.60$

The enterprise concerned claims to be a member of the low-risk group and the insurance company and the board of directors of the enterprise agree that the board will pay the premium of the B_1 group (which is less than the premium of the other two groups) during the first year of their commitment. At the end of the year the enterprise will be reclassified if necessary.

At the end of the first year the total claim for the enterprise indeed turns out to have exceeded K. Determine the probabilities of the enterprise being member of the various groups. **(10 marks)**

Question 8

The length of bus routes from any particular transit system will typically vary from one route to another. The study by a City council gives the following information on lengths in (km) for one particular system:

Length	Frequency
5 - < 10	6
10- < 15	23
15 - < 20	30
20 - < 25	35
25 - < 30	32
30 - < 35	48
35 - < 40	42
40 - < 45	40
45 - < 50	28

Find the:
  a. Mean
  b. Median
  c. Mode
  d. Variance and standard deviation   **(3+5+5+7 marks)**

Question 9

(a)   Over a long period of time it has been observed that a given marksman can hit a target on a single trial with probability equal to .8. Suppose he fires four shots at the target.

   (i)   What is the probability that he will hit the target exactly two times?
   (ii)   What is the probability that he will hit the target at least once?   **(2+3 marks)**

(b)   Use the cumulative binominal table for $n = 5$ and $p = .6$ to find the probabilities of these events:
   (i) Exactly three successes
   (ii)Three or more successes   **(5 marks)**

(c)   A case of wine has 12 bottles, 3 of which contain spoiled wine. A sample of 4 bottles is randomly selected from the case.
   (i) Find the probability distribution for x, the number of bottles of spoiled wine in the sample
                                                            (10 marks)

Question 10

(a)   Let x be a normally distributed random variable with a mean of 10 and a standard deviation of 2. Find the probability that x lies between 11 and 13.6.   **(4 marks)**

(b)   Studies show that gasoline use for compact cars sold in the United States is normally distributed, with a mean of 25.5 miles per gallon (mpg) and a standard deviation of 4.5 mpg. What percentage of compacts gets 30 mpg or more?   **(4 marks)**

(c)   The scores on a national achievement test were approximately normally distributed, with a mean of 540 and a standard deviation of 110.

(i) If you achieve a score of 680, how far, in standard deviations, did your score depart from the mean?
(ii) What percentage of those who took the examination scored higher than you?   **(3+3 marks)**

(d) The probability that a patient recovers from a rare disease is 0.4. If 100 people are known to have contracted this disease, what is the probability that less than 30 survive? Use the Normal approximation to the Binomial to answer this question.   **(6 marks)**

Question 11

In an investigation into the effect of the landing speed on the number of landings made with the same set of main landing tyres on the Boeing 737, the following data is obtained:

Observation	Number of landings	Landing speed
1	84	112
2	85	114
3	84	116
4	80	118
5	81	120
6	76	122
7	78	124
8	71	126
9	72	128
10	68	130
11	69	132
12	64	134

(a) Which is a dependent variable and which is the independent variable?
(b) Find the least squares regression line by choosing appropriate dependent and independent variables based on your answer in part (a).
(c) Interpret the meaning of the values of 'a' and 'b' calculated in (b).
(d) Plot the scatter diagram and the regression line.   **(2+10+5+3 marks)**

END OF EXAM!!

Table A.3 Standard Normal Curve Areas $\Phi(z) = P(Z \leq z)$

Standard normal density function

Shaded area = $\Phi(z)$
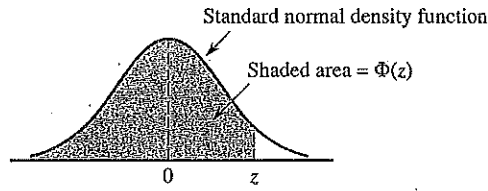
z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
−3.4	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0002
−3.3	.0005	.0005	.0005	.0004	.0004	.0004	.0004	.0004	.0004	.0003
−3.2	.0007	.0007	.0006	.0006	.0006	.0006	.0006	.0005	.0005	.0005
−3.1	.0010	.0009	.0009	.0009	.0008	.0008	.0008	.0008	.0007	.0007
−3.0	.0013	.0013	.0013	.0012	.0012	.0011	.0011	.0011	.0010	.0010
−2.9	.0019	.0018	.0017	.0017	.0016	.0016	.0015	.0015	.0014	.0014
−2.8	.0026	.0025	.0024	.0023	.0023	.0022	.0021	.0021	.0020	.0019
−2.7	.0035	.0034	.0033	.0032	.0031	.0030	.0029	.0028	.0027	.0026
−2.6	.0047	.0045	.0044	.0043	.0041	.0040	.0039	.0038	.0037	.0036
−2.5	.0062	.0060	.0059	.0057	.0055	.0054	.0052	.0051	.0049	.0038
−2.4	.0082	.0080	.0078	.0075	.0073	.0071	.0069	.0068	.0066	.0064
−2.3	.0107	.0104	.0102	.0099	.0096	.0094	.0091	.0089	.0087	.0084
−2.2	.0139	.0136	.0132	.0129	.0125	.0122	.0119	.0116	.0113	.0110
−2.1	.0179	.0174	.0170	.0166	.0162	.0158	.0154	.0150	.0146	.0143
−2.0	.0228	.0222	.0217	.0212	.0207	.0202	.0197	.0192	.0188	.0183
−1.9	.0287	.0281	.0274	.0268	.0262	.0256	.0250	.0244	.0239	.0233
−1.8	.0359	.0352	.0344	.0336	.0329	.0322	.0314	.0307	.0301	.0294
−1.7	.0446	.0436	.0427	.0418	.0409	.0401	.0392	.0384	.0375	.0367
−1.6	.0548	.0537	.0526	.0516	.0505	.0495	.0485	.0475	.0465	.0455
−1.5	.0668	.0655	.0643	.0630	.0618	.0606	.0594	.0582	.0571	.0559
−1.4	.0808	.0793	.0778	.0764	.0749	.0735	.0722	.0708	.0694	.0681
−1.3	.0968	.0951	.0934	.0918	.0901	.0885	.0869	.0853	.0838	.0823
−1.2	.1151	.1131	.1112	.1093	.1075	.1056	.1038	.1020	.1003	.0985
−1.1	.1357	.1335	.1314	.1292	.1271	.1251	.1230	.1210	.1190	.1170
−1.0	.1587	.1562	.1539	.1515	.1492	.1469	.1446	.1423	.1401	.1379
−0.9	.1841	.1814	.1788	.1762	.1736	.1711	.1685	.1660	.1635	.1611
−0.8	.2119	.2090	.2061	.2033	.2005	.1977	.1949	.1922	.1894	.1867
−0.7	.2420	.2389	.2358	.2327	.2296	.2266	.2236	.2206	.2177	.2148
−0.6	.2743	.2709	.2676	.2643	.2611	.2578	.2546	.2514	.2483	.2451
−0.5	.3085	.3050	.3015	.2981	.2946	.2912	.2877	.2843	.2810	.2776
−0.4	.3446	.3409	.3372	.3336	.3300	.3264	.3228	.3192	.3156	.3121
−0.3	.3821	.3783	.3745	.3707	.3669	.3632	.3594	.3557	.3520	.3482
−0.2	.4207	.4168	.4129	.4090	.4052	.4013	.3974	.3936	.3897	.3859
−0.1	.4602	.4562	.4522	.4483	.4443	.4404	.4364	.4325	.4286	.4247
−0.0	.5000	.4960	.4920	.4880	.4840	.4801	.4761	.4721	.4681	.4641

(continued)

Table A.3 Standard Normal Curve Areas *(cont.)* $\Phi(z) = P(Z \le z)$

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9278	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998