

UNIVERSITY OF ESWATINI



RESIT EXAMINATION PAPER 2018

- TITLE OF PAPER :** **PROBABILITY THEORY I**
- COURSE CODE :** **STA211**
- TIME ALLOWED :** **TWO (2) HOURS**
- INSTRUCTIONS :** **ANSWER ANY THREE QUESTIONS.**
- REQUIREMENTS :** **SCIENTIFIC CALCULATOR AND
STATISTICAL TABLES.**

Question 1

- a) Let A and B be events in a probability space. The probability of event A is $P(A) = 1/3$ and the conditional probability of B given A^c is $P(B|A^c) = 1/4$. Find the probability that A or B occurs.
(5 Marks)
- b) Let E and F be two events for which the probability that at least one of them occurs is $3/4$. Find the probability that neither E nor F occurs.
(5 Marks)
- c) One tosses a fair coin twice. The two events of interest are: $A = \{\text{first toss is a head}\}$ and $B = \{\text{second toss is a head}\}$. Are A and B independent? And are they disjoint?
(5 Marks)
- d) In a certain country it is established that 0.5% of the population suffers from a certain disease. For this disease there exists a test that gives the correct diagnosis for 80% of healthy persons and for 98% of sick persons. A person is tested and found sick. Find the probability that the diagnosis is wrong, i.e. that the person is actually healthy.
(5 Marks)

Question 2

- a) On January 28, 1986 the space shuttle Challenger exploded about one minute after the launch. The cause of the disaster was explosion of the main fuel tank, caused by flames of hot gas erupting from one of the solid rocket boosters. These rocket boosters are manufactured in segments, joint together with O-rings. Each rocket booster has three O-rings and per launch two rocket boosters are used, so in total six O-rings each time. Based on data on the number of failed O-rings, available from previous launches, it was found that the probability p that an individual O-ring fails depends on the launch temperature t (in degrees Fahrenheit) according to

$$p = \frac{\exp(a + bt)}{1 + \exp(a + bt)}$$

with $a = 5.085$ and $b = -0.1156$. Hence, p increases with decreasing launch temperature. At the time of the fatal launch of the Challenger, t was extremely low: 31 degrees Fahrenheit. Although the above formula is based on data for which $t > 50$ degrees Fahrenheit, let us use this formula also for $t = 31$ degrees Fahrenheit.

Find the probability of at least one O-ring failing during the 1986 Challenger launch.

(8 Marks)

- b) A machine fastens plastic screw-on caps onto containers of motor oil. If the machine applies more torque than the cap can withstand, the cap will break. Both the applied torque and the strength of the caps vary. The capping machine torque is a normally distributed random

variable with mean 0.79Nm and standard deviation 0.10Nm. The cap strength, being the torque that would break the cap, is also a normally distributed random variable with mean 1.13Nm and standard deviation 0.14Nm. Assume that the cap strength and the applied torque are independent.

What is the probability that a cap will break while being fastened by the capping machine?

(5 Marks)

- c) Let X be an exponentially distributed random variable with parameter $\lambda = 1/5$. Find the conditional probability $P(x < 5 | 3 < x < 6)$.

(7 Marks)

Question 3

- a) The random variable X is uniformly distributed on the interval $(0, 1)$. Derive the PDF of the random variable $Y = -\ln X$.
- b) Consider two independent random variables X_1 and X_2 , distributed exponentially with $\lambda = 1$. That is,

$$f_X(x) = \begin{cases} e^{-x}, & x \geq 0 \\ 0, & \text{otherwise} \end{cases}$$

Calculate the PDF of $X_1 + X_2$.

(20 Marks)

Question 4

- a) The random variable W is uniformly distributed on the interval $(\pi, 2\pi)$. What can you say about $E[\sin(W)]$ and $\sin[E(W)]$? Which one is bigger? And if they are equal, are they also equal to zero?
- b) A pulse of light has energy X that is a random variable with parameter λ , i.e., its PDF is

(8 Marks)

$$f_X(x) = \lambda^2 x e^{-\lambda x}, \text{ for } x \geq 0.$$

This pulse illuminates an ideal photon-counting detector whose output N is a Poisson-distributed random variable with mean x when $X = x$, i.e., its conditional PMF is

$$p_{N|X}(n|x) = \frac{\lambda^n e^{-\lambda}}{n!}, \text{ for } n = 0, 1, 2, \dots$$

Find $E[N]$ and $\text{Var}[N]$

(12 Marks)

Question 5

(a) Suppose that the discrete random variable X has the probability function

$$P(X = x) = (1 - \theta)^{x-1}\theta, \quad x = 1, 2, \dots$$

Show that X has moment generating function

$$M_X(t) = \frac{e^t\theta}{1 - e^t(1 - \theta)}, \quad t < -\ln(1 - \theta)$$

Hence show that the expected value of X is $1/\theta$ and that the variance of X is $(1 - \theta)\theta^{-2}$

(10 Marks)

(b) The random variable X has a Chi-squared distribution with k degrees of freedom ($k = 1, 2, 3, \dots$), which has moment generating function (mgf) $m(t) = (1 - 2t)^{-k/2}$ for $t < 1/2$.

i. Using the $m(t)$, find the mean and variance of X .

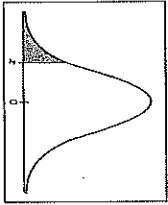
(5 Marks)

ii. In the case $k = 4$, the probability density function is given by

$$f(x) = \frac{1}{4}xe^{-1/2}, \quad x > 0$$

Using integration, confirm that the mgf of the Chi-squared distribution with 4 degrees of freedom is $m(t) = (1 - 2t)^{-2}$ for $t < 1/2$.

(5 Marks)



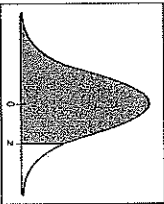
The table entry for z is the area to the left of z.

TABLE 5 Areas of a Standard Normal Distribution

(a) Table of Areas to the Left of z

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
-3.4	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0002
-3.3	.0005	.0005	.0005	.0004	.0004	.0004	.0004	.0004	.0004	.0003
-3.2	.0007	.0007	.0006	.0006	.0006	.0006	.0006	.0005	.0005	.0005
-3.1	.0010	.0009	.0009	.0008	.0008	.0008	.0008	.0007	.0007	.0007
-3.0	.0013	.0013	.0013	.0012	.0012	.0011	.0011	.0011	.0010	.0010
-2.9	.0015	.0015	.0015	.0014	.0014	.0013	.0013	.0012	.0012	.0011
-2.8	.0018	.0018	.0018	.0017	.0016	.0015	.0015	.0014	.0014	.0013
-2.7	.0020	.0020	.0020	.0019	.0018	.0017	.0017	.0016	.0015	.0015
-2.6	.0023	.0023	.0023	.0022	.0021	.0020	.0020	.0019	.0018	.0018
-2.5	.0025	.0025	.0025	.0024	.0023	.0022	.0022	.0021	.0020	.0020
-2.4	.0027	.0027	.0027	.0026	.0025	.0024	.0024	.0023	.0022	.0022
-2.3	.0029	.0029	.0029	.0028	.0027	.0026	.0026	.0025	.0024	.0024
-2.2	.0031	.0031	.0031	.0030	.0029	.0028	.0028	.0027	.0026	.0026
-2.1	.0033	.0033	.0033	.0032	.0031	.0030	.0030	.0029	.0028	.0028
-2.0	.0035	.0035	.0035	.0034	.0033	.0032	.0032	.0031	.0030	.0030
-1.9	.0037	.0037	.0037	.0036	.0035	.0034	.0034	.0033	.0032	.0032
-1.8	.0039	.0039	.0039	.0038	.0037	.0036	.0036	.0035	.0034	.0034
-1.7	.0041	.0041	.0041	.0040	.0039	.0038	.0038	.0037	.0036	.0036
-1.6	.0043	.0043	.0043	.0042	.0041	.0040	.0040	.0039	.0038	.0038
-1.5	.0045	.0045	.0045	.0044	.0043	.0042	.0042	.0041	.0040	.0040
-1.4	.0047	.0047	.0047	.0046	.0045	.0044	.0044	.0043	.0042	.0042
-1.3	.0049	.0049	.0049	.0048	.0047	.0046	.0046	.0045	.0044	.0044
-1.2	.0051	.0051	.0051	.0050	.0049	.0048	.0048	.0047	.0046	.0046
-1.1	.0053	.0053	.0053	.0052	.0051	.0050	.0050	.0049	.0048	.0048
-1.0	.0054	.0054	.0054	.0053	.0052	.0051	.0051	.0050	.0049	.0049
-0.9	.0055	.0055	.0055	.0054	.0053	.0052	.0052	.0051	.0050	.0050
-0.8	.0056	.0056	.0056	.0055	.0054	.0053	.0053	.0052	.0051	.0051
-0.7	.0057	.0057	.0057	.0056	.0055	.0054	.0054	.0053	.0052	.0052
-0.6	.0058	.0058	.0058	.0057	.0056	.0055	.0055	.0054	.0053	.0053
-0.5	.0059	.0059	.0059	.0058	.0057	.0056	.0056	.0055	.0054	.0054
-0.4	.0060	.0060	.0060	.0059	.0058	.0057	.0057	.0056	.0055	.0055
-0.3	.0061	.0061	.0061	.0060	.0059	.0058	.0058	.0057	.0056	.0056
-0.2	.0062	.0062	.0062	.0061	.0060	.0059	.0059	.0058	.0057	.0057
-0.1	.0063	.0063	.0063	.0062	.0061	.0060	.0060	.0059	.0058	.0058
0.0	.0064	.0064	.0064	.0063	.0062	.0061	.0061	.0060	.0059	.0059
0.1	.0065	.0065	.0065	.0064	.0063	.0062	.0062	.0061	.0060	.0060
0.2	.0066	.0066	.0066	.0065	.0064	.0063	.0063	.0062	.0061	.0061
0.3	.0067	.0067	.0067	.0066	.0065	.0064	.0064	.0063	.0062	.0062
0.4	.0068	.0068	.0068	.0067	.0066	.0065	.0065	.0064	.0063	.0063
0.5	.0069	.0069	.0069	.0068	.0067	.0066	.0066	.0065	.0064	.0064
0.6	.0070	.0070	.0070	.0069	.0068	.0067	.0067	.0066	.0065	.0065
0.7	.0071	.0071	.0071	.0070	.0069	.0068	.0068	.0067	.0066	.0066
0.8	.0072	.0072	.0072	.0071	.0070	.0069	.0069	.0068	.0067	.0067
0.9	.0073	.0073	.0073	.0072	.0071	.0070	.0070	.0069	.0068	.0068
1.0	.0074	.0074	.0074	.0073	.0072	.0071	.0071	.0070	.0069	.0069
1.1	.0075	.0075	.0075	.0074	.0073	.0072	.0072	.0071	.0070	.0070
1.2	.0076	.0076	.0076	.0075	.0074	.0073	.0073	.0072	.0071	.0071
1.3	.0077	.0077	.0077	.0076	.0075	.0074	.0074	.0073	.0072	.0072
1.4	.0078	.0078	.0078	.0077	.0076	.0075	.0075	.0074	.0073	.0073
1.5	.0079	.0079	.0079	.0078	.0077	.0076	.0076	.0075	.0074	.0074
1.6	.0080	.0080	.0080	.0079	.0078	.0077	.0077	.0076	.0075	.0075
1.7	.0081	.0081	.0081	.0080	.0079	.0078	.0078	.0077	.0076	.0076
1.8	.0082	.0082	.0082	.0081	.0080	.0079	.0079	.0078	.0077	.0077
1.9	.0083	.0083	.0083	.0082	.0081	.0080	.0080	.0079	.0078	.0078
2.0	.0084	.0084	.0084	.0083	.0082	.0081	.0081	.0080	.0079	.0079
2.1	.0085	.0085	.0085	.0084	.0083	.0082	.0082	.0081	.0080	.0080
2.2	.0086	.0086	.0086	.0085	.0084	.0083	.0083	.0082	.0081	.0081
2.3	.0087	.0087	.0087	.0086	.0085	.0084	.0084	.0083	.0082	.0082
2.4	.0088	.0088	.0088	.0087	.0086	.0085	.0085	.0084	.0083	.0083
2.5	.0089	.0089	.0089	.0088	.0087	.0086	.0086	.0085	.0084	.0084
2.6	.0090	.0090	.0090	.0089	.0088	.0087	.0087	.0086	.0085	.0085
2.7	.0091	.0091	.0091	.0090	.0089	.0088	.0088	.0087	.0086	.0086
2.8	.0092	.0092	.0092	.0091	.0090	.0089	.0089	.0088	.0087	.0087
2.9	.0093	.0093	.0093	.0092	.0091	.0090	.0090	.0089	.0088	.0088
3.0	.0094	.0094	.0094	.0093	.0092	.0091	.0091	.0090	.0089	.0089
3.1	.0095	.0095	.0095	.0094	.0093	.0092	.0092	.0091	.0090	.0090
3.2	.0096	.0096	.0096	.0095	.0094	.0093	.0093	.0092	.0091	.0091
3.3	.0097	.0097	.0097	.0096	.0095	.0094	.0094	.0093	.0092	.0092
3.4	.0098	.0098	.0098	.0097	.0096	.0095	.0095	.0094	.0093	.0093

For values of z less than -3.09, use 0.0044 to approximate the area.



The table entry for z is the area to the left of z.

TABLE 5(a) continued

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
0.8	.7881	.7910	.7938	.7965	.7992	.8019	.8046	.8072	.8098	.8123
0.9	.8149	.8175	.8201	.8227	.8252	.8277	.8301	.8325	.8349	.8373
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9932	.9934	.9936	.9938
2.5	.9939	.9940	.9941	.9943	.9945	.9946	.9948	.9951	.9952	.9954
2.6	.9955	.9956	.9957	.9958	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9980	.9981	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990
3.1	.9990	.9991	.9991	.9992	.9992	.9992	.9992	.9992	.9993	.9993
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9997	.9997
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998

For z values greater than 3.49, use 1.0000 to approximate the area.

TABLE 5 continued

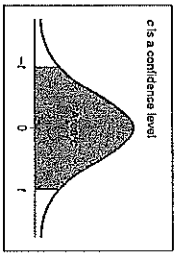
(b) Confidence Interval

Level of Confidence C	Critical Value z _C
0.70, or 70%	1.04
0.75, or 75%	1.15
0.80, or 80%	1.28
0.85, or 85%	1.44
0.90, or 90%	1.645
0.95, or 95%	1.96
0.98, or 98%	2.33
0.99, or 99%	2.58

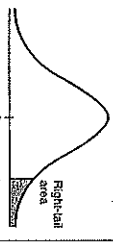
TABLE 5 continued

(c) Hypothesis Testing: Critical Values z_α

Level of Significance	α = 0.05	α = 0.01
Critical value z _α for a left-tailed test	-1.645	-2.33
Critical value z _α for a right-tailed test	1.645	2.33
Critical values z _{α/2} for a two-tailed test	±1.96	±2.58



One-tail area



Two-tail area

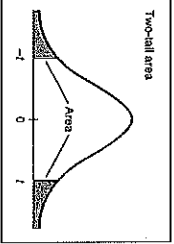


TABLE 6 Critical Values for Student's *t* Distribution

Table with columns for degrees of freedom (df) and confidence levels (c). Rows represent one-tail and two-tail areas.

For degrees of freedom not in the table, use the closest df that is smaller.

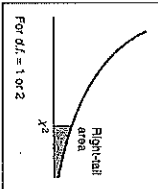
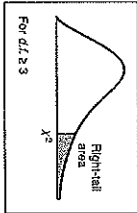


TABLE 7 The χ^2 Distribution

Table with columns for degrees of freedom (df) and right-tail areas. Rows represent right-tail areas from 0.995 to 0.005.

Source: From H. L. Nevitt, *Statistics*, June 1964. Reprinted by permission of the Biometrics Trustees.

TABLE 8 continued

Right-tail area	Degrees of freedom numerator, $d.f_n$								
	1	2	3	4	5	6	7	8	9
0.100	3.38	3.01	2.81	2.69	2.61	2.55	2.51	2.47	2.44
0.050	5.12	4.28	3.86	3.63	3.48	3.37	3.29	3.23	3.18
0.025	7.21	5.71	5.08	4.72	4.48	4.32	4.20	4.10	4.03
0.010	10.56	8.02	6.99	6.42	6.06	5.80	5.61	5.47	5.35
0.001	22.98	16.39	13.90	12.58	11.71	11.13	10.70	10.37	10.11
0.100	3.29	2.92	2.73	2.61	2.52	2.46	2.41	2.38	2.35
0.050	4.96	4.10	3.71	3.46	3.33	3.22	3.14	3.07	3.02
0.025	6.94	5.46	4.83	4.47	4.24	4.07	3.95	3.85	3.78
0.010	10.04	7.56	6.55	5.99	5.64	5.39	5.20	5.06	4.94
0.001	21.04	14.91	12.55	11.28	10.48	9.93	9.52	9.20	8.95
0.100	3.43	2.88	2.66	2.54	2.45	2.39	2.34	2.30	2.27
0.050	4.84	3.98	3.59	3.35	3.20	3.09	3.01	2.95	2.90
0.025	6.72	5.28	4.69	4.28	4.04	3.88	3.76	3.68	3.63
0.010	9.95	7.21	6.22	5.67	5.32	5.07	4.89	4.74	4.63
0.001	19.69	13.81	11.55	10.35	9.58	9.05	8.66	8.35	8.12
0.100	3.18	2.81	2.61	2.48	2.39	2.33	2.28	2.24	2.21
0.050	4.75	3.88	3.49	3.26	3.11	3.00	2.91	2.85	2.80
0.025	6.55	5.10	4.47	4.12	3.89	3.73	3.61	3.51	3.44
0.010	9.53	6.93	5.95	5.41	5.06	4.82	4.64	4.50	4.39
0.001	18.64	12.97	10.80	9.63	8.89	8.38	8.00	7.71	7.48
0.100	3.14	2.76	2.56	2.43	2.35	2.29	2.23	2.20	2.16
0.050	4.67	3.81	3.41	3.18	3.03	2.92	2.83	2.77	2.71
0.025	6.41	4.97	4.35	4.00	3.77	3.60	3.48	3.39	3.31
0.010	9.07	6.70	5.74	5.21	4.86	4.62	4.44	4.30	4.19
0.001	17.82	12.31	10.21	9.07	8.35	7.85	7.49	7.21	6.98
0.100	3.10	2.73	2.53	2.39	2.31	2.24	2.19	2.15	2.12
0.050	4.60	3.74	3.34	3.11	2.96	2.85	2.76	2.70	2.65
0.025	6.30	4.86	4.24	3.89	3.66	3.50	3.38	3.29	3.21
0.010	8.96	6.51	5.56	5.04	4.69	4.45	4.28	4.14	4.03
0.001	17.14	11.78	9.73	8.62	7.92	7.44	7.08	6.80	6.58
0.100	3.07	2.70	2.49	2.36	2.27	2.21	2.16	2.12	2.09
0.050	4.54	3.68	3.29	3.06	2.90	2.79	2.71	2.64	2.59
0.025	6.20	4.77	4.15	3.80	3.58	3.41	3.29	3.20	3.12
0.010	8.68	6.36	5.42	4.89	4.56	4.32	4.14	4.00	3.89
0.001	16.59	11.34	9.34	8.25	7.57	7.09	6.74	6.47	6.26
0.100	3.05	2.67	2.46	2.33	2.24	2.18	2.13	2.09	2.06
0.050	4.49	3.63	3.24	3.01	2.85	2.74	2.66	2.59	2.54
0.025	6.12	4.69	4.08	3.73	3.50	3.34	3.22	3.12	3.05
0.010	8.53	6.23	5.29	4.77	4.44	4.20	4.03	3.89	3.78
0.001	16.12	10.97	9.01	7.94	7.27	6.80	6.46	6.19	5.98

FOIA

TABLE 8 continued

Right-tail area	Degrees of freedom numerator, $d.f_n$									
	10	12	15	20	25	30	40	50	60	100
0.100	2.42	2.38	2.34	2.30	2.27	2.25	2.23	2.22	2.21	2.18
0.050	3.14	3.07	3.01	2.94	2.89	2.86	2.83	2.80	2.79	2.75
0.025	3.96	3.87	3.77	3.67	3.60	3.56	3.51	3.47	3.45	3.39
0.010	5.26	5.11	4.96	4.81	4.71	4.65	4.57	4.52	4.48	4.40
0.001	9.89	9.57	9.24	8.90	8.69	8.55	8.37	8.25	8.19	8.00
0.100	2.32	2.28	2.24	2.20	2.17	2.15	2.13	2.12	2.11	2.08
0.050	2.98	2.91	2.85	2.77	2.73	2.70	2.66	2.64	2.62	2.58
0.025	3.72	3.62	3.52	3.42	3.35	3.31	3.26	3.22	3.20	3.16
0.010	4.95	4.71	4.56	4.41	4.31	4.25	4.17	4.12	4.08	4.00
0.001	8.75	8.45	8.15	7.80	7.60	7.47	7.30	7.19	7.12	6.94
0.100	2.25	2.21	2.17	2.12	2.10	2.08	2.05	2.04	2.03	2.00
0.050	2.95	2.79	2.72	2.65	2.60	2.57	2.53	2.51	2.49	2.45
0.025	3.59	3.43	3.33	3.23	3.16	3.12	3.06	3.03	3.00	2.94
0.010	4.54	4.40	4.25	4.10	4.01	3.94	3.86	3.81	3.78	3.69
0.001	7.97	7.63	7.32	7.01	6.81	6.68	6.52	6.42	6.35	6.16
0.100	2.15	2.15	2.10	2.05	2.03	2.01	1.99	1.97	1.96	1.93
0.050	2.75	2.69	2.62	2.54	2.50	2.47	2.43	2.40	2.38	2.34
0.025	3.37	3.28	3.18	3.07	3.01	2.96	2.91	2.87	2.85	2.79
0.010	4.30	4.16	4.01	3.86	3.76	3.70	3.62	3.57	3.54	3.45
0.001	7.29	7.00	6.71	6.40	6.22	6.05	5.83	5.63	5.59	5.44
0.100	2.14	2.10	2.05	2.01	1.98	1.96	1.93	1.92	1.90	1.88
0.050	2.67	2.60	2.53	2.46	2.41	2.38	2.34	2.31	2.30	2.25
0.025	3.25	3.15	3.05	2.95	2.88	2.84	2.78	2.74	2.72	2.66
0.010	4.10	3.96	3.82	3.68	3.57	3.51	3.43	3.38	3.34	3.25
0.001	6.80	6.52	6.22	5.93	5.75	5.63	5.47	5.37	5.30	5.14
0.100	2.10	2.05	2.01	1.96	1.93	1.91	1.89	1.87	1.86	1.83
0.050	2.60	2.53	2.46	2.39	2.34	2.31	2.27	2.24	2.22	2.18
0.025	3.15	3.05	2.95	2.84	2.78	2.73	2.67	2.64	2.61	2.55
0.010	3.94	3.80	3.66	3.51	3.41	3.35	3.27	3.22	3.18	3.09
0.001	6.46	6.13	5.85	5.56	5.38	5.25	5.10	5.00	4.94	4.77
0.100	2.06	2.02	1.97	1.92	1.89	1.87	1.85	1.83	1.82	1.79
0.050	2.54	2.48	2.40	2.33	2.28	2.25	2.20	2.16	2.15	2.07
0.025	3.06	2.96	2.86	2.76	2.69	2.64	2.58	2.55	2.52	2.46
0.010	3.80	3.67	3.52	3.37	3.28	3.21	3.13	3.08	3.05	2.96
0.001	6.08	5.81	5.54	5.25	5.07	4.95	4.80	4.64	4.64	4.43
0.100	2.03	1.99	1.94	1.89	1.86	1.84	1.81	1.78	1.78	1.72
0.050	2.49	2.42	2.35	2.28	2.23	2.19	2.15	2.12	2.11	2.06
0.025	2.98	2.89	2.79	2.68	2.61	2.57	2.51	2.47	2.45	2.38
0.010	3.69	3.55	3.41	3.26	3.16	3.10	3.02	2.97	2.94	2.84
0.001	5.81	5.55	5.27	4.99	4.82	4.70	4.54	4.45	4.39	4.28

FOIA

TABLE 8 continued

Right-tail area	Degrees of freedom numerator, $d.f._n$								
	1	2	3	4	5	6	7	8	9
0.100	5.03	2.64	2.44	2.31	2.22	2.15	2.10	2.06	2.03
0.050	4.45	3.39	3.20	3.06	2.81	2.70	2.61	2.55	2.49
0.025	6.04	4.62	4.01	3.66	3.44	3.28	3.15	3.06	2.98
0.010	8.40	6.11	5.19	4.67	4.34	4.10	3.93	3.79	3.68
0.001	15.72	10.66	8.73	7.68	7.02	6.56	6.22	5.96	5.75
0.100	3.01	2.62	2.42	2.29	2.20	2.13	2.08	2.04	2.00
0.050	4.41	3.35	3.16	2.93	2.77	2.66	2.58	2.51	2.46
0.025	5.98	4.56	3.95	3.61	3.38	3.22	3.10	3.01	2.93
0.010	8.29	6.01	5.09	4.58	4.25	4.01	3.84	3.71	3.60
0.001	15.38	10.39	8.49	7.45	6.81	6.35	6.02	5.76	5.56
0.100	2.99	2.61	2.40	2.27	2.18	2.11	2.06	2.02	1.98
0.050	4.38	3.32	3.13	2.90	2.74	2.63	2.54	2.48	2.42
0.025	5.92	4.51	3.90	3.56	3.33	3.17	3.05	2.96	2.88
0.010	8.18	5.93	5.01	4.50	4.17	3.94	3.77	3.63	3.52
0.001	15.08	10.16	8.28	7.27	6.62	6.18	5.85	5.59	5.39
0.100	2.97	2.59	2.38	2.25	2.16	2.09	2.04	2.00	1.96
0.050	4.35	3.29	3.10	2.87	2.71	2.60	2.51	2.45	2.39
0.025	5.87	4.46	3.86	3.51	3.28	3.13	3.01	2.91	2.84
0.010	8.10	5.85	4.94	4.43	4.10	3.87	3.70	3.56	3.46
0.001	14.82	9.95	8.10	7.10	6.46	6.02	5.69	5.44	5.24
0.100	2.96	2.57	2.36	2.23	2.14	2.08	2.02	1.98	1.95
0.050	4.32	3.27	3.07	2.84	2.68	2.57	2.49	2.42	2.37
0.025	5.83	4.42	3.82	3.48	3.25	3.09	2.97	2.87	2.80
0.010	8.02	5.78	4.87	4.37	4.04	3.81	3.64	3.51	3.40
0.001	14.59	9.77	7.94	6.95	6.32	5.88	5.56	5.31	5.11
0.100	2.85	2.56	2.35	2.22	2.13	2.07	2.01	1.97	1.93
0.050	4.30	3.24	3.05	2.82	2.66	2.55	2.46	2.40	2.34
0.025	5.79	4.38	3.78	3.44	3.22	3.05	2.93	2.84	2.76
0.010	7.95	5.72	4.82	4.31	3.99	3.76	3.59	3.46	3.35
0.001	14.38	9.61	7.80	6.81	6.19	5.75	5.44	5.19	4.99
0.100	2.94	2.55	2.34	2.21	2.11	2.05	1.99	1.95	1.92
0.050	4.28	3.42	3.03	2.80	2.64	2.53	2.44	2.37	2.32
0.025	5.75	4.35	3.75	3.41	3.18	3.02	2.90	2.81	2.73
0.010	7.88	5.66	4.76	4.25	3.94	3.71	3.54	3.41	3.30
0.001	14.20	9.47	7.67	6.70	6.08	5.65	5.33	5.08	4.89
0.100	2.93	2.54	2.33	2.19	2.10	2.04	1.98	1.94	1.91
0.050	4.25	3.40	3.01	2.78	2.62	2.51	2.42	2.36	2.30
0.025	5.72	4.32	3.72	3.38	3.15	2.99	2.87	2.78	2.70
0.010	7.82	5.61	4.72	4.22	3.90	3.67	3.50	3.36	3.25
0.001	14.03	9.34	7.55	6.59	5.98	5.55	5.23	4.99	4.80

TABLE 8 continued

Right-tail area	Degrees of freedom numerator, $d.f._n$														
	10	12	15	20	25	30	40	50	60	80	100	120	150	200	1000
0.100	2.00	1.96	1.91	1.86	1.83	1.81	1.78	1.76	1.75	1.73	1.72	1.72	1.72	1.72	1.69
0.050	2.45	2.38	2.31	2.23	2.18	2.15	2.10	2.08	2.06	2.01	1.97	1.94	1.92	1.90	1.87
0.025	2.82	2.82	2.72	2.62	2.55	2.50	2.44	2.41	2.38	2.32	2.26	2.22	2.19	2.16	2.13
0.010	3.59	3.46	3.31	3.16	3.07	3.00	2.92	2.87	2.83	2.75	2.66	2.61	2.58	2.56	2.54
0.001	5.88	5.32	5.05	4.78	4.60	4.48	4.33	4.24	4.18	4.12	4.02	3.97	3.92	3.87	3.82
0.100	1.98	1.93	1.89	1.84	1.80	1.78	1.75	1.74	1.73	1.71	1.70	1.70	1.70	1.68	1.66
0.050	2.41	2.34	2.27	2.19	2.14	2.11	2.06	2.04	2.02	1.97	1.92	1.89	1.87	1.85	1.83
0.025	2.87	2.77	2.67	2.56	2.49	2.44	2.38	2.35	2.32	2.26	2.20	2.16	2.13	2.11	2.09
0.010	3.51	3.27	3.23	3.08	2.98	2.92	2.84	2.78	2.75	2.66	2.58	2.53	2.50	2.48	2.46
0.001	5.59	5.13	4.87	4.59	4.42	4.30	4.15	4.06	4.00	3.94	3.85	3.79	3.75	3.72	3.69
0.100	1.96	1.91	1.88	1.81	1.78	1.77	1.73	1.71	1.70	1.67	1.64	1.64	1.64	1.62	1.60
0.050	2.38	2.31	2.23	2.16	2.11	2.07	2.03	2.00	1.98	1.93	1.88	1.84	1.82	1.80	1.78
0.025	2.82	2.72	2.62	2.51	2.44	2.39	2.33	2.29	2.27	2.20	2.14	2.10	2.07	2.05	2.03
0.010	3.43	3.30	3.15	3.00	2.91	2.84	2.76	2.71	2.67	2.59	2.50	2.44	2.40	2.37	2.35
0.001	5.22	4.97	4.70	4.43	4.26	4.14	3.99	3.90	3.84	3.74	3.63	3.57	3.53	3.50	3.47
0.100	1.94	1.89	1.84	1.79	1.76	1.74	1.71	1.70	1.69	1.67	1.66	1.66	1.66	1.65	1.64
0.050	2.35	2.28	2.20	2.12	2.07	2.04	1.99	1.97	1.95	1.90	1.85	1.81	1.79	1.77	1.75
0.025	2.77	2.68	2.57	2.46	2.40	2.35	2.28	2.25	2.22	2.16	2.10	2.06	2.03	2.01	1.99
0.010	3.37	3.23	3.08	2.94	2.84	2.78	2.69	2.64	2.61	2.52	2.43	2.37	2.33	2.30	2.28
0.001	5.08	4.82	4.56	4.29	4.12	4.00	3.86	3.77	3.70	3.64	3.40	3.34	3.30	3.26	3.23
0.100	1.92	1.87	1.83	1.78	1.74	1.72	1.69	1.67	1.66	1.64	1.62	1.62	1.62	1.61	1.59
0.050	2.32	2.25	2.18	2.10	2.05	2.01	1.96	1.94	1.92	1.87	1.82	1.78	1.76	1.74	1.72
0.025	2.73	2.64	2.53	2.42	2.36	2.31	2.23	2.21	2.18	2.11	2.05	2.01	1.98	1.96	1.94
0.010	3.31	3.17	3.03	2.88	2.79	2.72	2.64	2.58	2.55	2.46	2.37	2.31	2.27	2.24	2.22
0.001	4.93	4.70	4.44	4.17	4.00	3.88	3.74	3.64	3.58	3.42	3.28	3.22	3.18	3.14	3.11
0.100	1.90	1.86	1.81	1.76	1.73	1.70	1.67	1.65	1.64	1.61	1.57	1.56	1.56	1.55	1.54
0.050	2.30	2.23	2.16	2.07	2.02	1.98	1.94	1.91	1.89	1.84	1.79	1.75	1.73	1.71	1.69
0.025	2.70	2.60	2.50	2.39	2.32	2.27	2.21	2.17	2.14	2.08	2.01	1.96	1.93	1.91	1.89
0.010	3.26	3.12	2.98	2.83	2.75	2.67	2.58	2.53	2.50	2.40	2.32	2.26	2.22	2.19	2.17
0.001	4.83	4.58	4.33	4.06	3.89	3.78	3.63	3.54	3.48	3.32	3.17	3.11	3.07	3.03	3.00
0.100	1.89	1.84	1.80	1.74	1.71	1.69	1.66	1.64	1.62	1.59	1.55	1.54	1.54	1.53	1.52
0.050	2.27	2.20	2.13	2.05	2.00	1.96	1.91	1.88	1.86	1.81	1.76	1.72	1.70	1.68	1.66
0.025	2.67	2.57	2.47	2.36	2.29	2.24	2.18	2.14	2.11	2.04	1.98	1.93	1.89	1.87	1.85
0.010	3.21	3.07	2.92	2.78	2.69	2.62	2.54	2.48	2.45	2.35	2.27	2.21	2.17	2.14	2.12
0.001	4.73	4.48	4.23	3.96	3.79	3.68	3.53	3.44	3.38	3.22	3.07	3.01	2.97	2.93	2.90
0.100	1.88	1.83	1.78	1.73	1.70	1.67	1.64	1.62	1.61	1.57	1.53	1.52	1.52	1.51	1.50
0.050	2.25	2.18	2.11	2.03	1.97	1.94	1.89	1.86	1.84	1.79	1.74	1.70	1.68	1.66	1.64
0.025	2.64	2.54	2.44	2.34	2.28	2.24	2.18	2.14	2.11	2.06	2.01	1.96	1.92	1.89	1.87
0.010	3.17	3.03	2.89	2.74	2.64	2.58	2.48	2.42	2.39	2.29	2.21	2.14	2.10	2.07	2.05
0.001	4.64	4.39	4.14	3.87	3.71	3.59	3.45	3.36	3.30	3.20	3.14	3.08	3.04	3.00	2.98