

UNIVERSITY OF ESWATINI

FINAL EXAMINATION PAPER 2018/2019

TITLE OF PAPER : TIME SERIES ANALYSIS

COURSE CODE : STA306

TIME ALLOWED : TWO (2) HOURS

REQUIREMENTS : CALCULATOR

INSTRUCTIONS : THIS PAPER HAS FIVE (5) QUESTIONS. ANSWER ANY THREE (3) QUESTIONS.

Question 1

[20 marks, 6+5+9]

Consider the time series

$$X_t + \frac{3}{10}X_{t-1} - \frac{1}{10}X_{t-2} = w_t - \frac{9}{10}w_{t-1} + \frac{7}{50}w_{t-2},$$

where w_t is a white noise random variable.

- Determine the values of p and q in the standard ARMA(p, q) framework so that there is no parameter redundancy in the model.
- Check whether the process is causal and invertible.
- Obtain the linear process form for this process.

Question 2

[20 marks, 2+2+2+3+2+3+2+4]

- Explain what plots would be expected for the autocorrelation function (ACF) and partial autocorrelation function (PACF) for the following processes.

- White noise
- AR(2)
- MA(1)
- ARMA(1,2)

- Consider the following three time series models where the error terms w_t are uncorrelated random errors with zero mean and constant variance.

- $X_t - 1.2X_{t-1} = w_t$.
- $X_t - 1.2X_{t-1} + 0.2X_{t-2} = w_t - 0.7w_{t-1}$.
- $X_t = w_t + 0.7w_{t-1} - 0.5w_{t-2}$

For each model, say whether it is stationary or not and specify p and q in the standard ARMA(p, q) framework.

- For a new time series X_t with length 20, the sample ACF and the sample PACF are tabulated below:

Lag	1	2	3	4	5
ACF	0.62	0.57	0.30	0.10	0.05
PACF	a_1	a_2	0.28	0.15	0.01

Find the values of a_1 and a_2 .

Question 3

[20 marks, 4+4+4+4+4]

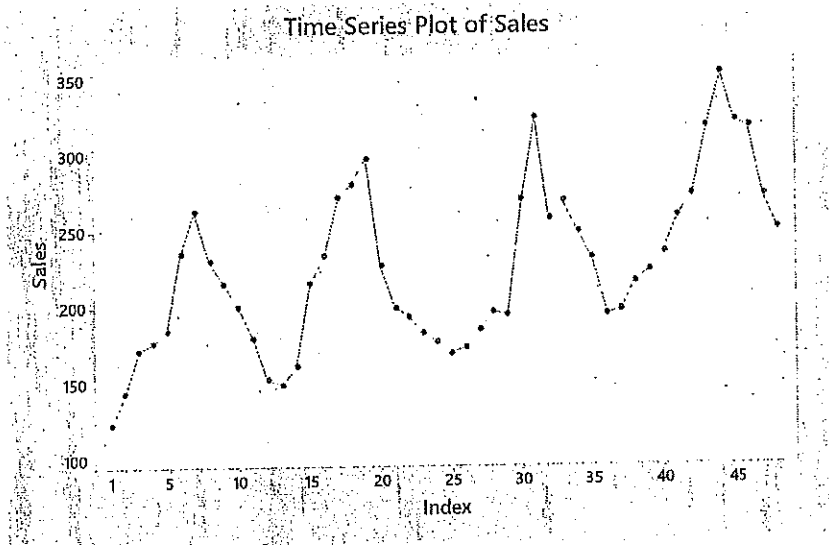
Write down a general model for each of the following series. Use the backshift operator and explain your notation.

- (a) $ARMA(1,1)_{12}$
- (b) $MA(2)_4$
- (c) $AR(1)$
- (d) $ARIMA(2,0,1) \times (0,1,0)_{12}$
- (e) $ARIMA(2,1,0)$

Question 4

[20 marks, 4+8+4+4]

A sports retailer recorded how many sets of golf clubs he sold from his shops each month for 4 years. The data is shown in the following plot. The retailer would like you to forecast how many sets of clubs he can expect to sell in each of the next three months.



- (a) Describe the main features of the time series shown by the plot.
- (b) What possible analyses could you do, using a suitable computer package such as R, to determine a stationary time series model for the data? Include brief details of diagnostic tests to check if the model is fitting well.
- (c) How could you forecast the sales for the next three months?
- (d) The retailer then tells you that the number of shops he sold from varied over the four year period. If you had the data on the number of shops open each month how might you change your analysis? What assumptions might be reasonable?

Question 5

[20 marks, 6+6+8]

Suppose that a model is set up for a seasonal time series Y_t so that the transformed series X_t , defined by

$$X_t = (1 - B^4)^3 Y_t,$$

where B is the backward shift operator, follows the AR(1) model

$$X_t = 0.7X_{t-1} + w_t,$$

where w_t is white noise with variance 1.

(a) Write down the abbreviated form of the model of Y_t SARIMA(p, d, q) \times (P, D, Q) $_S$, i.e. identify p, d, q, P, D, Q and S .

(b) Show that

$$Y_t = X_t + 3Y_{t-4} - 3Y_{t-8} + Y_{t-12},$$

for $t > 12$.

(c) If the first 13 observations of Y_t were

t	1	2	3	4	5	6	7	8	9	10	11	12	13
Y_t	10	12	15	16	9	13	15	17	8	12	14	15	9

find the one-step and two-step forecast means of Y_{14} and Y_{15} respectively.