UNIVERSITY OF SWAZILAND

MAIN EXAMINATION PAPER 2018

TITLE OF PAPER	: STATISTICAL MODELLING I
COURSE CODE	: STA 415
TIME ALLOWED	: TWO (2) HOURS
REQUIREMENTS	: CALCULATOR AND STATISTICAL TABLES
INSTRUCTIONS	: ANSWER ANY TREE QUESTIONS

Question 1

The following table and subsequent analysis are based on sample survey data on the usage of alcohol, cigarettes and marijuana among high school students;

Alcohol Use	Cigarette Use	Marijuana Use: Yes	Marijuana Use: No
Yes	Yes	911	538
	No	44	456
No	Yes	3	43
	No	2	279

R code

```
A<-c(1,1,1,1,0,0,0,0); ## 1--Alcohol use 0--otherwise
C<-c(1,1,0,0,1,1,0,0); ## 1---Cigarette use 0---otherwise
M<-c(1,0,1,0,1,0,1,0); ## 1-Marijuana use 0-otherwise
count<-c(911,538,44,456,3,43,2,279);
 AC<-A*C; AM<-A*M; CM<-C*M; ACM<-A*C*M;

##Model (AM,CM,AC) fit
drug.log<-glm(count~A+C+M+AM+CM+AC,family=poisson(link="log"))
summary(drug.log)
```

```
## output
 Call: glm(formula = count ~ A + C + M + AM + CM + AC, family =
poisson(link = "log"))  Coefficients:
              Estimate  Std. Error   z value           Pr(>|z|)
(Intercept)  5.63342      0.05970   94.361             < 2e-16***
A             0.48772      0.07577    6.437           1.22e-10 ***
C            -1.88667      0.16270  -11.596            < 2e-16 ***
M            -5.30904      0.47520  -11.172            < 2e-16***
AM            2.98601      0.46468    6.426           <1.31e-10***
CM            2.84789      0.16384   17.382            < 2e-16 ***
AC            2.05453      0.17406   11.803            < 2e-16 ***
 Null deviance: 2851.46098,    Residual deviance:    0.37399
##Estimated covariance matrix between AM and CM

          AM             CM
AM   0.215925578 -0.004968391
CM -0.004968391 0.026843349
```

Let X; Y and Z denote the variables Alcohol, Cigarette and Marijuana use respectively.

a) Write down the log-linear regression model and identify the associated estimates.

(5 Marks)

b) Compute the estimated odds ratio between any two variables of Alcohol, Cigarette, and Marijuana use controlling for the third variable.

(5 Marks)

c) Construct the 95% confidence interval for the true odds ratio between Alcohol and Cigarette use controlling for Marijuana use.

(5 Marks)

d) Test if the true odds ratio between Alcohol and Marijuana use controlling for Cigarette use equals the true odds ratio between Cigarette and Marijuana use controlling for Alcohol use at $\alpha = 5\%$.

(8 Marks)

Question 2

In a prospective, randomized study to investigate the capacity of aspirin to prevent pregnancy-induced hypertension (=high blood pressure), 65 women were treated with a daily dose of either aspirin (34 women) or placebo (31 women) during the third trimester of pregnancy. The result is summarized in the Table 1 below;

Table 1: Contingency table for study comparing Aspirin and Placebo

	Hypertension	No hypertension	Total
Aspirin treatment	4	30	34
Placebo treatment	11	20	31
Total	15	50	65

In Table 2, the probability that the test statistic n_{11} from Fisher's exact test attains a specific value t is given (n_{11} = number of women treated with Aspirin with hypertension).

Table 2: Conditional probability that n_{11} attains value t for data in Table 1

t	0	1	2	3	4	5	6	7
$P_{H_0}(n_{11}=t\,\vert\ldots$	0	0	0.001	0.004	0.019	0.060	0.131	0.205

t	8	9	10	11	12	13	14	15
$P_{H_0}(n_{11}=t\,\vert\ldots$	0.230	0.186	0.107	0.043	0.012	0.002	0	0

a) Fisher's exact test evaluates the probability $P_{H_0}(n_{11}=t\,|\ldots)$ under a certain condition. What is this condition (what are the dots in this expression)? How are $P_{H_0}(n_{11}=t\,|\ldots)$ in Table 2 calculated? State a formula for this probability.

(3 Marks)

b) Based on Table 2, calculate the one-sided p-value for Fisher's exact test for testing the null hypothesis that treatment and hypertension are independent versus the alternative that aspirin has a hypertension preventing effect. Interpret the result.

(5 Marks)

c) Consider a two-sided test for the independence null hypothesis versus the alternative that aspirin has any (positive or negative) effect on hypertension for pregnant women. There are diffierent methods to define a two-sided p-value based on Fisher's exact test. A simple but uncommon way is to multiply the one-sided p-value by 2. Choose another method and describe exactly how the two-sided p-value is defined. Compute the two-sided p-value and interpret.

(5 Marks)

d) Considering again the one-sided test problem: Calculate the mid p-value and mention an advantage and a disadvantage of using mid p-values in contrast to usual p-values.

(7 Marks)

Question 3

Suppose that there are two categorical explanatory variables, sex (male or female) and handedness (right- or left-handed). Suppose that people, coming to a shopping centre, are investigated: their sex is registered and they are asked about being left- or right-handed. Let probabilities that a person, coming to the centre, is MR, ML, FR, and FL (MR means male, right-handed etc.) are θ_{11}, θ_{12}, θ_{21}, and θ_{22} respectively. Denote Y_{11}, Y_{12}, Y_{21}, and Y_{22} the number of MR, ML, FR, and FL

(i) among first 1000 people,
(ii) coming during the day.

Suppose that people come independently of each other, and that the total number of people, coming during the day, has the Poisson distribution with parameter λ.

a) Find the distribution of $Y = [Y_{11}, Y_{12}, Y_{21}, Y_{22}]$ in case (i) and in case (ii).

(10 Marks)

b) Suppose that design (i) is used, and results of the investigation are presented in the contingency table below. The question of interest is whether there is an association between sex and handedness. Examine this by testing the hypothesis that the variables are independent.

	Right-handed	Left-handed
Male	430	90
Female	440	40

(10 Marks)

Question 4

A hospital administrator wished to develop a regression model for predicting the degree of long-term recovery after discharge from the hospital for severely injured patients. The predictor variable to be utilized is number of days of hospitalization (X), and the response variable is a prognostic index for long-term recovery (Y), with large values of the index reflecting a good prognosis. Hence, it was decided to investigate the appropriateness of the two-parameter nonlinear exponential regression model.

$$Y_i = \Upsilon_0 \exp(\Upsilon_1 X_i) + \epsilon_i$$

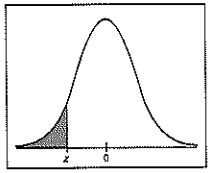
In the estimation of Υ_0 and Υ_1, which among the Least Squares Method and Gauss-Newton Method is more appropriate? Justify your choice of method over the other.

(20 Marks)

Question 5

For a classical linear model $= \boldsymbol{exp}\{\boldsymbol{X\beta} + \varepsilon\ \}$, where y, ε are n vectors, $\boldsymbol{\beta}$ has dimension p, X has n×p, and $\varepsilon_i{}'s$ are i.i.d. N(0,σ^2), derive the information matrix of $\boldsymbol{\beta}$.

(20 Marks)



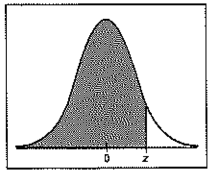
The table entry for z is the area to the left of z.

TABLE 5 Areas of a Standard Normal Distribution

(a) Table of Areas to the Left of z

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
−3.4	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0002
−3.3	.0005	.0005	.0005	.0004	.0004	.0004	.0004	.0004	.0004	.0003
−3.2	.0007	.0007	.0006	.0006	.0006	.0006	.0006	.0005	.0005	.0005
−3.1	.0010	.0009	.0009	.0009	.0008	.0008	.0008	.0008	.0007	.0007
−3.0	.0013	.0013	.0013	.0012	.0012	.0011	.0011	.0011	.0010	.0010
−2.9	.0019	.0018	.0018	.0017	.0016	.0016	.0015	.0015	.0014	.0014
−2.8	.0026	.0025	.0024	.0023	.0023	.0022	.0021	.0021	.0020	.0019
−2.7	.0035	.0034	.0033	.0032	.0031	.0030	.0029	.0028	.0027	.0026
−2.6	.0047	.0045	.0044	.0043	.0041	.0040	.0039	.0038	.0037	.0036
−2.5	.0062	.0060	.0059	.0057	.0055	.0054	.0052	.0051	.0049	.0048
−2.4	.0082	.0080	.0078	.0075	.0073	.0071	.0069	.0068	.0066	.0064
−2.3	.0107	.0104	.0102	.0099	.0096	.0094	.0091	.0089	.0087	.0084
−2.2	.0139	.0136	.0132	.0129	.0125	.0122	.0119	.0116	.0113	.0110
−2.1	.0179	.0174	.0170	.0166	.0162	.0158	.0154	.0150	.0146	.0143
−2.0	.0228	.0222	.0217	.0212	.0207	.0202	.0197	.0192	.0188	.0183
−1.9	.0287	.0281	.0274	.0268	.0262	.0256	.0250	.0244	.0239	.0233
−1.8	.0359	.0351	.0344	.0336	.0329	.0322	.0314	.0307	.0301	.0294
−1.7	.0446	.0436	.0427	.0418	.0409	.0401	.0392	.0384	.0375	.0367
−1.6	.0548	.0537	.0526	.0516	.0505	.0495	.0485	.0475	.0465	.0455
−1.5	.0668	.0655	.0643	.0630	.0618	.0606	.0594	.0582	.0571	.0559
−1.4	.0808	.0793	.0778	.0764	.0749	.0735	.0721	.0708	.0694	.0681
−1.3	.0968	.0951	.0934	.0918	.0901	.0885	.0869	.0853	.0838	.0823
−1.2	.1151	.1131	.1112	.1093	.1075	.1056	.1038	.1020	.1003	.0985
−1.1	.1357	.1335	.1314	.1292	.1271	.1251	.1230	.1210	.1190	.1170
−1.0	.1587	.1562	.1539	.1515	.1492	.1469	.1446	.1423	.1401	.1379
−0.9	.1841	.1814	.1788	.1762	.1736	.1711	.1685	.1660	.1635	.1611
−0.8	.2119	.2090	.2061	.2033	.2005	.1977	.1949	.1922	.1894	.1867
−0.7	.2420	.2389	.2358	.2327	.2296	.2266	.2236	.2206	.2177	.2148
−0.6	.2743	.2709	.2676	.2643	.2611	.2578	.2546	.2514	.2483	.2451
−0.5	.3085	.3050	.3015	.2981	.2946	.2912	.2877	.2843	.2810	.2776
−0.4	.3446	.3409	.3372	.3336	.3300	.3264	.3228	.3192	.3156	.3121
−0.3	.3821	.3783	.3745	.3707	.3669	.3632	.3594	.3557	.3520	.3483
−0.2	.4207	.4168	.4129	.4090	.4052	.4013	.3974	.3936	.3897	.3859
−0.1	.4602	.4562	.4522	.4483	.4443	.4404	.4364	.4325	.4286	.4247
−0.0	.5000	.4960	.4920	.4880	.4840	.4801	.4761	.4721	.4681	.4641

For values of z less than −3.49, use 0.000 to approximate the area.



The table entry for z is the area to the left of z.

TABLE 5(a) *continued*

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998

For z values greater than 3.49, use 1.000 to approximate the area.

TABLE 5 *continued*

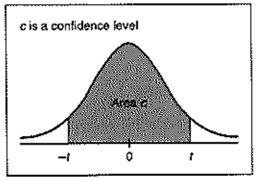
(b) Confidence Interval Critical Values z_c

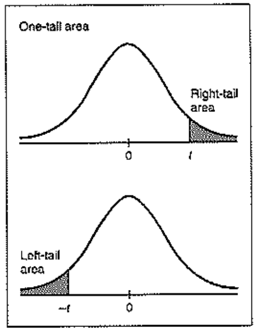
Level of Confidence c	Critical Value z_c
0.70, or 70%	1.04
0.75, or 75%	1.15
0.80, or 80%	1.28
0.85, or 85%	1.44
0.90, or 90%	1.645
0.95, or 95%	1.96
0.98, or 98%	2.33
0.99, or 99%	2.58

TABLE 5 *continued*

(c) Hypothesis Testing, Critical Values z_0

Level of Significance	$\alpha = 0.05$	$\alpha = 0.01$
Critical value z_0 for a left-tailed test	−1.645	−2.33
Critical value z_0 for a right-tailed test	1.645	2.33
Critical values $\pm z_0$ for a two-tailed test	±1.96	±2.58





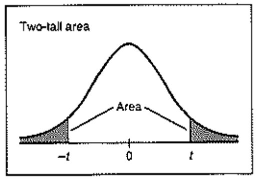
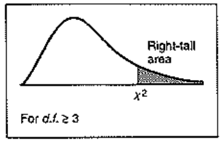


TABLE 6 Critical Values for Student's *t* Distribution

one-tail area	0.250	0.125	0.100	0.075	0.050	0.025	0.010	0.005	0.0005
two-tail area	0.500	0.250	0.200	0.150	0.100	0.050	0.020	0.010	0.0010
d.f. \ *c*	0.500	0.750	0.800	0.850	0.900	0.950	0.980	0.990	0.999
1	1.000	2.414	3.078	4.165	6.314	12.706	31.821	63.657	636.619
2	0.816	1.604	1.886	2.282	2.920	4.303	6.965	9.925	31.599
3	0.765	1.423	1.638	1.924	2.353	3.182	4.541	5.841	12.924
4	0.741	1.344	1.533	1.778	2.132	2.776	3.747	4.604	8.610
5	0.727	1.301	1.476	1.699	2.015	2.571	3.365	4.032	6.869
6	0.718	1.273	1.440	1.650	1.943	2.447	3.143	3.707	5.959
7	0.711	1.254	1.415	1.617	1.895	2.365	2.998	3.499	5.408
8	0.706	1.240	1.397	1.592	1.860	2.306	2.896	3.355	5.041
9	0.703	1.230	1.383	1.574	1.833	2.262	2.821	3.250	4.781
10	0.700	1.221	1.372	1.559	1.812	2.228	2.764	3.169	4.587
11	0.697	1.214	1.363	1.548	1.796	2.201	2.718	3.106	4.437
12	0.695	1.209	1.356	1.538	1.782	2.179	2.681	3.055	4.318
13	0.694	1.204	1.350	1.530	1.771	2.160	2.650	3.012	4.221
14	0.692	1.200	1.345	1.523	1.761	2.145	2.624	2.977	4.140
15	0.691	1.197	1.341	1.517	1.753	2.131	2.602	2.947	4.073
16	0.690	1.194	1.337	1.512	1.746	2.120	2.583	2.921	4.015
17	0.689	1.191	1.333	1.508	1.740	2.110	2.567	2.898	3.965
18	0.688	1.189	1.330	1.504	1.734	2.101	2.552	2.878	3.922
19	0.688	1.187	1.328	1.500	1.729	2.093	2.539	2.861	3.883
20	0.687	1.185	1.325	1.497	1.725	2.086	2.528	2.845	3.850
21	0.686	1.183	1.323	1.494	1.721	2.080	2.518	2.831	3.819
22	0.686	1.182	1.321	1.492	1.717	2.074	2.508	2.819	3.792
23	0.685	1.180	1.319	1.489	1.714	2.069	2.500	2.807	3.768
24	0.685	1.179	1.318	1.487	1.711	2.064	2.492	2.797	3.745
25	0.684	1.198	1.316	1.485	1.708	2.060	2.485	2.787	3.725
26	0.684	1.177	1.315	1.483	1.706	2.056	2.479	2.779	3.707
27	0.684	1.176	1.314	1.482	1.703	2.052	2.473	2.771	3.690
28	0.683	1.175	1.313	1.480	1.701	2.048	2.467	2.763	3.674
29	0.683	1.174	1.311	1.479	1.699	2.045	2.462	2.756	3.659
30	0.683	1.173	1.310	1.477	1.697	2.042	2.457	2.750	3.646
35	0.682	1.170	1.306	1.472	1.690	2.030	2.438	2.724	3.591
40	0.681	1.167	1.303	1.468	1.684	2.021	2.423	2.704	3.551
45	0.680	1.165	1.301	1.465	1.679	2.014	2.412	2.690	3.520
50	0.679	1.164	1.299	1.462	1.676	2.009	2.403	2.678	3.496
60	0.679	1.162	1.296	1.458	1.671	2.000	2.390	2.660	3.460
70	0.678	1.160	1.294	1.456	1.667	1.994	2.381	2.648	3.435
80	0.678	1.159	1.292	1.453	1.664	1.990	2.374	2.639	3.416
100	0.677	1.157	1.290	1.451	1.660	1.984	2.364	2.626	3.390
500	0.675	1.152	1.283	1.442	1.648	1.965	2.334	2.586	3.310
1000	0.675	1.151	1.282	1.441	1.646	1.962	2.330	2.581	3.300
∞	0.674	1.150	1.282	1.440	1.645	1.960	2.326	2.576	3.291

For degrees of freedom *d.f.* not in the table, use the closest *d.f.* that is *smaller.*



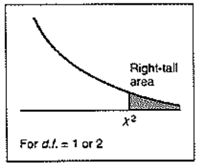


TABLE 7 The χ^2 Distribution

	Right-tail Area									
d.f.	.995	.990	.975	.950	.900	.100	.050	.025	.010	.005
1	0.0^4393	0.0^3157	0.0^3982	0.0^2393	0.0158	2.71	3.84	5.02	6.63	7.88
2	0.0100	0.0201	0.0506	0.103	0.211	4.61	5.99	7.38	9.21	10.60
3	0.072	0.115	0.216	0.352	0.584	6.25	7.81	9.35	11.34	12.84
4	0.207	0.297	0.484	0.711	1.064	7.78	9.49	11.14	13.28	14.86
5	0.412	0.554	0.831	1.145	1.61	9.24	11.07	12.83	15.09	16.75
6	0.676	0.872	1.24	1.64	2.20	10.64	12.59	14.45	16.81	18.55
7	0.989	1.24	1.69	2.17	2.83	12.02	14.07	16.01	18.48	20.28
8	1.34	1.65	2.18	2.73	3.49	13.36	15.51	17.53	20.09	21.96
9	1.73	2.09	2.70	3.33	4.17	14.68	16.92	19.02	21.67	23.59
10	2.16	2.56	3.25	3.94	4.87	15.99	18.31	20.48	23.21	25.19
11	2.60	3.05	3.82	4.57	5.58	17.28	19.68	21.92	24.72	26.76
12	3.07	3.57	4.40	5.23	6.30	18.55	21.03	23.34	26.22	28.30
13	3.57	4.11	5.01	5.89	7.04	19.81	22.36	24.74	27.69	29.82
14	4.07	4.66	5.63	6.57	7.79	21.06	23.68	26.12	29.14	31.32
15	4.60	5.23	6.26	7.26	8.55	22.31	25.00	27.49	30.58	32.80
16	5.14	5.81	6.91	7.96	9.31	23.54	26.30	28.85	32.00	34.27
17	5.70	6.41	7.56	8.67	10.09	24.77	27.59	30.19	33.41	35.72
18	6.26	7.01	8.23	9.39	10.86	25.99	28.87	31.53	34.81	37.16
19	6.84	7.63	8.91	10.12	11.65	27.20	30.14	32.85	36.19	38.58
20	7.43	8.26	8.59	10.85	12.44	28.41	31.41	34.17	37.57	40.00
21	8.03	8.90	10.28	11.59	13.24	29.62	32.67	35.48	38.93	41.40
22	8.64	9.54	10.98	12.34	14.04	30.81	33.92	36.78	40.29	42.80
23	9.26	10.20	11.69	13.09	14.85	32.01	35.17	38.08	41.64	44.18
24	9.89	10.86	12.40	13.85	15.66	33.20	36.42	39.36	42.98	45.56
25	10.52	11.52	13.12	14.61	16.47	34.38	37.65	40.65	44.31	46.93
26	11.16	12.20	13.84	15.38	17.29	35.56	38.89	41.92	45.64	48.29
27	11.81	12.88	14.57	16.15	18.11	36.74	40.11	43.19	46.96	49.64
28	12.46	13.56	15.31	16.93	18.94	37.92	41.34	44.46	48.28	50.99
29	13.21	14.26	16.05	17.71	19.77	39.09	42.56	45.72	49.59	52.34
30	13.79	14.95	16.79	18.49	20.60	40.26	43.77	46.98	50.89	53.67
40	20.71	22.16	24.43	26.51	29.05	51.80	55.76	59.34	63.69	66.77
50	27.99	29.71	32.36	34.76	37.69	63.17	67.50	71.42	76.15	79.49
60	35.53	37.48	40.48	43.19	46.46	74.40	79.08	83.30	88.38	91.95
70	43.28	45.44	48.76	51.74	55.33	85.53	90.53	95.02	100.4	104.2
80	51.17	53.54	57.15	60.39	64.28	96.58	101.9	106.6	112.3	116.3
90	59.20	61.75	65.65	69.13	73.29	107.6	113.1	118.1	124.1	128.3
100	67.33	70.06	74.22	77.93	82.36	118.5	124.3	129.6	135.8	140.2

Source: From H. L. Herter, *Biometrika,* June 1964. Printed by permission of the Biometrika Trustees.

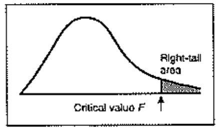


TABLE 8 Critical Values For F Distribution

Degrees of freedom denominator, $d.f._D$	Right-tail area	Degrees of freedom numerator, $d.f._N$: 1	2	3	4	5	6	7	8	9
	0.100	39.86	49.50	53.59	55.83	57.24	58.20	58.91	59.44	59.86
	0.050	161.45	199.50	215.71	224.58	230.16	233.99	236.77	238.88	240.54
1	0.025	647.79	799.50	864.16	899.58	921.85	937.11	948.22	956.66	963.28
	0.010	4052.2	4999.5	5403.4	5624.6	5763.6	5859.0	5928.4	5981.1	6022.5
	0.001	405284	500000	540379	562500	576405	585937	592873	598144	602284
	0.100	8.53	9.00	9.16	9.24	9.29	9.33	9.35	9.37	9.38
	0.050	18.51	19.00	19.16	19.25	19.30	19.33	19.35	19.37	19.38
2	0.025	38.51	39.00	39.17	39.25	39.30	39.33	39.36	39.37	39.39
	0.010	98.50	99.00	99.17	99.25	99.30	99.33	99.36	99.37	99.39
	0.001	998.50	999.00	999.17	999.25	999.30	999.33	999.36	999.37	999.39
	0.100	5.54	5.46	5.39	5.34	5.31	5.28	5.27	5.25	5.24
	0.050	10.13	9.55	9.28	9.12	9.01	8.94	8.89	8.85	8.81
3	0.025	17.44	16.04	15.44	15.10	14.88	14.73	14.62	14.54	14.47
	0.010	34.12	30.82	29.46	28.71	28.24	27.91	27.67	27.49	27.35
	0.001	167.03	148.50	141.11	137.10	134.58	132.85	131.58	130.62	129.86
	0.100	4.54	4.32	4.19	4.11	4.05	4.01	3.98	3.95	3.94
	0.050	7.71	6.94	6.59	6.39	6.26	6.16	6.09	6.04	6.00
4	0.025	12.22	10.65	9.98	9.60	9.36	9.20	9.07	8.98	8.90
	0.010	21.20	18.00	16.69	15.98	15.52	15.21	14.98	14.80	14.66
	0.001	74.14	61.25	56.18	53.44	51.71	50.53	49.66	49.00	48.47
	0.100	4.06	3.78	3.62	3.52	3.45	3.40	3.37	3.34	3.32
	0.050	6.61	5.79	5.41	5.19	5.05	4.95	4.88	4.82	4.77
5	0.025	10.01	8.43	7.76	7.39	7.15	6.98	6.85	6.76	6.68
	0.010	16.26	13.27	12.06	11.39	10.97	10.67	10.46	10.29	10.16
	0.001	47.18	37.12	33.20	31.09	29.75	28.83	28.16	27.65	27.24
	0.100	3.78	3.46	3.29	3.18	3.11	3.05	3.01	2.98	2.96
	0.050	5.99	5.14	4.76	4.53	4.39	4.28	4.21	4.15	4.10
6	0.025	8.81	7.26	6.60	6.23	5.99	5.82	5.70	5.60	5.52
	0.010	13.75	10.92	9.78	9.15	8.75	8.47	8.26	8.10	7.98
	0.001	35.51	27.00	23.70	21.92	20.80	20.03	19.46	19.03	18.69
	0.100	3.59	3.26	3.07	2.96	2.88	2.83	2.78	2.75	2.72
	0.050	5.59	4.74	4.35	4.12	3.97	3.87	3.79	3.73	3.68
7	0.025	8.07	6.54	5.89	5.52	5.29	5.12	4.99	4.90	4.82
	0.010	12.25	9.55	8.45	7.85	7.46	7.19	6.99	6.84	6.72
	0.001	29.25	21.69	18.77	17.20	16.21	15.52	15.02	14.63	14.33
	0.100	3.46	3.11	2.92	2.81	2.73	2.67	2.62	2.59	2.56
	0.050	5.32	4.46	4.07	3.84	3.69	3.58	3.50	3.44	3.39
8	0.025	7.57	6.06	5.42	5.05	4.82	4.65	4.53	4.43	4.36
	0.010	11.26	8.65	7.59	7.01	6.63	6.37	6.18	6.03	5.91
	0.001	25.41	18.49	15.83	14.39	13.48	12.86	12.40	12.05	11.77

TABLE 8 *continued*

Degrees of freedom denominator, $d.f._D$	Right-tail area	Degrees of freedom numerator, $d.f._N$: 10	12	15	20	25	30	40	50	60	120	1000
	0.100	60.19	60.71	61.22	61.74	62.05	62.26	62.53	62.69	62.79	63.06	63.30
	0.050	241.88	243.91	245.95	248.01	249.26	250.10	251.14	251.77	252.20	253.25	254.19
1	0.025	968.63	976.71	984.87	993.10	998.08	1001.4	1005.6	1008.1	1009.8	1014.0	1017.7
	0.010	6055.8	6106.3	6157.3	6208.7	6239.8	6260.6	6286.8	6302.5	6313.0	6339.4	6362.7
	0.001	605621	610668	615764	620908	624017	626099	628712	630285	631337	633972	636301
	0.100	9.39	9.41	9.42	9.44	9.45	9.46	9.47	9.47	9.47	9.48	9.49
	0.050	19.40	19.41	19.43	19.45	19.46	19.46	19.47	19.48	19.48	19.49	19.49
2	0.025	39.40	39.41	39.43	39.45	39.46	39.46	39.47	39.48	39.48	39.49	39.50
	0.010	99.40	99.42	99.43	99.45	99.46	99.47	99.47	99.48	99.48	99.49	99.50
	0.001	999.40	999.42	999.43	999.45	999.46	999.47	999.47	999.48	999.48	999.49	999.50
	0.100	5.23	5.22	5.20	5.18	5.17	5.17	5.16	5.15	5.15	5.14	5.13
	0.050	8.79	8.74	8.70	8.66	8.63	8.62	8.59	8.58	8.57	8.55	8.53
3	0.025	14.42	14.34	14.25	14.17	14.12	14.08	14.04	14.01	13.99	13.95	13.91
	0.010	27.23	27.05	26.87	26.69	26.58	26.50	26.41	26.35	26.32	26.22	26.14
	0.001	129.25	128.32	127.37	126.42	125.84	125.45	124.96	124.66	124.47	123.97	123.53
	0.100	3.92	3.90	3.87	3.84	3.83	3.82	3.80	3.80	3.79	3.78	3.76
	0.050	5.96	5.91	5.86	5.80	5.77	5.75	5.72	5.70	5.69	5.66	5.63
4	0.025	8.84	8.75	8.66	8.56	8.50	8.46	8.41	8.38	8.36	8.31	8.26
	0.010	14.55	14.37	14.20	14.02	13.91	13.84	13.75	13.69	13.65	13.56	13.47
	0.001	48.05	47.41	46.76	46.10	45.70	45.43	45.09	44.88	44.75	44.40	44.09
	0.100	3.30	3.27	3.24	3.21	3.19	3.17	3.16	3.15	3.14	3.12	3.11
	0.050	4.74	4.68	4.62	4.56	4.52	4.50	4.46	4.44	4.43	4.40	4.37
5	0.025	6.62	6.52	6.43	6.33	6.27	6.23	6.18	6.14	6.12	6.07	6.02
	0.010	10.05	9.89	9.72	9.55	9.45	9.38	9.29	9.24	9.20	9.11	9.03
	0.001	26.92	26.42	25.91	25.39	25.08	24.87	24.60	24.44	24.33	24.06	23.82
	0.100	2.94	2.90	2.87	2.84	2.81	2.80	2.78	2.77	2.76	2.74	2.72
	0.050	4.06	4.00	3.94	3.87	3.83	3.81	3.77	3.75	3.74	3.70	3.67
6	0.025	5.46	5.37	5.27	5.17	5.11	5.07	5.01	4.98	4.96	4.90	4.86
	0.010	7.87	7.72	7.56	7.40	7.30	7.23	7.14	7.09	7.06	6.97	6.89
	0.001	18.41	17.99	17.56	17.12	16.85	16.67	16.44	16.31	16.21	15.98	15.77
	0.100	2.70	2.67	2.63	2.59	2.57	2.56	2.54	2.52	2.51	2.49	2.47
	0.050	3.64	3.57	3.51	3.44	3.40	3.38	3.34	3.32	3.30	3.27	3.23
7	0.025	4.76	4.67	4.57	4.47	4.40	4.36	4.31	4.28	4.25	4.20	4.15
	0.010	6.62	6.47	6.31	6.16	6.06	5.99	5.91	5.86	5.82	5.74	5.66
	0.001	14.08	13.71	13.32	12.93	12.69	12.53	12.33	12.20	12.12	11.91	11.72
	0.100	2.54	2.50	2.46	2.42	2.40	2.38	2.36	2.35	2.34	2.32	2.30
	0.050	3.35	3.28	3.22	3.15	3.11	3.08	3.04	3.02	3.01	2.97	2.93
8	0.025	4.30	4.20	4.10	4.00	3.94	3.89	3.84	3.81	3.78	3.73	3.68
	0.010	5.81	5.67	5.52	5.36	5.26	5.20	5.12	5.07	5.03	4.95	4.87
	0.001	11.54	11.19	10.84	10.48	10.26	10.11	9.92	9.80	9.73	9.53	9.36

TABLE 8 *continued*

Degrees of freedom denominator, $d.f._D$	Right-tail area	Degrees of freedom numerator, $d.f._N$: 1	2	3	4	5	6	7	8	9
	0.100	3.36	3.01	2.81	2.69	2.61	2.55	2.51	2.47	2.44
	0.050	5.12	4.26	3.86	3.63	3.48	3.37	3.29	3.23	3.18
9	0.025	7.21	5.71	5.08	4.72	4.48	4.32	4.20	4.10	4.03
	0.010	10.56	8.02	6.99	6.42	6.06	5.80	5.61	5.47	5.35
	0.001	22.86	16.39	13.90	12.56	11.71	11.13	10.70	10.37	10.11
	0.100	3.29	2.92	2.73	2.61	2.52	2.46	2.41	2.38	2.35
	0.050	4.96	4.10	3.71	3.48	3.33	3.22	3.14	3.07	3.02
10	0.025	6.94	5.46	4.83	4.47	4.24	4.07	3.95	3.85	3.78
	0.010	10.04	7.56	6.55	5.99	5.64	5.39	5.20	5.06	4.94
	0.001	21.04	14.91	12.55	11.28	10.48	9.93	9.52	9.20	8.96
	0.100	3.23	2.86	2.66	2.54	2.45	2.39	2.34	2.30	2.27
	0.050	4.84	3.98	3.59	3.36	3.20	3.09	3.01	2.95	2.90
11	0.025	6.72	5.26	4.63	4.28	4.04	3.88	3.76	3.66	3.59
	0.010	9.65	7.21	6.22	5.67	5.32	5.07	4.89	4.74	4.63
	0.001	19.69	13.81	11.56	10.35	9.58	9.05	8.66	8.35	8.12
	0.100	3.18	2.81	2.61	2.48	2.39	2.33	2.28	2.24	2.21
	0.050	4.75	3.89	3.49	3.26	3.11	3.00	2.91	2.85	2.80
12	0.025	6.55	5.10	4.47	4.12	3.89	3.73	3.61	3.51	3.44
	0.010	9.33	6.93	5.95	5.41	5.06	4.82	4.64	4.50	4.39
	0.001	18.64	12.97	10.80	9.63	8.89	8.38	8.00	7.71	7.48
	0.100	3.14	2.76	2.56	2.43	2.35	2.28	2.23	2.20	2.16
	0.050	4.67	3.81	3.41	3.18	3.03	2.92	2.83	2.77	2.71
13	0.025	6.41	4.97	4.35	4.00	3.77	3.60	3.48	3.39	3.31
	0.010	9.07	6.70	5.74	5.21	4.86	4.62	4.44	4.30	4.19
	0.001	17.82	12.31	10.21	9.07	8.35	7.86	7.49	7.21	6.98
	0.100	3.10	2.73	2.52	2.39	2.31	2.24	2.19	2.15	2.12
	0.050	4.60	3.74	3.34	3.11	2.96	2.85	2.76	2.70	2.65
14	0.025	6.30	4.86	4.24	3.89	3.66	3.50	3.38	3.29	3.21
	0.010	8.86	6.51	5.56	5.04	4.69	4.46	4.28	4.14	4.03
	0.001	17.14	11.78	9.73	8.62	7.92	7.44	7.08	6.80	6.58
	0.100	3.07	2.70	2.49	2.36	2.27	2.21	2.16	2.12	2.09
	0.050	4.54	3.68	3.29	3.06	2.90	2.79	2.71	2.64	2.59
15	0.025	6.20	4.77	4.15	3.80	3.58	3.41	3.29	3.20	3.12
	0.010	8.68	6.36	5.42	4.89	4.56	4.32	4.14	4.00	3.89
	0.001	16.59	11.34	9.34	8.25	7.57	7.09	6.74	6.47	6.26
	0.100	3.05	2.67	2.46	2.33	2.24	2.18	2.13	2.09	2.06
	0.050	4.49	3.63	3.24	3.01	2.85	2.74	2.66	2.59	2.54
16	0.025	6.12	4.69	4.08	3.73	3.50	3.34	3.22	3.12	3.05
	0.010	8.53	6.23	5.29	4.77	4.44	4.20	4.03	3.89	3.78
	0.001	16.12	10.97	9.01	7.94	7.27	6.80	6.46	6.19	5.98

TABLE 8 *continued*

Degrees of freedom denominator, $d.f._D$	Right-tail area	Degrees of freedom numerator, $d.f._N$: 10	12	15	20	25	30	40	50	60	120	1000
	0.100	2.42	2.38	2.34	2.30	2.27	2.25	2.23	2.22	2.21	2.18	2.16
	0.050	3.14	3.07	3.01	2.94	2.89	2.86	2.83	2.80	2.79	2.75	2.71
9	0.025	3.96	3.87	3.77	3.67	3.60	3.56	3.51	3.47	3.45	3.39	3.34
	0.010	5.26	5.11	4.96	4.81	4.71	4.65	4.57	4.52	4.48	4.40	4.32
	0.001	9.89	9.57	9.24	8.90	8.69	8.55	8.37	8.26	8.19	8.00	7.84
	0.100	2.32	2.28	2.24	2.20	2.17	2.16	2.13	2.12	2.11	2.08	2.06
	0.050	2.98	2.91	2.85	2.77	2.73	2.70	2.66	2.64	2.62	2.58	2.54
10	0.025	3.72	3.62	3.52	3.42	3.35	3.31	3.26	3.22	3.20	3.14	3.09
	0.010	4.85	4.71	4.56	4.41	4.31	4.25	4.17	4.12	4.08	4.00	3.92
	0.001	8.75	8.45	8.13	7.80	7.60	7.47	7.30	7.19	7.12	6.94	6.78
	0.100	2.25	2.21	2.17	2.12	2.10	2.08	2.05	2.04	2.03	2.00	1.98
	0.050	2.85	2.79	2.72	2.65	2.60	2.57	2.53	2.51	2.49	2.45	2.41
11	0.025	3.53	3.43	3.33	3.23	3.16	3.12	3.06	3.03	3.00	2.94	2.89
	0.010	4.54	4.40	4.25	4.10	4.01	3.94	3.86	3.81	3.78	3.69	3.61
	0.001	7.92	7.63	7.32	7.01	6.81	6.68	6.52	6.42	6.35	6.18	6.02
	0.100	2.19	2.15	2.10	2.06	2.03	2.01	1.99	1.97	1.96	1.93	1.91
	0.050	2.75	2.69	2.62	2.54	2.50	2.47	2.43	2.40	2.38	2.34	2.30
12	0.025	3.37	3.28	3.18	3.07	3.01	2.96	2.91	2.87	2.85	2.79	2.73
	0.010	4.30	4.16	4.01	3.86	3.76	3.70	3.62	3.57	3.54	3.45	3.37
	0.001	7.29	7.00	6.71	6.40	6.22	6.09	5.93	5.83	5.76	5.59	5.44
	0.100	2.14	2.10	2.05	2.01	1.98	1.96	1.93	1.92	1.90	1.88	1.85
	0.050	2.67	2.60	2.53	2.46	2.41	2.38	2.34	2.31	2.30	2.25	2.21
13	0.025	3.25	3.15	3.05	2.95	2.88	2.84	2.78	2.74	2.72	2.66	2.60
	0.010	4.10	3.96	3.82	3.66	3.57	3.51	3.43	3.38	3.34	3.25	3.18
	0.001	6.80	6.52	6.23	5.93	5.75	5.63	5.47	5.37	5.30	5.14	4.99
	0.100	2.10	2.05	2.01	1.96	1.93	1.91	1.89	1.87	1.86	1.83	1.80
	0.050	2.60	2.53	2.46	2.39	2.34	2.31	2.27	2.24	2.22	2.18	2.14
14	0.025	3.15	3.05	2.95	2.84	2.78	2.73	2.67	2.64	2.61	2.55	2.50
	0.010	3.94	3.80	3.66	3.51	3.41	3.35	3.27	3.22	3.18	3.09	3.02
	0.001	6.40	6.13	5.85	5.56	5.38	5.25	5.10	5.00	4.94	4.77	4.62
	0.100	2.06	2.02	1.97	1.92	1.89	1.87	1.85	1.83	1.82	1.79	1.76
	0.050	2.54	2.48	2.40	2.33	2.28	2.25	2.20	2.18	2.16	2.11	2.07
15	0.025	3.06	2.96	2.86	2.76	2.69	2.64	2.59	2.55	2.52	2.46	2.40
	0.010	3.80	3.67	3.52	3.37	3.28	3.21	3.13	3.08	3.05	2.96	2.88
	0.001	6.08	5.81	5.54	5.25	5.07	4.95	4.80	4.70	4.64	4.47	4.33
	0.100	2.03	1.99	1.94	1.89	1.86	1.84	1.81	1.79	1.78	1.75	1.72
	0.050	2.49	2.42	2.35	2.28	2.23	2.19	2.15	2.12	2.11	2.06	2.02
16	0.025	2.99	2.89	2.79	2.68	2.61	2.57	2.51	2.47	2.45	2.38	2.32
	0.010	3.69	3.55	3.41	3.26	3.16	3.10	3.02	2.97	2.93	2.84	2.76
	0.001	5.81	5.55	5.27	4.99	4.82	4.70	4.54	4.45	4.39	4.23	4.08

TABLE 8 *continued*

Degrees of freedom denominator, $d.f._D$	Right-tail area	Degrees of freedom numerator, $d.f._N$ 1	2	3	4	5	6	7	8	9
	0.100	3.03	2.64	2.44	2.31	2.22	2.15	2.10	2.06	2.03
	0.050	4.45	3.59	3.20	2.96	2.81	2.70	2.61	2.55	2.49
17	0.025	6.04	4.62	4.01	3.66	3.44	3.28	3.16	3.06	2.98
	0.010	8.40	6.11	5.19	4.67	4.34	4.10	3.93	3.79	3.68
	0.001	15.72	10.66	8.73	7.68	7.02	6.56	6.22	5.96	5.75
	0.100	3.01	2.62	2.42	2.29	2.20	2.13	2.08	2.04	2.00
	0.050	4.41	3.55	3.16	2.93	2.77	2.66	2.58	2.51	2.46
18	0.025	5.98	4.56	3.95	3.61	3.38	3.22	3.10	3.01	2.93
	0.010	8.29	6.01	5.09	4.58	4.25	4.01	3.84	3.71	3.60
	0.001	15.38	10.39	8.49	7.46	6.81	6.35	6.02	5.76	5.56
	0.100	2.99	2.61	2.40	2.27	2.18	2.11	2.06	2.02	1.98
	0.050	4.38	3.52	3.13	2.90	2.74	2.63	2.54	2.48	2.42
19	0.025	5.92	4.51	3.90	3.56	3.33	3.17	3.05	2.96	2.88
	0.010	8.18	5.93	5.01	4.50	4.17	3.94	3.77	3.63	3.52
	0.001	15.08	10.16	8.28	7.27	6.62	6.18	5.85	5.59	5.39
	0.100	2.97	2.59	2.38	2.25	2.16	2.09	2.04	2.00	1.96
	0.050	4.35	3.49	3.10	2.87	2.71	2.60	2.51	2.45	2.39
20	0.025	5.87	4.46	3.86	3.51	3.29	3.13	3.01	2.91	2.84
	0.010	8.10	5.85	4.94	4.43	4.10	3.87	3.70	3.56	3.46
	0.001	14.82	9.95	8.10	7.10	6.46	6.02	5.69	5.44	5.24
	0.100	2.96	2.57	2.36	2.23	2.14	2.08	2.02	1.98	1.95
	0.050	4.32	3.47	3.07	2.84	2.68	2.57	2.49	2.42	2.37
21	0.025	5.83	4.42	3.82	3.48	3.25	3.09	2.97	2.87	2.80
	0.010	8.02	5.78	4.87	4.37	4.04	3.81	3.64	3.51	3.40
	0.001	14.59	9.77	7.94	6.95	6.32	5.88	5.56	5.31	5.11
	0.100	2.95	2.56	2.35	2.22	2.13	2.06	2.01	1.97	1.93
	0.050	4.30	3.44	3.05	2.82	2.66	2.55	2.46	2.40	2.34
22	0.025	5.79	4.38	3.78	3.44	3.22	3.05	2.93	2.84	2.76
	0.010	7.95	5.72	4.82	4.31	3.99	3.76	3.59	3.45	3.35
	0.001	14.38	9.61	7.80	6.81	6.19	5.76	5.44	5.19	4.99
	0.100	2.94	2.55	2.34	2.21	2.11	2.05	1.99	1.95	1.92
	0.050	4.28	3.42	3.03	2.80	2.64	2.53	2.44	2.37	2.32
23	0.025	5.75	4.35	3.75	3.41	3.18	3.02	2.90	2.81	2.73
	0.010	7.88	5.66	4.76	4.26	3.94	3.71	3.54	3.41	3.30
	0.001	14.20	9.47	7.67	6.70	6.08	5.65	5.33	5.09	4.89
	0.100	2.93	2.54	2.33	2.19	2.10	2.04	1.98	1.94	1.91
	0.050	4.26	3.40	3.01	2.78	2.62	2.51	2.42	2.36	2.30
24	0.025	5.72	4.32	3.72	3.38	3.15	2.99	2.87	2.78	2.70
	0.010	7.82	5.61	4.72	4.22	3.90	3.67	3.50	3.36	3.26
	0.001	14.03	9.34	7.55	6.59	5.98	5.55	5.23	4.99	4.80

TABLE 8 *continued*

Degrees of freedom denominator, $d.f._D$	Right-tail area	Degrees of freedom numerator, $d.f._N$ 10	12	15	20	25	30	40	50	60	120	1000
	0.100	2.00	1.96	1.91	1.86	1.83	1.81	1.78	1.76	1.75	1.72	1.69
	0.050	2.45	2.38	2.31	2.23	2.18	2.15	2.10	2.08	2.06	2.01	1.97
17	0.025	2.92	2.82	2.72	2.62	2.55	2.50	2.44	2.41	2.38	2.32	2.26
	0.010	3.59	3.46	3.31	3.16	3.07	3.00	2.92	2.87	2.83	2.75	2.66
	0.001	5.58	5.32	5.05	4.78	4.60	4.48	4.33	4.24	4.18	4.02	3.87
	0.100	1.98	1.93	1.89	1.84	1.80	1.78	1.75	1.74	1.72	1.69	1.66
	0.050	2.41	2.34	2.27	2.19	2.14	2.11	2.06	2.04	2.02	1.97	1.92
18	0.025	2.87	2.77	2.67	2.56	2.49	2.44	2.38	2.35	2.32	2.26	2.20
	0.010	3.51	3.37	3.23	3.08	2.98	2.92	2.84	2.78	2.75	2.66	2.58
	0.001	5.39	5.13	4.87	4.59	4.42	4.30	4.15	4.06	4.00	3.84	3.69
	0.100	1.96	1.91	1.86	1.81	1.78	1.76	1.73	1.71	1.70	1.67	1.64
	0.050	2.38	2.31	2.23	2.16	2.11	2.07	2.03	2.00	1.98	1.93	1.88
19	0.025	2.82	2.72	2.62	2.51	2.44	2.39	2.33	2.30	2.27	2.20	2.14
	0.010	3.43	3.30	3.15	3.00	2.91	2.84	2.76	2.71	2.67	2.58	2.50
	0.001	5.22	4.97	4.70	4.43	4.26	4.14	3.99	3.90	3.84	3.68	3.53
	0.100	1.94	1.89	1.84	1.79	1.76	1.74	1.71	1.69	1.68	1.64	1.61
	0.050	2.35	2.28	2.20	2.12	2.07	2.04	1.99	1.97	1.95	1.90	1.85
20	0.025	2.77	2.68	2.57	2.46	2.40	2.35	2.29	2.25	2.22	2.16	2.09
	0.010	3.37	3.23	3.09	2.94	2.84	2.78	2.69	2.64	2.61	2.52	2.43
	0.001	5.08	4.82	4.56	4.29	4.12	4.00	3.86	3.77	3.70	3.54	3.40
	0.100	1.92	1.87	1.83	1.78	1.74	1.72	1.69	1.67	1.66	1.62	1.59
	0.050	2.32	2.25	2.18	2.10	2.05	2.01	1.96	1.94	1.92	1.87	1.82
21	0.025	2.73	2.64	2.53	2.42	2.36	2.31	2.25	2.21	2.18	2.11	2.05
	0.010	3.31	3.17	3.03	2.88	2.79	2.72	2.64	2.58	2.55	2.46	2.37
	0.001	4.95	4.70	4.44	4.17	4.00	3.88	3.74	3.64	3.58	3.42	3.28
	0.100	1.90	1.86	1.81	1.76	1.73	1.70	1.67	1.65	1.64	1.60	1.57
	0.050	2.30	2.23	2.15	2.07	2.02	1.98	1.94	1.91	1.89	1.84	1.79
22	0.025	2.70	2.60	2.50	2.39	2.32	2.27	2.21	2.17	2.14	2.08	2.01
	0.010	3.26	3.12	2.98	2.83	2.73	2.67	2.58	2.53	2.50	2.40	2.32
	0.001	4.83	4.58	4.33	4.06	3.89	3.78	3.63	3.54	3.48	3.32	3.17
	0.100	1.89	1.84	1.80	1.74	1.71	1.69	1.66	1.64	1.62	1.59	1.55
	0.050	2.27	2.20	2.13	2.05	2.00	1.96	1.91	1.88	1.86	1.81	1.76
23	0.025	2.67	2.57	2.47	2.36	2.29	2.24	2.18	2.14	2.11	2.04	1.98
	0.010	3.21	3.07	2.93	2.78	2.69	2.62	2.54	2.48	2.45	2.35	2.27
	0.001	4.73	4.48	4.23	3.96	3.79	3.68	3.53	3.44	3.38	3.22	3.08
	0.100	1.88	1.83	1.78	1.73	1.70	1.67	1.64	1.62	1.61	1.57	1.54
	0.050	2.25	2.18	2.11	2.03	1.97	1.94	1.89	1.86	1.84	1.79	1.74
24	0.025	2.64	2.54	2.44	2.33	2.26	2.21	2.15	2.11	2.08	2.01	1.94
	0.010	3.17	3.03	2.89	2.74	2.64	2.58	2.49	2.44	2.40	2.31	2.22
	0.001	4.64	4.39	4.14	3.87	3.71	3.59	3.45	3.36	3.29	3.14	2.99